(1) $\lim_{n\to\infty} \frac{(3x-2)^{n+1}}{n+1} \cdot \frac{n}{(3x-2)^n} = (3x-2)\left(\frac{n}{n+1}\right) = 3x-2$

$-1 < 3x-2 < 1$

$\frac{1}{3} < x < 1$

$\boxed{x = \frac{1}{3}}$

$\frac{(-1)^n}{n}$ = Alternati Converges

$\boxed{x = 1}$

$\frac{1}{n}$ = p-series diverges

$$\boxed{\frac{1}{3} \le x < 1}$$

(2) $\lim_{n\to\infty} \frac{(n+1)(x+3)^{n+1}}{5^{n+1}} \cdot \frac{5^n}{n(x+3)^n} = (x+3)\left(\frac{n+1}{5n}\right) = \frac{1}{5}(x+3)$

$-1 < \frac{1}{5}(x+3) < 1$

$-5 < x+3 < 5$

$-8 < x < 2$

$\boxed{x = 2}$

$n(1)^n$

nth term Test diverges

$\boxed{x = -8}$

$n(-1)^n$

nth Term Test diverges

$$\boxed{-8 < x < 2}$$

(3) $\lim_{n\to\infty} \frac{(4x-5)^{2(n+1)+1}}{(n+1)^{\frac{3}{2}}} \cdot \frac{n^{\frac{3}{2}}}{(4x-5)^{2n+1}} = \frac{(4x-5)^2}{1} \cdot \frac{n^{\frac{3}{2}}}{n^{\frac{3}{2}} \ldots}$

$-1 < (4x-5)^2 < 1$

$-1 < 4x-5 < 1$

$4 < 4x < 6$

$1 < x < \frac{3}{2}$

$\boxed{x = 1}$

$\frac{(-1)^{2n+1}}{n^{\frac{3}{2}}} = \frac{-1}{n^{\frac{3}{2}}}$

converges, p-series

$\boxed{x = \frac{3}{2}}$

$\frac{1}{n^{\frac{3}{2}}}$

converges, p-series

$$\boxed{1 \le x \le \frac{3}{2}}$$

(4) $\boxed{\text{p-series}}$ $p = \frac{1}{2} < 1$ $\boxed{\text{diverges}}$

(5) $b_n = \dfrac{e^n}{e^{2n}}$ $\rightarrow$ converges, geometric $r = \frac{1}{e} < 1$

$$\lim_{n\to\infty} \frac{e^n}{1+e^{2n}} \cdot \frac{e^{2n}}{e^n} = \frac{e^{3n}}{e^{3n}+e^n} = 1 \neq 0$$ same behavior

Original series $\boxed{\text{converges}}$ $\boxed{\text{limit Comparison}}$

(6) $$\sum_{n=1}^{\infty} \frac{3^{n-1}+1}{3^n} = \frac{3^{n-1}}{3^n} + \frac{1}{3^n} = \frac{1}{3} + \frac{1}{3^n}$$

$$\lim_{n\to\infty}\left(\frac{1}{3}+\frac{1}{3^n}\right) = \frac{1}{3} \neq 0$$ $\boxed{\text{diverges}}$ $\boxed{\text{nth Term Test}}$

(7) $$\sum_{n=1}^{\infty} (-1)^{n+1}\left(\frac{1+n}{n^2}\right) = (-1)^{n+1}\left(\frac{1}{n^2}\right) + (-1)^{n+1}\left(\frac{1}{n}\right)$$

$\downarrow$ converges, alternating

$\downarrow$ converges, alternating

Converge + Converge = $\boxed{\text{Converge}}$

(8) $$\lim_{n\to\infty} \frac{(n+1)^2\, 2^{n+1}}{3^{n+1}} \cdot \frac{3^n}{n^2\, 2^n} = \frac{2\ n^2 \cdots}{3\ n^2 \cdots} = \frac{2}{3}$$

$\boxed{\text{Ratio Test}}$ $\lim = \frac{2}{3} < 1$ $\boxed{\text{Converges}}$

(9) After ratio test

$-7 < x < -1$

$\boxed{x = -7}$

$\frac{(-3)^n}{n\,3^n} = \frac{(-1)^n}{n}$ conv, Alternate

$\boxed{x = -1}$

$\frac{3^n}{n\,3^n} = \frac{1}{n}$

diverges pseries

$\boxed{-7 \le x < -1}$

(10) After ratio test

$0 < x < \frac{2}{3}$

$\boxed{x = 0}$

$\frac{(-1)^{n-1}(-1)^n}{n^2} = \frac{1}{n^2}$

converges, p-series

$\boxed{x = \frac{2}{3}}$

$\frac{(-1)^{n-1}(1)^n}{n^2} = \frac{(-1)^{n-1}}{n^2}$

converse, Alternating

$\boxed{0 \le x \le \frac{2}{3}}$

(11) After ratio Test

$-1 < x < 1$

$\boxed{x = -1}$

$\frac{(-1)^n}{\sqrt{n}}$

converges, alternating

$\boxed{x = 1}$

$\frac{1}{\sqrt{n}}$

diverges, p-series

$\boxed{-1 \le x < 1}$

(12) After ratio Test

$\frac{-1}{10} < x < \frac{1}{10}$

$\boxed{x = \frac{-1}{10}}$

$\frac{(-1)^n}{\ln(n)}$

converges, Alternating

$\boxed{x = \frac{1}{10}}$

$\frac{1}{\ln n}$ $b_n = \frac{1}{n}$

diverges p-series

$\frac{1}{\ln n} > \frac{1}{n}$

diverges, direct comparison

$\boxed{\frac{-1}{10} \le x < \frac{1}{10}}$

(13) After ratio Test

$\frac{-1}{e} < x < \frac{1}{e}$

$\boxed{x = \frac{1}{e}}$

$\frac{1}{n^e}$

converges, p-series

$\boxed{x = \frac{-1}{e}}$

$\frac{(-1)^n}{n^e}$

converges Alternating

$\boxed{\frac{-1}{e} \le x \le \frac{1}{e}}$

(14) After ratio Test

$-3 < x < 3$

$\boxed{x = -3}$

$$\frac{(-1)^n}{n\sqrt{n}} = \frac{(-1)^n}{n^{\frac{3}{2}}}$$

Converges, Alternating

$\boxed{x = 3}$

$$\frac{1}{n\sqrt{n}} = \frac{1}{n^{\frac{3}{2}}}$$

Converges – p-series

$\boxed{-3 \leq x \leq 3}$