Unit 2 Domain

I. Definition of a Function (2A)

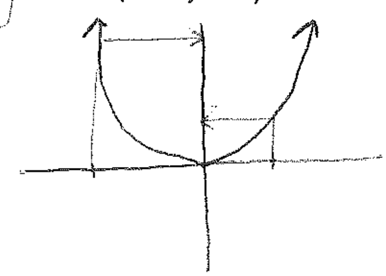
A function is a rule such that every x-value has <u>exactly one</u> y-value

one and only one

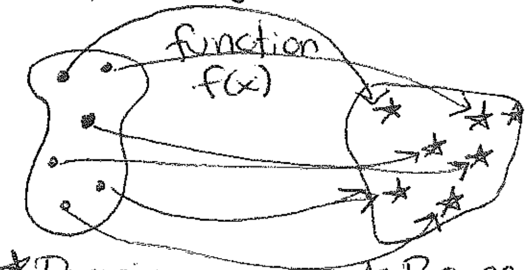
A. Visualizations

1. Algebraically

$y=f(x)$ → $f(x) = x^2$



2. Map Diagram



★ Domain
★ Input
★ Pre-Image

★ Range
★ Output
★ Image

Domain: all possible x-values / imput / pre-image

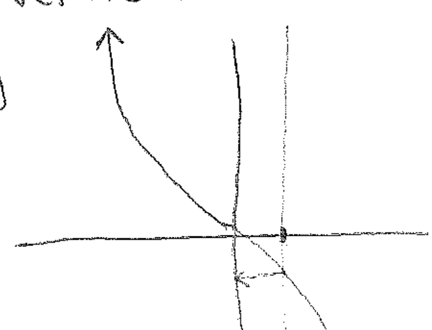
Range: all possible y-values / output / image

equation that relates the x & y-values

B. How to Determine if a Relation is a Function?

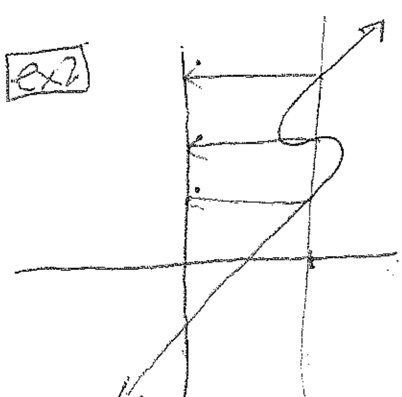
1. Vertical Line Test

ex1



pass, b/c every x-value has exactly one y-value
∴ function

ex2



fail, an x-value has more than one (3) different y-values
∴ not a function

2. Ordered Pairs

$\boxed{ex1}$ $(-1,3)(4,5)(3,5)(0,2)$

pass, every x-value has exactly one y-value

∴ function

$\boxed{ex2}$ $(10,4)(3,8)(10,7)(1,2)$

fail, an x-value (10) has 2 different y-values

∴ not a function

3. Mathematically, given an Equation

→ given an equation, solve for y {or f(x)} and determine if every x-value has exatly one y-valve after you plug it in.

$\boxed{ex1}$ $y - x^2 + 4 = 0$

$y = x^2 - 4$

pass, if you plug in an x-value, you will get one y-value

∴ function

$\boxed{ex2}$ $x - y^2 + 4 = 0$

$\sqrt{y^2} = \sqrt{x+4}$

$y = \pm\sqrt{x+4}$

$\sqrt{r^2} = \sqrt{4}$

$r = \pm 2$

fail, if you plug in an x-valu you will get 2 y-values!

∴ not a function

$\boxed{ex3}$ $|2y| + 3 = x$

$|2y| = x - 3$

$2y = x - 3$ $\qquad 2y = -(x-3)$

$y = \frac{x-3}{2}$ $\qquad 2y = -x + 3$

$y = \frac{-x+3}{2}$

$|r| = 8$

$r = 8$ $\qquad r = -8$

∴ two part function

if you plug in an x-valve, you get 2 different y-values

∴ not a function

$\boxed{ex4}$ $|x+2| + y = 7$

$y = 7 - |x+2|$

if you plug in an x you get one y ∴ pass

∴ function

II Evaluating Functions (2E)

A. To "evaluate" a function at a specific value, means to "plug" in the number.

$\boxed{ex1}$ Evaluate $f(x)=2x^2-9$ for $f(3)$ and $f(x+3)$

$f(3)=2(3)^2-9=2\cdot 9-9=\boxed{9}$

$f(x+3)=2(x+3)^2-9=2(x^2+6x+9)-9$

$=2x^2+12x+18-9=\boxed{2x^2+12x+9}$

B. The Difference Quotient ← you will use this in Calculus!!

$$\frac{f(a+h)-f(a)}{h}$$

$\boxed{ex2}$ Evaluate $f(x)=3x^2+1$ for the difference quotient $\frac{f(a+h)-f(a)}{h}$

Trick: ① find $f(a+h)$
② then find $f(a)$
③ then put it together as $\frac{f(a+h)-f(a)}{h}$

① $f(a+h)=3(a+h)^2+1$
$=3(a^2+2ah+h^2)+1$
$3a^2+6ah+3h^2+1$ ✓

② $f(a)=3(a)^2+1=$
$3a^2+1$ ✓

③ Put it together

$$\frac{3a^2+6ah+3h^2+1-(3a^2+1)}{h}$$

$$=\frac{\cancel{3a^2}+6ah+3h^2+\cancel{1}-\cancel{3a^2}-\cancel{1}}{h}=\frac{6ah+3h^2}{h}$$

$$=\frac{\cancel{h}(6a+3h)}{\cancel{h}}=\boxed{6a+3h}$$

$\boxed{\text{ex3}}$ Evaluate $f(x) = \frac{1}{x+1}$ for the difference quotient $\frac{f(a+h) - f(a)}{h}$

$$f(a+h) = \frac{1}{(a+h)+1} = \frac{1}{a+h+1}$$

$$f(a) = \frac{1}{a+1}$$

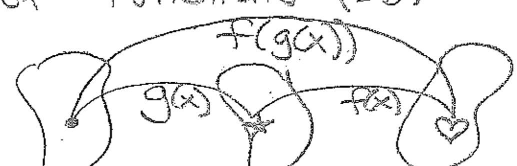
oooh! Unit 1 ü

common denom.

$$\frac{f(a+h) - f(a)}{h} = \frac{\frac{1}{a+h+1} - \frac{1}{a+1}}{h} = \frac{\frac{(a+1) - (a+h+1)}{(a+h+1)(a+1)}}{h} =$$

$$\frac{\cancel{a}+\cancel{1}-\cancel{a}-h-\cancel{1}}{(a+h+1)(a+1)} \cdot \frac{1}{h} = \frac{-\cancel{h}}{\cancel{h}(a+h+1)(a+1)} = \boxed{\frac{-1}{(a+h+1)(a+1)}}$$

III Compositions of Functions (2D)

A. Visual 

and $f(g(x))$ is also written as $f \circ g$.

$\boxed{\text{ex1}}$ Let $f(x) = x^2$ and $g(x) = x-3$, find $f \circ g$ and $g \circ f$.

$$f \circ g = f(g(x)) = f(x-3) = (x-3)^2 = \boxed{x^2 - 6x + 9}$$

$$g \circ f = g(f(x)) = g(x^2) = \boxed{x^2 - 3}$$

$\boxed{\text{ex2}}$ If $f(x) = \sqrt{x}$ and $g(x) = \sqrt{2-x}$ and $h(x) = x^2$, find $f \circ g \circ h$.

$$f \circ g \circ h = f(g(h(x))) = f(g(x^2)) = f(\sqrt{2-x^2}) =$$

$$= \sqrt{\sqrt{2-x^2}} = \boxed{\sqrt[4]{2-x^2}}$$

IV. Solving Linear, Polynomial & Rational Inequalities (2c)

A. Linear - in the form $mx+b$ ← 1st power only

$\boxed{\text{ex1}}$ Solve and write in set and interval notation

$2x+5 \geq 4x-9$ ← x is to the 1st power so just isolate x.

$-2x \geq -14$

$x \leq 7$ ← when you divide by a negative, switch the sign.

$$\boxed{\begin{array}{l}\text{Set: } \{x \mid x \leq 7\} \\ \text{interval: } (-\infty, 7]\end{array}}$$

basically the x's have exponents

B. Polynomial - in the form $a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0$

$\boxed{\text{ex2}}$ Solve and write in interval and set notation.

$x^2+7x+10<0$ ← <0 means negative

① factor $(x+5)(x+2)<0$ ∴ the product is negative so either $+ \cdot - = -$ or $- \cdot + = -$

② Find critical points

$x+5=0$ $x+2=0$

$x=-5$ $x=-2$ ← Are NOT the answer!

③ set up intervals

the sign is $<$ so the intervals use ( )

	$(-\infty,-5)$	$(-5,-2)$	$(-2,\infty)$
test point	-236	-4	0
$(x+5)$	$-$	$+$	$+$
$(x+2)$	$-$	$-$	$+$
$(x+5)(x+2)$	$+$	$-$	$+$
<0 ?	☹	☺	☹

$$\boxed{\begin{array}{l}\text{interval: } (-5,-2) \\ \text{Set: } \{x \mid -5<x<-2\}\end{array}}$$

$\boxed{\text{ex3}}$ Solve and write in set & interval notation

compare to 0 always

$x^2 - 3x \geq 18$

$x^2 - 3x - 18 \geq 0$

$(x-6)(x+3) \geq 0$

$x = 6$ $x = -3$ critical points

≥ 0 product is positive

$- \cdot - = +$

$+ \cdot + = +$

the sign is $\geq$ so the intervals use [ ]

	$(-\infty, -3]$	$[-3, 6]$	$[6, \infty)$
test point	-4	2	8
$(x-6)$	$-$	$-$	$+$
$(x+3)$	$-$	$+$	$+$
$(x-6)(x+3)$	$+$	$-$	$+$
≥ 0	YES!	NO!	YES!

interval: $(-\infty, -3] \cup [6, \infty)$

set: $\{x \mid x \leq -3 \text{ or } x \geq 6\}$

C. Rational - ratio of two functions {fraction}

$\boxed{\text{ex4}}$ Solve and write is set & interval notation

$x \neq 1$

$$\frac{x+3}{x-1} \leq 2$$

$$\frac{x+3}{x-1} - 2 \leq 0$$

$$\frac{x+3}{x-1} - \frac{2(x-1)}{x-1} \leq 0$$

$$\frac{x+3-2x+2}{x-1} \leq 0$$

$$\frac{-x+5}{x-1} \leq 0$$

want quotient to be negative

$-x+5=0$ $x-1=0$

$x=5$ $x=1$ critical points

should be [ ] but $x \neq 1$ so ( ) with 1

	$(-\infty, 1)$	$(1, 5]$	$[5, \infty)$
tp	-4	4	$1{,}235$
$-x+5$	$+$	$+$	$-$
$x-1$	$-$	$+$	$+$
$\frac{-x+5}{x-1}$	$-$	$+$	$-$
≤ 0?	✓	✗	✓

interval: $(-\infty, 1) \cup [5, \infty)$

set: $\{x \mid x < 1 \text{ or } x \geq 5\}$

I Domain of Functions (2B)

Domain is the set of all possible x-values of a function

A. Restricted Domain

A function will have a domain of all real numbers, $\mathbb{R}$ or $(-\infty, \infty)$, unless it has a restricted domain from one of the following:

1. Denominator $\neq 0$

ex1 $f(x) = \frac{x+3}{x+1}$ ← restriction, $\neq 0$

$x+1 \neq 0$
$x \neq -1$

Domain: $\boxed{\begin{array}{c} \{x \mid x \neq -1\} \\ (-\infty, -1) \cup (-1, \infty) \end{array}}$ set / interval

everything except -1

2. Radicand ≥ 0 {even roots only!}

ex2 $f(x) = \sqrt{2x+1}$ restriction, radicand ≥ 0

$2x+1 \geq 0$
$x \geq -\frac{1}{2}$

Domain: $\boxed{\begin{array}{c} \{x \mid x \geq -\frac{1}{2}\} \\ [-\frac{1}{2}, \infty) \end{array}}$ set / interval

3. Given restrictions

ex3 $f(x) = 3x^2+1, \quad 1 < x < 5$ ← Given restriction

Domain would be $\mathbb{R}$ ... but

Domain: $\boxed{\begin{array}{c} \{x \mid 1 < x < 5\} \\ (1, 5) \end{array}}$ set / interval

B. Tricky Problems

$\boxed{\text{ex4}}$ Find the domain of

$f(x) = \dfrac{2}{x^2-16}$, $0 \le x < 10$ ← Given restriction

↑ restriction, denominator $\neq 0$

$x^2 - 16 \neq 0$

$(x-4)(x+4) \neq 0$

$x \neq 4$ $\quad \cancel{x \neq -4}$

↑ *extraneous solution* b/c we know $0 \le x < 10$

Domain: $\boxed{\{x \mid 0 \le x < 10 \text{ and } x \neq 4\} \quad [0,4) \cup (4,10)}$

$\boxed{\text{ex5}}$ Find the domain of $f(x) = \sqrt{x^2+9x+18}$

↑ restriction, radicand ≥ 0

$x^2 + 9x + 18 \ge 0$ polynomial inequality!

$(x+6)(x+3) \ge 0$ $\quad + \cdot + = +$, $\quad - \cdot - = +$

c.p. $x=-6$ $\quad x=-3$

	$(-\infty,-6]$	$[-6,-3]$	$[-3,\infty)$
t.p.	-7	-4	0
$(x+6)(x+3)$	$- \cdot - = +$	$+ \cdot - = -$	$+ \cdot + = +$
≥ 0?	✓	☹	✓

Domain: $\boxed{(-\infty,-6] \cup [-3,\infty) \quad \{x \mid x \le -6 \text{ or } x \ge -3\}}$

$\boxed{\text{ex6}}$ Find the domain of $f(x) = \dfrac{2x^2+3}{\sqrt{x^2+3x}}$ ← restriction, ≥ 0; restriction, $\neq 0$

fix: denominator > 0

$x^2 + 3x > 0$

$x(x+3) > 0$

c.p. $x=0$ $\quad x=-3$

	$(-\infty,-3)$	$(-3,0)$	$(0,\infty)$
t.p.	-4	-2	2
$x(x+3)$	$- \cdot - = +$	$- \cdot + = -$	$+ \cdot + = +$
>0?	✓	☹	✓

Domain: $\boxed{(-\infty,-3) \cup (0,\infty) \quad \{x \mid x < -3 \text{ or } x > 0\}}$

VI Deconstructing Composite Functions (2D)

Goal: To find individual functions that will make up the given composite function.

A. Forwards {composition}

ex1 Find $f \circ g$ if $f(x) = 2x^2$ and $g(x) = x+1$

$f \circ g = f(g(x)) = f(x+1) = \boxed{2(x+1)^2}$

B. Backwards {deconstruction}

ex1 Find $f(x)$ and $g(x)$ such that $f(g(x)) = \sqrt[4]{x+9}$

→ Use the order of operations

① take x and add 9 ← $g(x)$

② 4th root the result ← $f(x)$

→ when finding $f(g(x))$, you find $g(x)$ first then plug that result into $f(x)$ ... so

① $g(x) = x+9$

② $f(x) = \sqrt[4]{x}$

x always represents "the result" from the step before.

→ check

$f(g(x)) = f(x+9) = \sqrt[4]{x+9}$ ✓

usually notation of a composition is an uppercase

ex2 Find $f(x)$ and $g(x)$ such that $F(x) = f(g(x)) = \dfrac{x^2-5}{3}$

combine { ① square x
② subtract 5 from the result } → $g(x) = x^2-5$

③ divide the result by 3 → $f(x) = \dfrac{x}{3}$

Check:

$f(g(x)) = f(x^2-5) = \dfrac{x^2-5}{3}$ ✓

$\boxed{\text{ex 3}}$ Find $f(x)$, $g(x)$ and $h(x)$ such that

$Q(x) = f(g(h(x)))$ if $Q(x) = \dfrac{1}{x^2+1}$

① square x $\longrightarrow$ $h(x) = x^2$

② add 1 to the result $\longrightarrow$ $g(x) = x+1$

③ 1 over the result $\longrightarrow$ $f(x) = \dfrac{1}{x}$

★ Check: $f(g(h(x))) = f(g(x^2)) = f(x^2+1) = \dfrac{1}{x^2+1}$ ✓

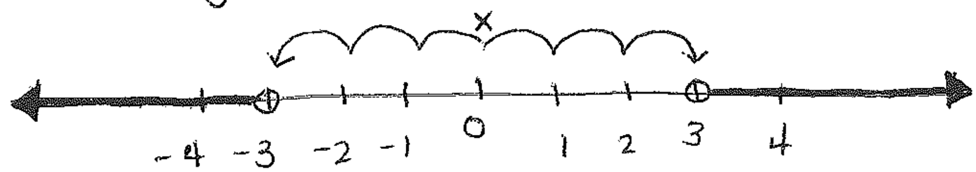
VII Solving Absolute Value Inequalities (2H, 2I)

Absolute value is the "distance from 0"

$\boxed{\text{ex1}}$ $|x| > 3$ mean " x's distance from 0 is greater than 3."

↑ say

matter:

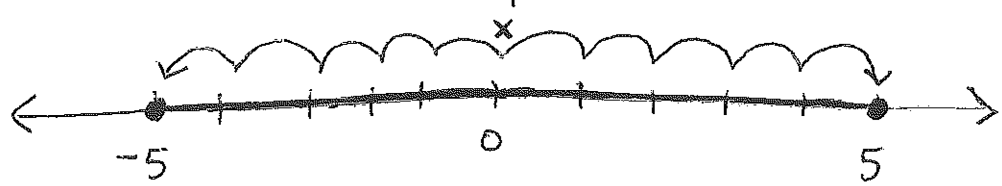


math: $\{x \mid x < -3 \text{ or } x > 3\}$ is the solution set

$\boxed{\text{ex2}}$ $|x| \leq 5$ mean " x's distance from 0 is less than or equal to 5."

↑ say

matter:



math: $\{x \mid -5 \leq x \leq 5\}$

General Rule

If $|x| \geq c$	then $x \leq -c$ or $x \geq c$		Greater than OR
If $|x| \leq c$	then $-c \leq x \leq c$		Less than AND

"GO LA"

$\boxed{\text{ex3}}$ Solve $|2x| < 14$

" 2x's distance from 0 is less than 14"

$-14 < 2x < 14$

$-7 < x < 7$

$$\boxed{\{x \mid -7 < x < 7\}}$$

$\boxed{\text{ex4}}$ $|3x+2| \geq 4$

" 3x+2's distance from 0 is greater than or equal to 4"

$3x+2 \leq -4$ or $3x+2 \geq 4$

$3x \leq -6$ $\quad$ $3x \geq 2$

$x \leq -2$ $\quad$ $x \geq \frac{2}{3}$

$$\boxed{\{x \mid x \leq -2 \text{ or } x \geq \tfrac{2}{3}\}}$$

$\boxed{\text{ex5}}$ $\frac{1}{4}|4x+8|-2<3$ ★ isolate the absolute value first!!

$$\frac{1}{4}|4x+8|<5$$

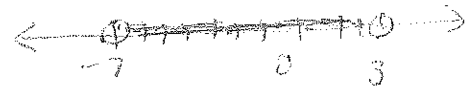
$$|4x+8|<20$$

$$-20<4x+8<20$$

$$-28<4x<12$$

$$-7<x<3$$

$\boxed{\{x \mid -7<x<3\}}$ $\boxed{(-7,3)}$



Be careful, when you isolate the absolute value and get:

$|x| < -\#$

positive # — can't be less than — a negative #

$\emptyset$

$|x| > -\#$

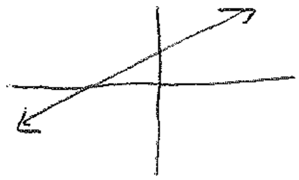
positive # — always greater than — negative number

$\mathbb{R}$

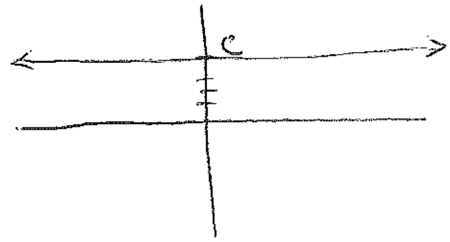
VIII Translations of Parent Functions (2F)

A. Parent Functions

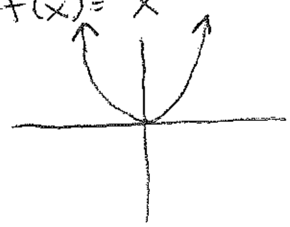
1. $f(x) = mx + b$



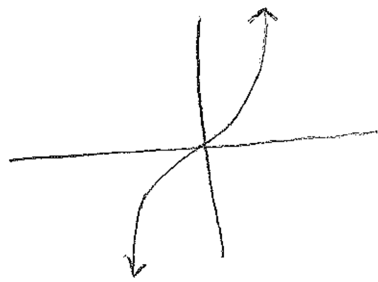
2. $f(x) = c$ ← constant



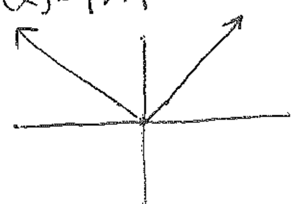
3. $f(x) = x^2$



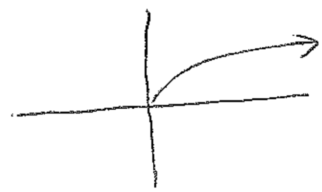
4. $f(x) = x^3$



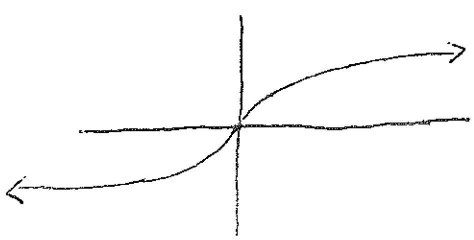
5. $f(x) = |x|$



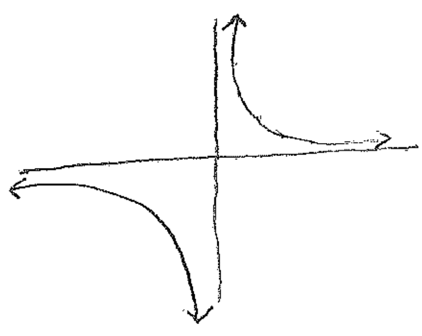
6. $f(x) = \sqrt{x}$



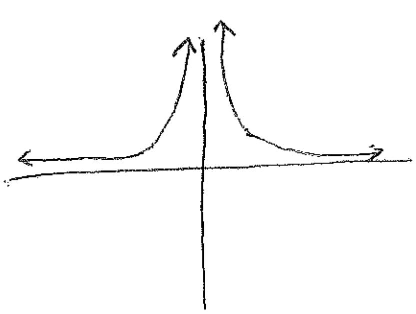
7. $f(x) = \sqrt[3]{x}$



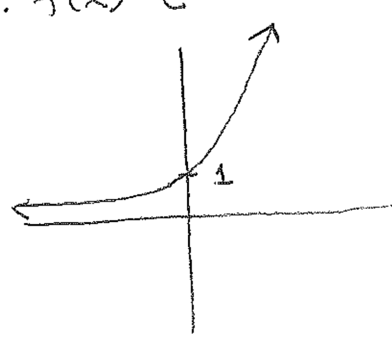
8. $f(x) = \frac{1}{x}$



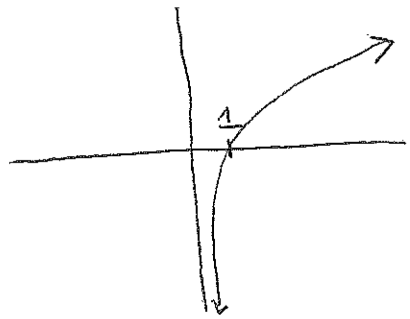
9. $f(x) = \frac{1}{x^2}$



10. $f(x) = e^x$



11. $f(x) = \ln(x)$



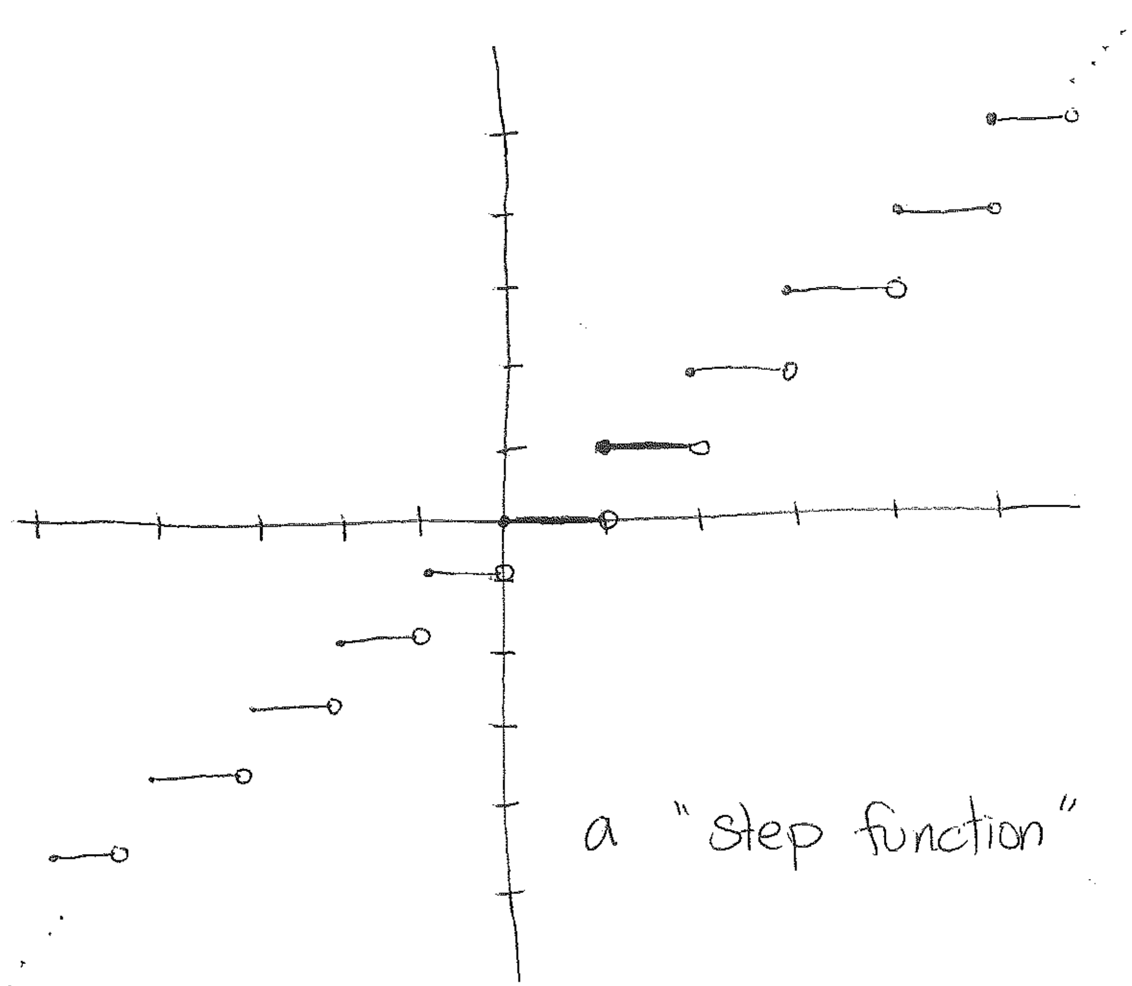
12. $f(x) = [x]$

the "greatest integer" function

aka the "floor" function

→ means the greatest integer that is <u>less than</u> or <u>equal to</u> the number

x	$y \to [x]$
5	5
4.5	4
3.1	3
2.999	2
2.1	2
1.5	1
.5	0
0	0
-.5	-1
-1	-1
-1.3	-2
-1.57	-2
-3.01	-4
-3.999	-4



a "step function"

B. Translating

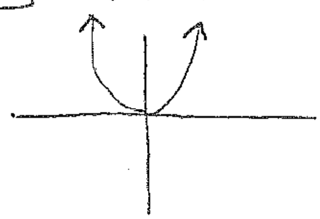
1. Reflection

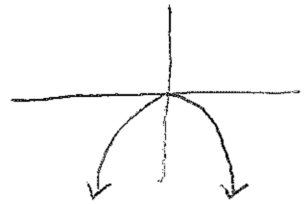
outside

To reflect across the xaxis : $-f(x)$

To reflect across the yaxis : $f(-x)$ inside

$\boxed{\text{ex 1}}$ $f(x) = x^2$ ➡ $f(x) = -x^2$ outside

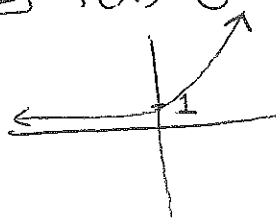


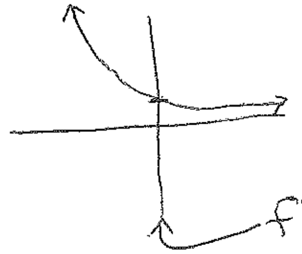


← flip across x-axis

"inside"

$\boxed{\text{ex 2}}$ $f(x) = e^x$ ➡ $f(x) = e^{-x}$





flip across y-axis

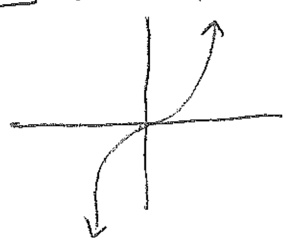
2. Vertical Shift

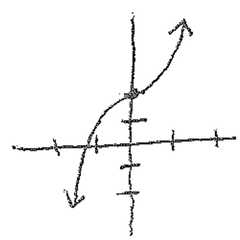
outside

To shift up: $f(x) + c$

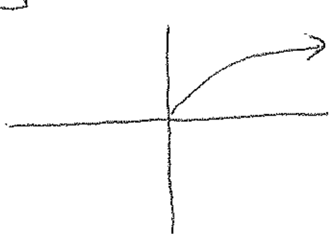
To shift down: $f(x) - c$ outside

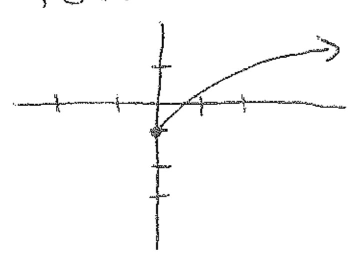
$\boxed{\text{ex 3}}$ $f(x) = x^3$ ➡ $f(x) = x^3 + 2$ ← outside





$\boxed{\text{ex 4}}$ $f(x) = \sqrt{x}$ ➡ $f(x) = \sqrt{x} - 1$





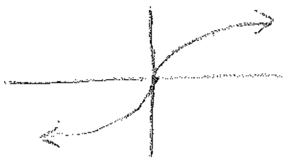
3. Horizontal Shift

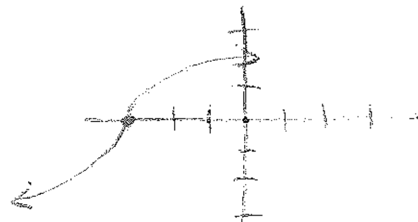
To shift to the <u>left</u> : $f(x+c)$ ← inside

To shift to the <u>right</u> : $f(x-c)$ ← inside

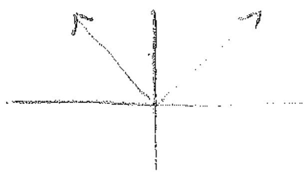
be careful, its opposite of what you would think

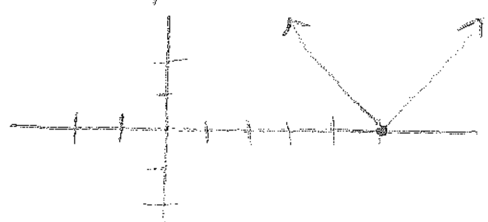
$\boxed{ex5}$ $f(x) = \sqrt[3]{x}$ ➡ $f(x) = \sqrt[3]{x+3}$





$\boxed{ex6}$ $f(x) = |x|$ ➡ $f(x) = |x-5|$





Cheet Sheet

Reflect	$-f(x) \Rightarrow$ across x-axis $f(-x) \Rightarrow$ across y-axis
Vertical Shift	$f(x)+c \Rightarrow$ up $f(x)-c \Rightarrow$ down
Horizontal Shift	$f(x+c) \Rightarrow$ left $f(x-c) \Rightarrow$ right

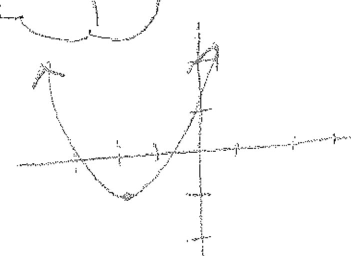
4. Combination Problems

$\boxed{ex7}$ Graph $f(x) = (x+2)^2 - 1$

left 2   down 1

parent function $f(x) = x^2$

→ pick a point from the parent function & move it!
finish the graph from there

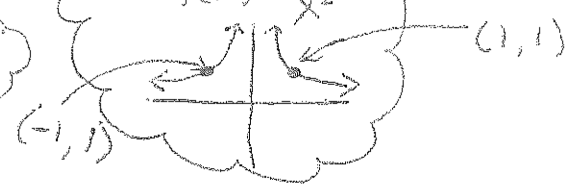


ex8 Graph $f(x) = -\frac{1}{x^2} + 2$

reflect across x-axis

up 2

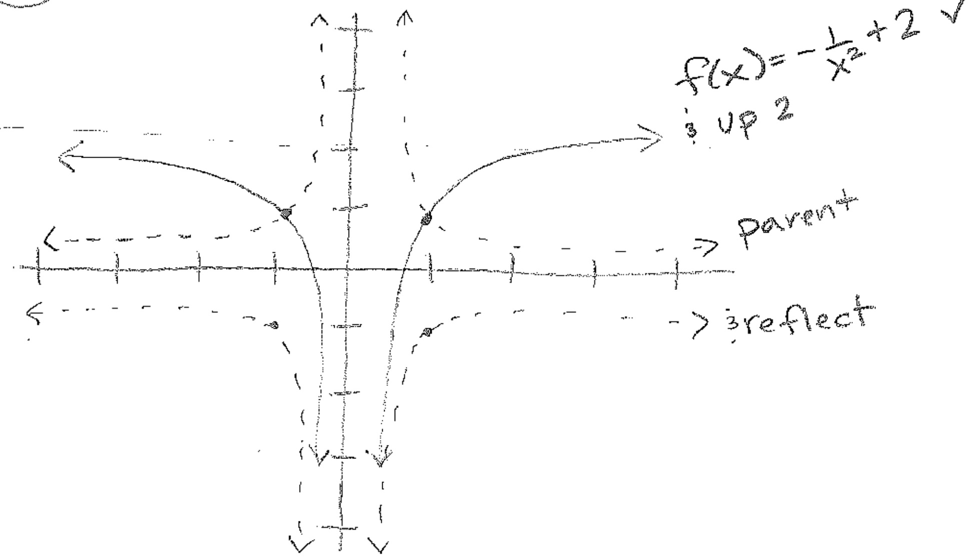
parent function $f(x) = \frac{1}{x^2}$



→ Use the order of operations!

★ 1st: reflect

★ 2nd: up 2



Honors

B. Transforming Parent Functions

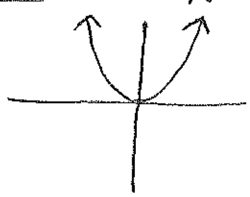
1. Reflections

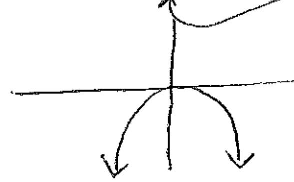
outside

To reflect across the <u>x-axis</u>: $-f(x)$

To reflect across the <u>y-axis</u>: $f(-x)$ inside

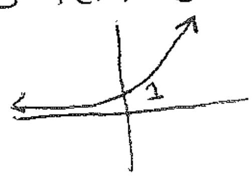
$\boxed{\text{ex 1}}$ $f(x) = x^2$ → $f(x) = -x^2$ "outside"

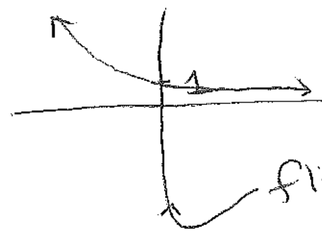




← flip across x-axis

$\boxed{\text{ex2}}$ $f(x) = e^x$ → $f(x) = e^{-x}$ inside





flip across y-axis

2. Vertical Shifts

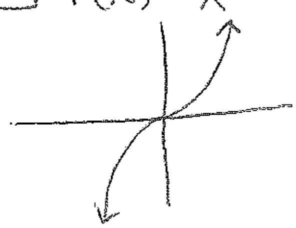
outside

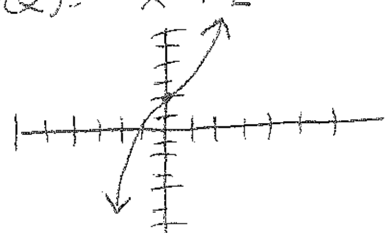
To shift <u>up</u>: $f(x) + c$ ← constant

To shift <u>down</u>: $f(x) - c$

outside

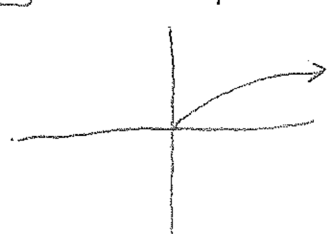
$\boxed{\text{ex3}}$ $f(x) = x^3$ → $f(x) = x^3 + 2$ ← outside

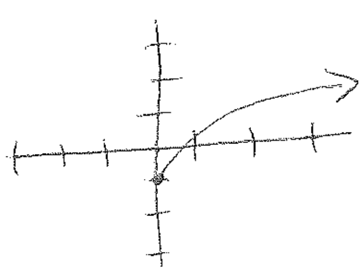




outside

$\boxed{\text{ex4}}$ $f(x) = \sqrt{x}$ → $f(x) = \sqrt{x} - 1$





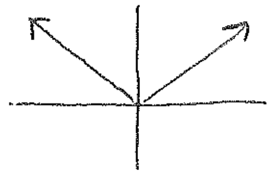
3. Horizontal Shifts

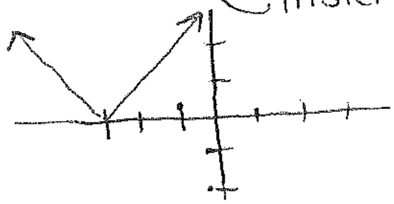
To shift to the <u>right</u>: $f(x-c)$ ← inside

To shift to the <u>left</u>: $f(x+c)$ ← inside

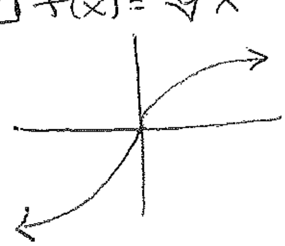
be careful! its backwards!

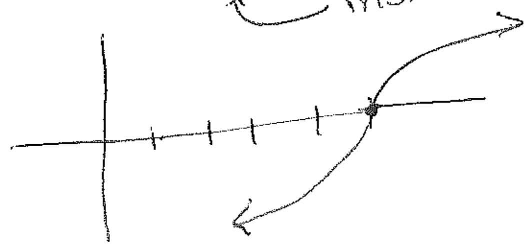
$\boxed{ex5}$ $f(x)=|x|$ ➡ $f(x)=|x+3|$ ← inside





$\boxed{ex6}$ $f(x)=\sqrt[3]{x}$ ➡ $f(x)=\sqrt[3]{x-5}$ ← inside





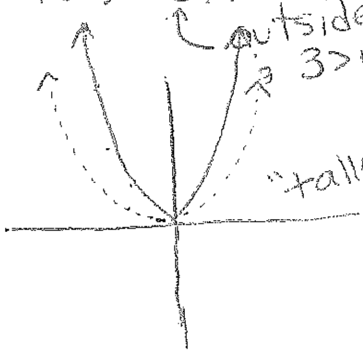
4. Vertical Stretch/Shrink

To <u>stretch</u> vertically: $c \cdot f(x)$ if $c>1$ ← outside

To <u>shrink</u> vertically: $c\, f(x)$ if $0<c<1$

$\boxed{ex7}$ $f(x)=3x^2$ ← outside

$3>1$ ∴ stretch

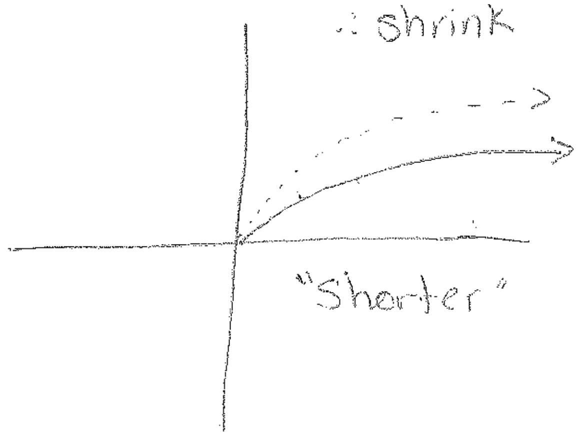


"taller"

$\boxed{ex8}$ $f(x)=\frac{1}{4}\sqrt{x}$ ← outside

$0<c<1$

∴ shrink



"shorter"

5. Horizontal Stretch/Shrink

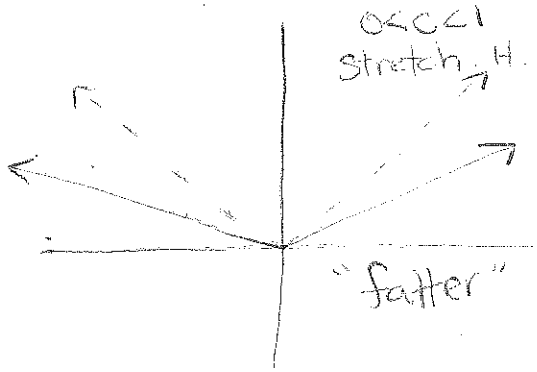
To **stretch** horizontally: $f(cx)$ if $0 < c < 1$ — inside

To **shrink** horizontally: $f(cx)$ if $c > 1$

be careful its backwards

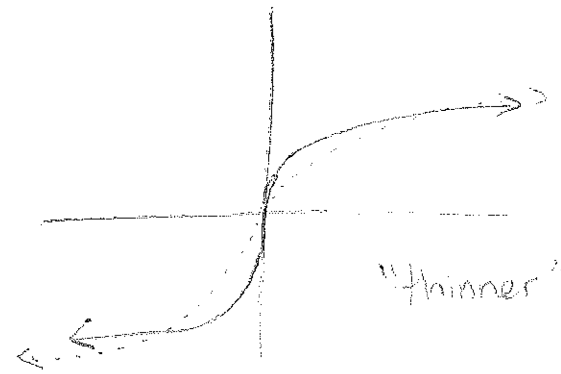
$\boxed{\text{ex 9}}$ $f(x) = \left|\frac{1}{2}x\right|$

inside
$0<c<1$
Stretch H.



$\boxed{\text{ex 10}}$ $f(x) = \sqrt[3]{2x}$

shrink!



6. Combination Problems

$\boxed{\text{ex 11}}$ Graph $f(x) = -\frac{1}{x^2} + 2$

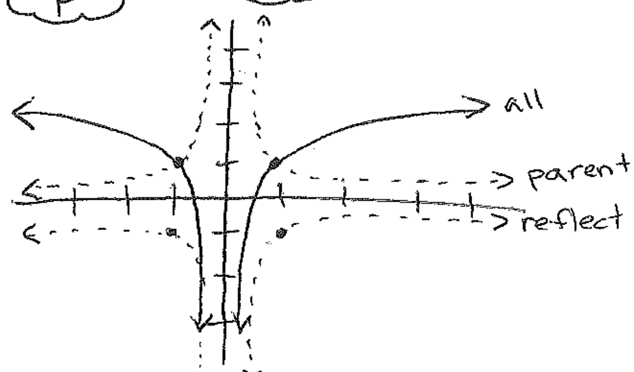
reflect across x-axis

up 2

→ follow the order of operations

1st: reflect

2nd: up 2



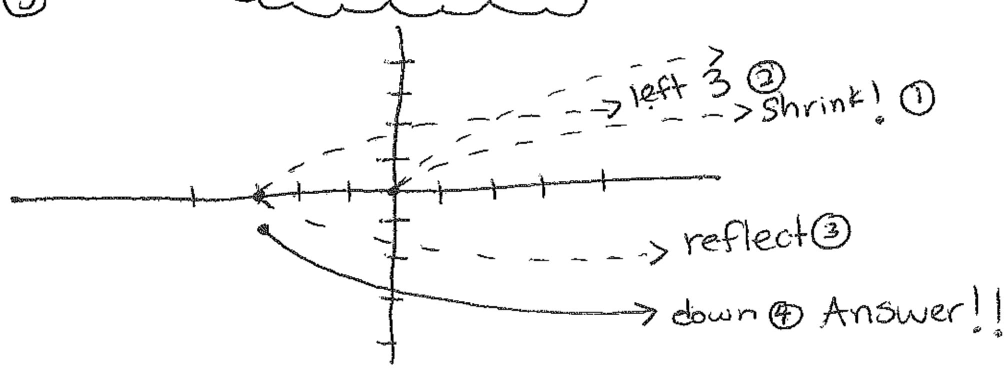
$\boxed{\text{ex 12}}$ Graph $f(x) = -\sqrt{2x+3} - 1$

reflect across the x-axis 3

left 3 2

down 1 4

Shrink Horizontally 1



Cheat Sheet

Reflect	$-f(x) \Rightarrow$ across x-axis $f(-x) \Rightarrow$ across y-axis
Vertical shift	$f(x)+c \Rightarrow$ up $f(x)-c \Rightarrow$ down
Horizontal shift	$f(x+c) \Rightarrow$ left $f(x-c) \Rightarrow$ right
Vertical	$c\,f(x)$ if $c>1$ stretch : away from x-axis $c\,f(x)$ if $0<c<1$ shrink : closer to x-axis
Horizontal	$f(cx)$ if $0<c<1$ stretch : away from y-axis $f(cx)$ if $c>1$ shrink : closer to y-axis

C. Graphing Piecewise Defined Functions

$\boxed{\text{ex 1}}$ $f(x) = \begin{cases} -2x+3 & x \geq 2 \quad \{\text{right}\} \\ x^2 & -1 \leq x < 2 \quad \{\text{middle}\} \\ \frac{1}{2}x+3 & x < -1 \quad \{\text{left}\} \end{cases}$

3 functions on <u>one</u> graph

<u>range</u>
tells you <u>what</u> each piece looks like
{♬}

<u>Domain</u>
tells you <u>when</u> each parent functio[n] starts and stops on the x-axis.
"walls" {time}

☆ To graph ☆

① identify each shape

$-2x+3 \Rightarrow$ line, slope $= -\frac{2}{1}$

$x^2 \Rightarrow$ parabola

$\frac{1}{2}x+3 \Rightarrow$ line, slope $= \frac{1}{2}$

② identify "walls", <u>when</u> does the graph change over?

@ $x = -1$

@ $x = 2$

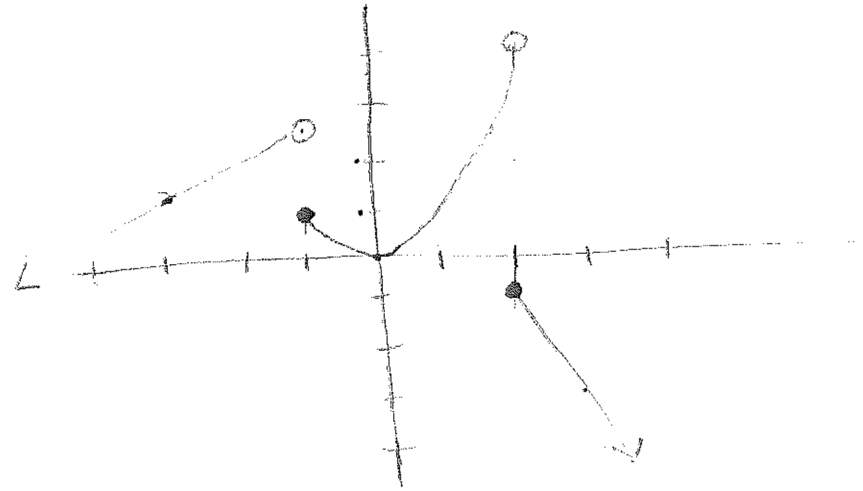
③ identify where each piece begins and ends
{plug in your "wall" values into the corresponding ranges}

@ $x=-1$ $\quad \frac{1}{2}(-1)+3 = 2\frac{1}{2}$ $\quad (-1, 2\frac{1}{2})$ ○ b/c $x < -1$

$(-1)^2 = 1$ $\quad (-1, 1)$ ● b/c $-1 \leq x < 2$

@ $x = 2$ $\quad (2)^2 = 4$ $\quad (2, 4)$ ○ b/c $-1 \leq x < 2$

$-2(2)+3 = -1$ $\quad (2, -1)$ ● b/c $x \geq 2$



$\boxed{ex2}$ $f(x) = \begin{cases} \sqrt{x} + 1 & x \geq 4 \quad (R) \\ -2x & -2 < x < 4 \quad (M) \\ (x+1)^3 & x \leq -2 \quad (L) \end{cases}$

<u>walls</u>
$x = -2$
$x = 4$

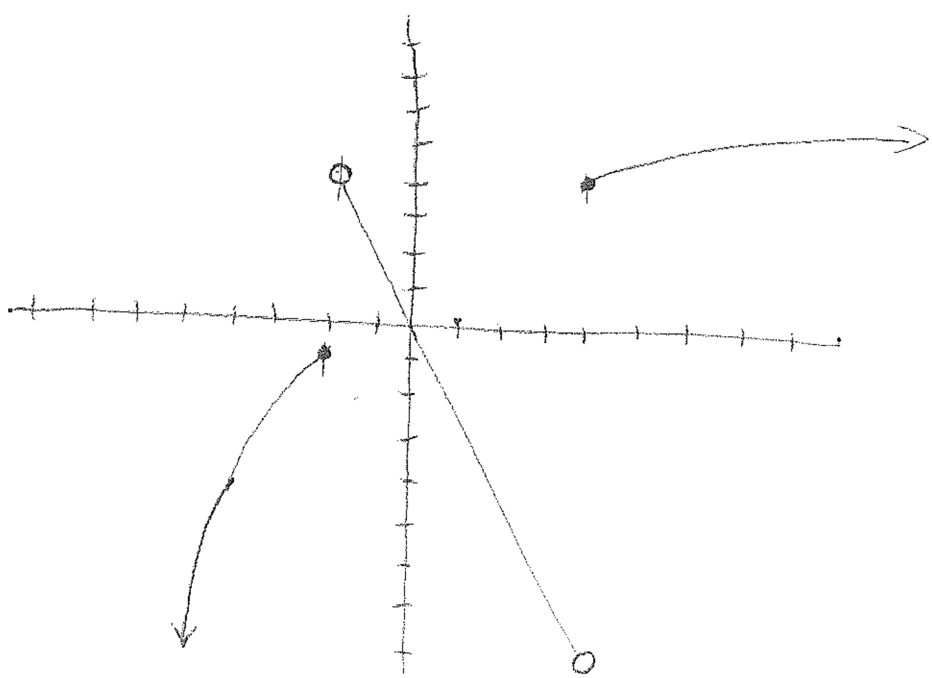
up 1

line
slope $= \frac{-2}{1}$

left 1

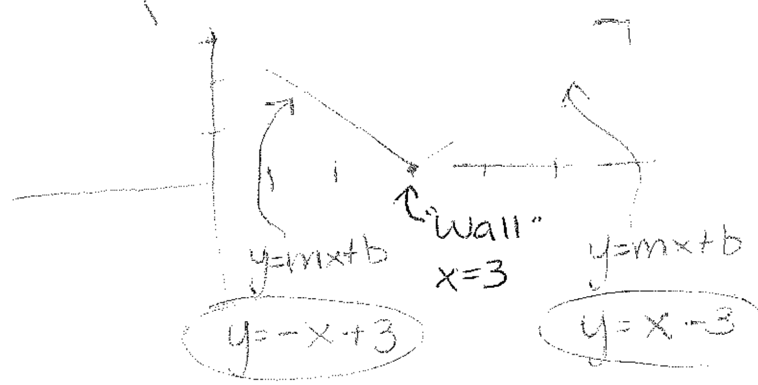
@ $x = -2$ $\quad (-2+1)^3 = -1 \quad (-2, -1)$ ●
$\quad -2(-2) = +4 \quad (-2, +4)$ ○

@ $x = 4$ $\quad -2(4) = -8 \quad (4, -8)$ ○
$\quad \sqrt{4} + 1 = 3 \quad (4, 3)$ ●



IX Writing Absolute Value Functions as Piecewise (2G)

Looking at
$f(x) = |x-3|$
↑ right 3



$$f(x)=\begin{cases} x-3 & x \geq 3 \\ -x+3 & x < 3 \end{cases}$$

$-x+3$ ← $-(x-3)$

Note: only one "or equals to" sign per wall value.

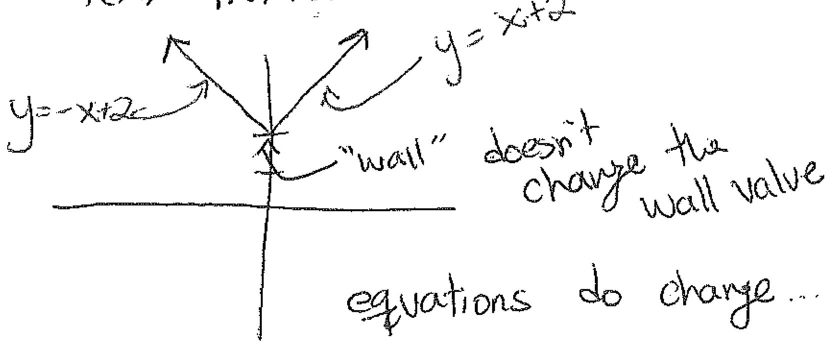
pattern: 1st is the same $x \geq$ wall
2nd is the opposite $x <$ wall

why? $|5| = 5$ answer is the same as the value you put in...

$|-5| = 5$ answer is the opposite as the value you put in.
$-(-5) = 5$
"fix"

Question: Does a vertical shift change the wall value?
Does it change your equations?

$f(x) = |x|+2$



equations do change... up 2

$\boxed{ex1}$ Write as a piecewise $f(x) = |2x+7|$

① "wall" $2x+7=0$

$x = -\frac{7}{2}$

② Test a point less than $-\frac{7}{2}$ and greater than $-\frac{7}{2}$

★ greater than. $x \geq -\frac{7}{2}$ ... test 5... $|2(5)+7| = |17| = 17$

same ∴ equation is the same

use $2x+7$

★ less than: $x < -\frac{7}{2}$ ... test -10... $|2(-10)+7| = |-13| = 13$

opposite ∴ equation is the opposite

use $-(2x+7) = -2x-7$

$$\boxed{f(x) = \begin{cases} 2x+7 & x \geq -\frac{7}{2} \\ -2x-7 & x < -\frac{7}{2} \end{cases}}$$

doesn't affect "wall"

$\boxed{ex2}$ Write as a piecewise : $f(x) = |4x+2| + 3$

$4x+2=0$

$4x=-2$

$x = -\frac{1}{2}$

$$f(x) = \begin{cases} \overset{\text{pos same}}{(4x+2)} + 3 & x \geq -\frac{1}{2} \\ \overset{\text{neg} \therefore \text{opposite}}{-(4x+2)} + 3 & x < -\frac{1}{2} \end{cases}$$

← check 3

← check -3

$$\boxed{f(x) = \begin{cases} 4x+5 & x \geq -\frac{1}{2} \\ -4x+1 & x < -\frac{1}{2} \end{cases}}$$

ex3 Write as a piecewise $f(x) = |x+1| + |4x-16|$

$x=-1$ $x=4$

2 "walls"

$\therefore$ 3 parts to the graph

$$f(x) = \begin{cases} \overset{\text{pos} \therefore \text{same}}{(x+1)} + \overset{\text{pos} \therefore \text{same}}{(4x-16)} & x \geq 4 \\ \overset{\text{pos} \therefore \text{same}}{(x+1)} + -\overset{\text{neg} \therefore \text{opposite}}{(4x-16)} & -1 \leq x < 4 \\ -\overset{\text{neg} \therefore \text{opp}}{(x+1)} + -\overset{\text{neg} \therefore \text{opp}}{(4x-16)} & x < -1 \end{cases}$$

Check 5

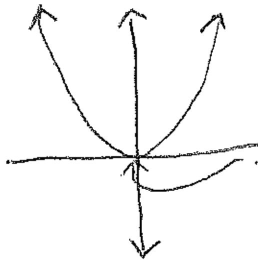
Check 0

Check -2

$$\boxed{f(x) = \begin{cases} 5x-15 & x \geq 4 \\ -3x+17 & -1 \leq x < 4 \\ -5x+15 & x < -1 \end{cases}}$$

X Symmetry (2J)

1. Symmetric about the <u>y-axis</u> ; aka <u>EVEN</u>



flip across y-axis ∴ it's the <u>same</u>

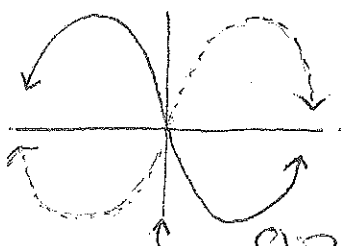
when $\boxed{f(x) = f(-x)}$

$\boxed{\text{ex 1}}$ Is $f(x) = x^4 + 1$ even?

$f(-x) = (-x)^4 + 1 = x^4 + 1$

same!! ∴ EVEN

2. Symmetric about the <u>origin</u> ; aka <u>ODD</u>



flip across y-axis <u>is the same as</u> a flip across the x-axis

when $\boxed{f(-x) = -f(x)}$

$\boxed{\text{ex 2}}$ Is $f(x) = 2x^3 + x$ odd?

$f(-x) = 2(-x)^3 + (-x) = -2x^3 - x$

$-f(x) = -(2x^3 + x) = -2x^3 - x$

same!! ∴ ODD

$\boxed{ex3}$ Is $f(x) = x^3 + x^2 + 1$ even, odd or neither?

$f(-x) = (-x)^3 + (-x)^2 + 1 = -x^3 + x^2 + 1$ (not even)

$-f(x) = -(x^3 + x^2 + 1) = -x^3 - x^2 - 1$ (not odd)

$\therefore$ NEITHER

$\boxed{ex4}$ Is $f(x) = \frac{1}{2}|x| + x^4$ even, odd or neither?

$f(-x) = \frac{1}{2}|-x| + (-x)^4 = \frac{1}{2}|x| + x^4$ EVEN!