

AP CALCULUS
VELOCITY & SPEED PRACTICE

Name Answer Key

Directions: No calculator! Use your notes!

- 1) A particle moves along a coordinate line in such a way that its position is given by $S(t) = 2\sqrt{t}(2 - 2t + t^2)$ for $t \geq 0$. For what times is the particle moving to the left?
- A) $0 < t < 2$
 B) The particle never moves to the left.
 C) $1 < t < 2$
 D) $2 < t < 5$
 E) $t > 5$

$$V(t) = (2\sqrt{t})(-2 + 2t) + (2 - 2t + t^2)(t^{-\frac{1}{2}})$$

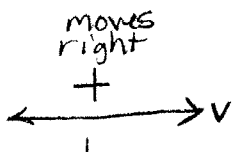
$$V(t) = t^{-\frac{1}{2}}(2t(-2 + 2t) + 2 - 2t + t^2)$$

$$V(t) = \frac{1}{\sqrt{t}}(-4t + 4t^2 + 2 - 2t + t^2)$$

$$V(t) = \frac{1}{\sqrt{t}}(5t^2 - 6t + 2)$$

$$0 = \frac{1}{\sqrt{t}}(5t^2 - 6t + 2)$$

No Sol. $b^2 - 4ac$
 $36 - 4(5)(2)$
 $36 - 40$
 -4
 Not factorable!
 No Sol.

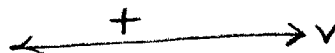


- 3) A particle moves along the x-axis in such a way that its position is given by $x(t) = 2t^3 + 24t - 4\cos(\frac{\pi}{2}t)$ for $t > 0$. When is the particle moving to the left?
- A) $t > 0$
 B) $0 < t < 2$
 C) The particle never moves to the left.
 D) $0 < t < \frac{\pi}{2}$
 E) $t > \frac{\pi}{2}$

$$v(t) = 6t^2 + 24 + 2\pi \sin(\frac{\pi}{2}t)$$

Always positive
 (And always larger than -2π)

The most negative this gets is -1 .



- 2) A particle moves along the x-axis so that its position at time t is given by $x(t) = 2t^2 - 12t + 9$. For what value of t is the particle at rest?
- A) 1
 B) 4
 C) 3
 D) 0
 E) 9

$v(t) = 4t - 12$
 $0 = 4t - 12$
 $t = 3$

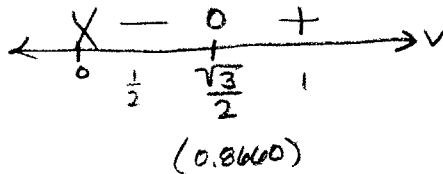
$v(t) = 0$

- 4) A particle moves along a horizontal axis so that its position is given by $x(t) = 4t^5 - 5t^3$ for any time t . How many times does the particle change direction?
- A) 1
 B) 2
 C) 3
 D) 0
 E) 5

$$v(t) = 20t^4 - 15t^2$$

$$0 = 5t^2(4t^2 - 3)$$

$t = 0$ $t = \sqrt{\frac{3}{4}}$
 $t = \frac{\sqrt{3}}{2}$



5) A particle moves along the x-axis so that its position at any time t is given by $x(t) = t^3 - 6t^2 - 3$. Which of the following *best* describes the motion of the particle for $0 < t < 1$?

- A) Moving to the right and speeding up.
- B) Moving to the left and speeding up.**
- C) Moving to the right and slowing down.
- D) Moving to the left at a constant speed.
- E) Moving to the left and slowing down.

$$v(t) = 4t^3 - 12t$$

$$0 = 4t(t^2 - 3)$$

$$t = 0 \quad t = \pm\sqrt{3}$$

← not in $0 < t < 1$!

$$a(t) = 12t^2 - 12$$

$$0 = 12(t^2 - 1)$$

$$t = \pm 1$$

6) A particle moves along the x-axis so that its position is given by $x(t) = 2 \cos(2t)$ for $0 \leq t \leq \pi$. Which statement *best* describes the motion of the particle for $\frac{\pi}{4} < t < \frac{\pi}{2}$?

- A) moving to the right at increasing speed
- B) moving to the left at decreasing speed**
- C) moving to the left at constant speed
- D) moving to the right at decreasing speed
- E) moving to the left at increasing speed

$$v(t) = -4 \sin(2t)$$

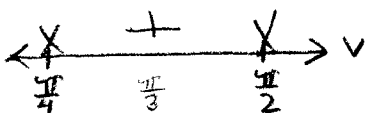
$$0 = -4 \sin(2t)$$

$$0 = \sin(2t)$$

$$2t = \frac{\pi}{2} \quad 2t = \frac{3\pi}{2}$$

$$t = \frac{\pi}{4} \quad t = \frac{3\pi}{4}$$

← not in domain $\frac{\pi}{4} < t < \frac{\pi}{2}$



$$a(t) = -8 \cos(2t)$$

$$0 = -8 \cos(2t)$$

$$0 = \cos(2t)$$

$$2t = \frac{\pi}{2} \quad 2t = \frac{3\pi}{2}$$

$$t = \frac{\pi}{4} \quad t = \frac{3\pi}{4}$$

← not in domain

7) A particle moves along the x-axis so that its position at any time t is given by $x(t) = t^3 - 6t^2 + 9t + 12$. During what times is the speed of the particle increasing?

- A) $t < 1$ or $t > 3$
- B) $1 < t < 2$ or $t > 3$**
- C) $t < 1$ or $2 < t < 3$
- D) $t < 2$ or $t > 3$
- E) $1 < t < 3$

$$v(t) = 3t^2 - 12t + 9$$

$$0 = 3(t^2 - 4t + 3)$$

$$0 = 3(t-3)(t-1)$$

$$t = 3 \quad t = 1$$

$$a(t) = 6t - 12$$

$$0 = 6t - 12$$

$$t = 2$$

8) A particle moves along the x-axis so that its position is given by $x(t) = 4t^3 - 3t^2$ for any time $t \geq 0$. During what time interval is the particle's position to the left of zero?

- A) $0 < t < \frac{3}{4}$
- B) $0 < t < \frac{4}{3}$
- C) $\frac{1}{2} < t < \frac{3}{4}$
- D) $1 < t < \frac{3}{4}$
- E) $0 < t < \frac{1}{2}$**

At $t=0$, $x(0) = 0$
Particle starts at the origin $(0,0)$.

$$v(t) = 12t^2 - 6t$$

$$0 = 6t(2t - 1)$$

$$t = 0 \quad t = \frac{1}{2}$$

moves left moves right

9) A particle moves on a straight line in such a way that its distance at any time t from a fixed point on the line is given by $S(t) = 4t - 3t^2$. What is the total distance traveled by the particle between $t = 0$ and $t = 2$?

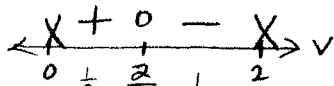
$$s(t) = t(4 - 3t)$$

- A) $\frac{16}{3}$
- B) $\frac{20}{3}$**
- C) $\frac{22}{3}$
- D) 6
- E) 4

$$v(t) = 4 - 6t$$

$$0 = 4 - 6t$$

$$t = \frac{2}{3}$$



$$\text{Dis} = \left| -4 - \frac{4}{3} \right| + \left| -\frac{12}{3} - \frac{4}{3} \right| + \left| -\frac{16}{3} \right|$$

$$\text{Dis} = \frac{4}{3} + \frac{16}{3} + \frac{16}{3} = \frac{36}{3} = 12$$

$$s\left(\frac{2}{3}\right) = \frac{2}{3}(4 - 2) = \frac{2}{3}(2) = \frac{4}{3}$$

$$s(0) = 0$$

$$s(2) = 2(4 - 6) = 2(-2) = -4$$

$$\text{Total Dis} = \frac{4}{3} + \frac{16}{3} = \frac{20}{3}$$

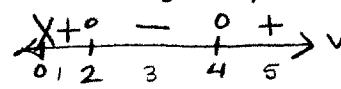
10. A particle moves along the x-axis in such a way that its position at time t for $t \geq 0$ is given by

$$x(t) = \frac{1}{3}t^3 - 3t^2 + 8t$$

$$v(t) = t^2 - 6t + 8$$

$$0 = (t - 4)(t - 2)$$

$$t = 4 \quad t = 2$$



$$\text{Dis} = \frac{20}{3}$$

$$\text{Dis} = \left| 6 - \frac{20}{3} \right| = \left| \frac{18}{3} - \frac{20}{3} \right| = \left| -\frac{2}{3} \right| = \frac{2}{3}$$

- A. When is the particle at rest?
- B. When is the particle moving to the right?
- C. Find all values of t for which the particle moving to the left.
- D. When does the particle change direction?
- E. What is the velocity of the particle at $t = 3$?
- F. What is the total distance that the particle traveled from time $t = 0$ to $t = 3$?

- A. $t = 2, 4$
- B. $t \in (0, 2) \cup (4, \infty)$
- C. $t \in (2, 4)$
- D. $t = 2, 4$
- E. $v(3) = (-1)(1) = -1$
- F. T.D. = $\frac{20}{3} + \frac{2}{3} = \frac{22}{3}$

$$x(0) = 0$$

$$x(2) = \frac{8}{3} - 12 + 16 = \frac{8}{3} + 4 = \frac{8}{3} + \frac{12}{3} = \frac{20}{3}$$

$$x(3) = 9 - 27 + 24 = 9 - 3 = 6$$

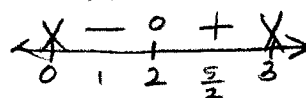
11. A particle moves along a line such that its position function is given by $s(t) = t^3 - 12t + 3$ for the time interval $[0, 3]$. Find the following:

$$v(t) = 3t^2 - 12$$

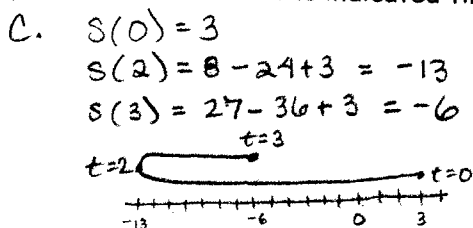
$$0 = 3(t^2 - 4)$$

$$t = \pm 2$$

- A. When the particle is at rest, moving to the left, and to the right.
- B. The intervals where the particle is speeding up and slowing down.
- C. Draw a picture that illustrates the motion of the particle.
- D. The total distance that the particle travels over the indicated time interval.



- A. At rest: $t = 2$
Left: $t \in (0, 2)$
Right: $t \in (2, 3)$
- B. speeding up: $t \in (2, 3)$
slowing down: $t \in (0, 2)$



$$a(t) = 6t$$

$$0 = 6t$$

$$t = 0$$

$$\text{D. T.D.} = 16 + 7 = 23 \text{ units}$$

