AP Calculus BC

KEY

INTEGRATION BY PARTS
Review & Renew

*Hints

1. $\int x \ln x\,dx$ $\frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$	11. $\int \cos x \ln(\sin x)\,dx$ $\sin x \ln(\sin x) - \sin x + C$
2. $\int x\sec^2 x\,dx$ $x\tan x + \ln\lvert\cos x\rvert + C$	12. $\int \frac{\ln x}{x^2}\,dx$ $-\frac{\ln x}{x} - \frac{1}{x} + C$
3. $\int x\cos 5x\,dx$ $\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$	13. $\int (x\sqrt{x+3})\,dx$ $\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$
4. $\int (\ln x)^2\,dx$ $x(\ln x)^2 - 2(x\ln x - x) + C$ $x(\ln x)^2 - 2x\ln x + 2x + C$	Identity or IBP 14. $\int \sin^2 x\,dx$ $\frac{1}{2}x - \frac{1}{4}\sin 2x + C$ $\frac{1}{2}x - \frac{1}{4}(2\sin x\cos x) + C$
5. $\int xe^{-x}\,dx$ $-xe^{-x} - e^{-x} + C$	15. $\int \frac{x}{e^x}\,dx$ $-xe^{-x} - e^{-x} + C$
6. $\int \sin^{-1} x\,dx$ $x\sin^{-1}x + (1-x^2)^{1/2} + C$	16. $\int e^x \sin x\,dx$ $\frac{1}{2}(e^x\sin x - e^x\cos x) + C$
7. $\int x\sin(3x)\,dx$ $-\frac{1}{3}x\cos 3x + \frac{1}{9}\sin(3x) + C$	17. $\int x^2 \sin x\,dx$ $-x^2\cos x + 2x\sin x + 2\cos x + C$
u-sub 8. $\int \ln(2x+1)\,dx$ $x\ln(2x+1) - x + \frac{\ln(2x+1)}{2} + C$ $x\ln(2x+1) - \frac{1}{2}(2x+1) + \frac{1}{2}\ln(2x+1) + C$	18. $\int \ln x\,dx$ $x\ln x - x + C$
9. $\int \cos(\ln x)\,dx$ $\frac{1}{2}(x\cos(\ln x) + x\sin(\ln x)) + C$	19. $\int xe^{5x}\,dx$ $\frac{1}{5}xe^{5x} - \frac{1}{25}e^{5x} + C$
10. $\int x^3 e^x\,dx$ $e^x(x^3 - 3x^2 + 6x - 6) + C$	20. $\int x^2 e^{5x}\,dx$

8) $\frac{1}{2}\left[(2x+1)\ln(2x+1) - (2x+1)\right] + C$ $\frac{1}{5}x^2e^{5x} - \frac{2}{25}xe^{5x} + \frac{2}{125}e^{5x} + C$

From: Calculus, Stewart, 5th ed.
and
Calculus, Hughes-Hallett, Gleason, McCallum, et al., 3rd ed.

Integration by Parts WS: ILATE

$-c^{Sc}_{-s}$

1) $\int x \ln x \, dx$ $\quad u = \ln x \quad dv = x\,dx$ $\quad du = \frac{1}{x}dx \quad v = \frac{x^2}{2}$

$$\ln x \cdot \frac{x^2}{2} - \int \frac{x^2}{2} \cdot \frac{1}{x}\,dx = \frac{x^2}{2}\cdot \ln x - \frac{1}{2}\int x\,dx$$

$$= \frac{x^2}{2}\ln x - \frac{1}{2}\cdot\frac{x^2}{2} + C = \frac{x^2}{2}\ln x - \frac{1}{4}x^2 + C$$

2) $\int x \sec^2 x \, dx$ $\quad u = x \quad dv = \sec^2 x\,dx$ $\quad du = dx \quad v = \tan x$

$$x\tan x - \int \tan x\,dx = x\tan x + \ln|\cos x| + C$$

$$\int \frac{\sin x}{\cos x}dx \quad \begin{matrix} u = \cos x \\ du = -\sin x\,dx \\ -du = \sin x\,dx \end{matrix}$$

$$-\int \frac{1}{u}du = -\ln|\cos x| + C$$

3) $\int x \cos 5x \, dx$ $\quad u = x \quad dv = \cos 5x\,dx$ $\quad du = dx \quad v = \frac{1}{5}\sin 5x$

$$\frac{1}{5}x\cdot\sin 5x - \frac{1}{5}\int \sin 5x\,dx \quad \begin{matrix} u = 5x \\ du = 5dx \\ \frac{1}{5}du = dx \end{matrix}$$

$$-\frac{1}{5}\cdot\frac{1}{5}\int \sin u\,du$$

$$\frac{1}{5}x\sin 5x + \frac{1}{25}\cos 5x + C$$

4) $\int (\ln x)^2 dx = \int (\ln x \cdot \ln x) dx$

$u = \ln x \quad dv = \ln x\, dx$

$du = \frac{1}{x} dx \quad v = x\ln x - x$

$\ln x(x\ln x - x) - \int (x\ln x - x)(\frac{1}{x}) dx$

$x(\ln x)^2 - x\ln x - \int (\ln x - 1) dx \qquad \int \ln x \cdot 1\, dx$

$x(\ln x)^2 - x\ln x - (x\ln x - x - x) + C$

$-2x$

$x(\ln x)^2 - x\ln x - x\ln x + 2x + C$

$x(\ln x)^2 - 2x\ln x + 2x + C$

5) $\int x e^{-x} dx$

u	dv
$+ x$	e^{-x}
$- 1$	$-e^{-x}$
0	e^{-x}

$-xe^{-x} - e^{-x} + C$

6) $\int \sin^{-1} x \cdot 1\, dx$

$u = \sin^{-1} x \quad dv = 1\, dx$

$du = \frac{1}{\sqrt{1-x^2}} \quad v = x$

$x\sin^{-1} x - \int x \cdot \frac{1}{\sqrt{1-x^2}} dx$

$u = 1 - x^2$

$du = -2x\, dx$

$-\frac{1}{2} du = x\, dx$

$x\sin^{-1} x + \frac{1}{2}\int u^{-1/2} du \rightarrow 2u^{1/2}$

$x\sin^{-1} x + \frac{1}{2} \cdot 2(1-x^2)^{1/2} + C$

$x\sin^{-1} x + (1-x^2)^{1/2} + C$

7) $\int x\sin(3x)\,dx$

u	dv
$+x$	$\sin(3x)$
-1	$-\frac{1}{3}\cos(3x)$
0	$-\frac{1}{9}\sin(3x)$

$$-\frac{1}{3}x\cos(3x)+\frac{1}{9}\sin(3x)+C$$

8) $\int \ln(2x+1)\,dx$

$u = 2x+1$

$du = 2x\,dx$

$\frac{1}{2}du = x\,dx$

$\int \ln u\,du$

$= u\ln u - u + C$

$= (2x+1)(\ln(2x+1)) - (2x+1) + C$

9) $\int \cos(\ln x)\,dx$

$u = \cos(\ln x)$ $\quad v = x$

$du = \frac{-\sin(\ln x)}{x}$ $\quad dv = 1\,dx$

$x\cos(\ln x) + \int \sin(\ln x)\,dx$

$u = \sin(\ln x)$ $\quad v = x$

$du = \frac{\cos(\ln x)}{x}$ $\quad dv = 1\,dx$

$$x\cos(\ln x) + \left(\sin(\ln x)\cdot x - \int \cos(\ln x)\,dx\right) = \int \cos(\ln x)\,dx$$

Wrap around!

$$\frac{x\cos(\ln x)+x\sin(\ln x)}{2} + C = \int \cos(\ln x)\,dx$$

10) $\int x^3 e^x \, dx$

	u	dv
+	x^3	e^x
−	$3x^2$	e^x
+	$6x$	e^x
−	6	e^x
	0	e^x

$$e^x(x^3 - 3x^2 + 6x - 6) + C$$

11) $\int \cos x \cdot \ln(\sin x) \, dx$

$u = \ln(\sin x) \quad dv = \cos x \, dx$

$du = \frac{1}{\sin x} \cdot \cos x \, dx \quad v = \sin x$

$$\ln(\sin x) \cdot \sin x - \int \sin x \cdot \frac{1}{\sin x} \cdot \cos x \, dx$$

$$\sin x \ln(\sin x) - \int \cos x \, dx$$

$$\sin x \ln(\sin x) - \sin x + C$$

12) $\int \frac{\ln x}{x^2} \, dx$

$u = \ln x \quad dv = x^{-2} dx$

$du = \frac{1}{x} dx \quad v = -x^{-1}$

$$\int (x^{-2} \cdot \ln x) \, dx$$

$$\ln x \cdot \frac{1}{x^2} \, dx$$

$$\ln x \cdot x^{-2} dx$$

$$-\frac{\ln x}{x} + \int \frac{1}{x} \cdot \frac{1}{x^{-2}} \, dx$$

$$-\frac{\ln x}{x} - \frac{1}{x} + C$$

~~13)~~ $\int x\sqrt{x+3}\,dx$ $\quad u=x+3 \quad x=u-3$ $\quad du=dx$

$\int (u-3)\cdot u^{1/2}\,du$

$\int u^{3/2} - 3u^{1/2}\,du$

$\int u^{3/2}du - 3\int u^{1/2}du$

$\frac{2}{5}u^{5/2} - 3\cdot\frac{2}{3}u^{3/2} + C$

$\frac{2}{5}(x+3)^{5/2} - 2(x+3)^{3/2} + C$

14) $\int \sin^2 x\,dx = \int \frac{1-\cos 2x}{2}\,dx$

$\int \frac{1}{2}\,dx - \frac{1}{2}\int \cos 2x\,dx$ $\quad u=2x \quad du=2dx \quad \frac{1}{2}du=dx$

$\frac{1}{2}x - \frac{1}{2}\cdot\frac{1}{2}\int \cos u\,du$

$\frac{1}{2}x - \frac{1}{4}\sin 2x + C$

15) $\int \frac{x}{e^x}\,dx = \int x\cdot e^{-x}\,dx$

u	dv
$+x$	e^{-x}
-1	$-e^{-x}$
0	e^{-x}

$-xe^{-x} - e^{-x} + C$

$\int e^{-x}dx$

$-e^{-x}$

16) $\int e^x \sin x \, dx$

$u = \sin x \quad dv = e^x dx$
$du = \cos x \, dx \quad v = e^x$

$e^x \sin x - \int e^x \cos x \, dx$

$u = \cos x \quad dv = e^x dx$
$du = -\sin x \, dx \quad v = e^x$

$e^x \sin x - (e^x \cos x + \int e^x \sin x \, dx)$

$$\int e^x \sin x \, dx = e^x \sin x - e^x \cos x - \int e^x \sin x \, dx$$

$$2\int e^x \sin x \, dx = e^x \sin x - e^x \cos x$$

$$\int e^x \sin x \, dx = \frac{1}{2}(e^x \sin x - e^x \cos x) + C$$

17) $\int x^2 \sin x \, dx$

	u	dv
+	x^2	$\sin x$
−	$2x$	$-\cos x$
+	2	$-\sin x$
	0	$\cos x$

$-x^2 \cos x + 2x \sin x + 2\cos x + C$

18) $\int \ln x \, dx$

$u = \ln x \quad dv = 1 \, dx$
$du = \frac{1}{x} dx \quad v = x$

$x \ln x - \int x \cdot \frac{1}{x} \, dx = x \ln x - x + C$

19) $\int x e^{5x} dx$

u	dv
$+ x$	e^{5x}
$- 1$	$\frac{1}{5} e^{5x}$
0	$\frac{1}{25} e^{5x}$

$$\frac{1}{5} x e^{5x} - \frac{1}{25} e^{5x} + C$$

20) $\int x^2 e^{5x} dx$

u	dv
$+ x^2$	e^{5x}
$- 2x$	$\frac{1}{5} e^{5x}$
$+ 2$	$\frac{1}{25} e^{5x}$
0	$\frac{1}{125} e^{5x}$

$$\frac{1}{5} x^2 e^{5x} - \frac{2}{25} x e^{5x} + \frac{2}{125} e^{5x} + C$$