(6.0) For Q.1 and 2, show your work and circle the best possible answer.

1.

If $y = xy + x^2 + 1$, then when $x = -1$, $\frac{dy}{dx}$ is

(A) $\frac{1}{2}$ (B) $-\frac{1}{2}$ (C) -1 (D) -2 (E) nonexistent

$y = \underset{u}{x}\underset{v}{y} + x^2 + 1$

$y' = \underset{u}{x} \cdot \underset{v'}{y'} + \underset{v}{y} \cdot \underset{u'}{1} + 2x$

$-xy' \quad -xy'$

$y' - xy' = y + 2x$

$\frac{y'(1\cancel{-x})}{1\cancel{-x}} = \frac{y+2x}{1-x}$

$y' = \frac{y+2x}{1-x}$

$y' = \frac{(1)+2(-1)}{1-(-1)}$

$y' = \frac{1-2}{1+1} = \frac{-1}{2}$

What's $y = ?$

$y = (-1)y + (-1)^2 + 1$

$y = -y + 1 + 1$

$+y \quad +y$

$2y = 2$

$y = 1$

2.

If $\ln(2x + y) = x + 1$, then $\frac{dy}{dx} =$

(A) -2 (B) $2x + y - 2$ (C) $2x + y$ (D) $4x + 2y - 2$ (E) [illegible]

$\ln(2x+y) = x+1$

$\frac{1}{2x+y} \cdot (2+y') = 1$

$2 + y' = 2x + y$

$\boxed{y' = 2x + y - 2}$

(12.0) Explain your answer choice for Q.3 in a complete sentence in the space provided

3.

$A = \frac{1}{2}bh$

positive db/dt

If the base b of a triangle is increasing at a rate of 3 inches per minute while its height h is decreasing at a rate of 3 inches per minute, which of the following must be true about the area A of the triangle?

dh/dt negative

(A) A is always increasing.
(B) A is always decreasing.
(C) A is decreasing only when $b < h$.
(D) A is decreasing only when $b > h$.
(E) A remains constant.

$$A = \frac{1}{2}bh$$

$$\frac{dA}{dt} = \frac{1}{2}\left(b \cdot \frac{dh}{dt} + \frac{db}{dt} \cdot h\right)$$

$$\frac{dA}{dt} = \frac{1}{2}(b \cdot (-3) + 3h)$$

$$\frac{dA}{dt} = \frac{3}{2}(-b+h)$$

After ~~...~~ taking the derivative of the area, substituting dh/dt and db/dt with their respective values and simplifying the result, we yeild $(-b+h)$ which will be negative if $b>h$

If $(-b+h)$ is negative then $\frac{dA}{dt}$ is negative and the area is decreasing

(12.0) For Q.4, show your work and circle the best possible answer.

4.

The volume of a sphere is decreasing at a constant rate of 3 cubic centimeters per second. At the instant when the radius of the sphere is decreasing at a rate of 0.25 centimeter per second, what is the radius of the sphere?

(The volume V of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

$(-)$ $(-)$ $r = ?$

(A) 0.141 cm (B) 0.244 cm (C) 0.250 cm (D) 0.489 cm (E) 0.977 cm

$$V = \frac{4}{3}\pi r^3$$

$$\frac{dV}{dt} = \frac{4}{\cancel{3}}\pi \cancel{3}r^2 \cdot \frac{dr}{dt}$$

$$-3 = 4\pi r^2\left(-\frac{1}{4}\right)$$

$$\frac{-3}{-\pi} = \frac{-\pi r^2}{-\pi}$$

$$0.977 = r^2$$

Oof.
I think a lot of us (me included) forgot to take abs. value!

(17.0) For Q.5, show your work and circle the best possible answer.

5.

$$\int_2^4 \frac{dx}{5-3x} =$$

$u = 5-3x$

$\frac{du}{dx} = -3$

$dx = \frac{du}{-3}$

$= \int \frac{1}{u} \frac{du}{-3}$

$= -\frac{1}{3} \int \frac{1}{u} du = -\frac{1}{3} \ln|u| = -\frac{1}{3} \ln|(5-3x)|\Big|_2^4$

$= -\frac{1}{3}\ln|5-3(4)| - -\frac{1}{3}\ln|5-3(2)|$

$= -\frac{1}{3}\ln|-7| + \underbrace{\frac{1}{3}\ln|-1|}_{=0}$

$= \frac{-\ln 7}{3}$

(A) $-\ln 7$ $\boxed{-\frac{\ln 7}{3}}$ (C) $\frac{\ln 7}{3}$ (D) $\ln 7$ (E) $3\ln 7$

(17.0) Explain your answer choice for Q.6 in a complete sentence in the space provided.

6.

Which of the following definite integrals has the same value as $\int_0^4 xe^{x^2}\,dx$?

(A) $\frac{1}{2}\int_0^4 e^u\,du$

$\frac{1}{2}\int_0^{16} e^u\,du$

~~(C)~~ $2\int_0^2 e^u\,du$

~~(D)~~ $2\int_0^4 e^u\,du$

~~(E)~~ $2\int_0^{16} e^u\,du$

$u = x^2$

$\frac{du}{dx} = 2x$

$dx = \frac{du}{2x}$

$\int \cancel{x} e^u \frac{du}{2\cancel{x}} = \frac{1}{2}\int e^u du$

$u = x^2$

$u = 0^2 = 0$ new boundaries

$u = 4^2 = 16$

After applying u-substitution, the coefficient of x cancels out, a factor of $\frac{1}{2}$ is moved to the outside of the integral and then evaluating $u = x^2$ for $x = 0$ and 4 yields new boundaries $u = 0$ and $u = 16$

(12.0) Show your work and box your answers for Q.7.

(NOTE: Part D counts towards UNIT 3 while Part C counts as a *Unit 2 Reassessment* for Standard 12.0)

7.

Two particles move along the x-axis. For $0 \le t \le 8$, the position of particle P at time t is given by $x_P(t) = \ln(t^2 - 2t + 10)$, while the velocity of particle Q at time t is given by $v_Q(t) = t^2 - 8t + 15$. Particle Q is at position $x = 5$ at time $t = 0$.

$(0,5)$

(c) Find the acceleration of particle Q at time $t = 2$. Is the speed of particle Q increasing, decreasing, or neither at time $t = 2$? Explain your reasoning.

$v_Q(t) = t^2 - 8t + 15$

$a_Q(t) = 2t - 8$

$a_Q(2) = 2(2) - 8$

$a_Q(2) = 4 - 8$

$a_Q(2) = -4$

$v_Q(2) = 2^2 - 8(2) + 15$

$= 4 - 16 + 15$

$= 4 - 1$

$v_Q(2) = 3$

$\boxed{\text{The speed of particle } Q \text{ is decreasing @ } t=2 \text{ Since its velocity is positive and its acceleration is negative.}}$

(d) Find the position of particle Q the first time it changes direction.

$v_Q = t^2 - 8t + 15$

$0 = (t-5)(t-3)$

$t = 3 \quad t = 5$

↑

First time it changes direction!

$\int_0^3 t^2 - 8t + 15 \, dt =$

$\frac{t^3}{3} - \frac{8t^2}{2} + 15t \Big|_0^3 =$

$\frac{3^{3}}{3} - \frac{8(3)^2}{2} + 15(3) - 0 =$

$9 - 36 + 45 = 18$

At $t=0$ Q is at $x=5$

$\boxed{\text{So at } t=3 \text{ Q is at } x=23}$

All done? Try these problems! They take implicit differentiation one step further and consider those functions written in terms of x and y and raise questions about their tangent lines! They have tangent lines after all, especially since they have ever-changing slopes.

1.

Which of the following is an equation of the line tangent to the graph of $x^2 - 3xy = 10$ at the point $(1, -3)$?

(A) $y + 3 = -11(x - 1)$

(B) $y + 3 = -\frac{7}{3}(x - 1)$

(C) $y + 3 = \frac{1}{3}(x - 1)$

(D) $y + 3 = \frac{7}{3}(x - 1)$

(E) $y + 3 = \frac{11}{3}(x - 1)$

$-{-3}$ 1

y x

$$2x - (3xy' + 3y) = 0$$
$$2(1) - (3(1)y' + 3(-3)) = 0$$
$$2 - (3y' - 9) = 0$$
$$-2 \qquad -2$$
$$-(3y' - 9) = -2$$
$$3y' - 9 = 2$$
$$3y' = 11$$
$$y' = \frac{11}{3} \leftarrow \text{slope!}$$

2.

The slope of the line tangent to the curve $y^2 - (xy + 1)^3 = 0$ at $(2, -1)$ is

(A) $-\frac{3}{2}$ (B) $-\frac{3}{4}$ (C) 0 $\frac{3}{4}$ (E) $\frac{3}{2}$

$$2y \cdot \frac{dy}{dx} + 3(xy+1)^2 \cdot \left(x\frac{dy}{dx} + 1 \cdot y\right) = 0$$
$$2(-1)y' + 3(2(-1)+1)^2(2y' - 1) = 0$$
$$-2y' + 3(-1)^2(2y' - 1) = 0$$
$$-2y' + 3(2y' - 1) = 0$$
$$-2y' + 6y' - 3 = 0$$

$$4y' - 3 = 0$$
$$4y' = 3$$
$$y' = \frac{3}{4}$$