UNIT 6: Testing Series for Convergence

Name ________________________________ PER_____ DATE______________

Lesson 1

DO NOW

Find the sum of the infinite series given below.

$$\sum_{n=1}^{\infty} \frac{3}{2^n}$$

NOTICE / WONDER

***n*th Term Test for Divergence (ONLY)**
If $\lim_{n\to\infty} a_n \neq 0$, then the series $\sum_{n=1}^{\infty} a_n$ diverges.
(think about it, it should make perfect sense!)

Note: This does **NOT** say that if $\lim_{n\to\infty} a_n = 0$, then the series **DOES** converge. This test can only be used to prove that a series diverges (hence the name.) If $\lim_{n\to\infty} a_n = 0$, then this test doesn't tell us anything, is inconclusive, doesn't work, fails, etc. . . . We MUST use another test. This test can be a GREAT time-saver. Always perform it **FIRST**, not second, but FIRST!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!!

Independent Practice

Determine whether the following series converge or diverge. If they converge, find their sum.

(a) $\sum_{n=1}^{\infty} \frac{2n+3}{3n-5}$ (b) $\sum_{n=1}^{\infty} \frac{n!}{2n!+1}$ (c) $\sum_{n=1}^{\infty} \frac{3^n - 2}{3^n}$ (d) $\sum_{n=2}^{\infty} \frac{1}{(1.1)^n}$

Number Talk

Find the sum of the infinite series given below.

$$\left(1-\frac{1}{2}\right)+\left(\frac{1}{2}-\frac{1}{3}\right)+\left(\frac{1}{3}-\frac{1}{4}\right)+\cdots$$

Series Type: ______________________________

Definition: ____

Independent Practice

Determine whether the following series converges or diverges. If they converge, find their sum.

(a) $\sum_{n=1}^{\infty} \left(\frac{1}{2n+1} - \frac{1}{2n+3}\right)$ (b) $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ (c) $\sum_{n=1}^{\infty} \frac{1}{n^2+4n+3}$

Guided Notes

Integral Test and *p*-Series

Integral Test

If f is (**D**ogs **C**uss in **P**rison!) for $x \geq 1$ AND $a_n = f(x)$,

then $\sum_{n=1}^{\infty} a_n$ and either BOTH converge or diverge.

Note 1: This does NOT mean that the series converges to !!!!!!

Note 2: The function need only be for all $x > k$ for some $k \geq 1$.

If the series converges to S, then the remainder, $R_n = |S - S_n|$ is bounded by

$0 \leq R_n \leq$ (Not on AP exam This means $S \in [S_n, S_n + R_n]$.

Determine whether the following series converge or diverge.

(a) $\sum_{n=1}^{\infty} \frac{n}{n^2+1}$

(b) $\sum_{n=1}^{\infty} \frac{1}{n^2+1}$

p-series

A series of the form $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots+\frac{1}{n^p}+\cdots$ is called a *p*-series, where *p* is a positive constant.

For $p=1$, the series $\sum_{n=1}^{\infty}\frac{1}{n}=1+\frac{1}{2}+\frac{1}{3}+\cdots+\frac{1}{n}+\cdots$ is called the **harmonic series**.

TPS-C

Based on your experience with *p*-series and their reliance on the number one, fill in chart below.

p-Series Test

The *p*-series $\sum_{n=1}^{\infty}\frac{1}{n^p}=\frac{1}{1^p}+\frac{1}{2^p}+\frac{1}{3^p}+\cdots+\frac{1}{n^p}+\cdots$, based on above

a) If $p=1$, ______________________________

b) If $p<1$, ______________________________

c) If $p>1$, ______________________________

Note: If the *p*-series converges, we cannot find its sum ☹. This is more often the case than not.

Independent Practice

Determine of the following converges or diverges:

(a) $\sum_{n=1}^{\infty}\frac{1}{n\sqrt{n}}$ (b) $\sum_{n=1}^{\infty}\frac{n}{\sqrt{n}}$ (c) $\sum_{n=1}^{\infty}\frac{\sqrt{n}}{n}$ (d) $\sum_{n=1}^{\infty}\frac{999999999}{n^{1.0000000001}}$