AP CALCULUS
U-SUBSTITUTION PRACTICE!

Name Answer Key

1) $\int (x^3-1)^9 x^2\,dx =$

A) $\frac{(x^3-1)^8}{8}+c$

B) $\frac{x^3(x^3-1)^{10}}{10}+c$

C) $\frac{(x^3-1)^{10}}{10}+c$

D) $\frac{x^3(x^3-1)^{10}}{30}+c$

(E) $\frac{(x^3-1)^{10}}{30}+c$

$u = x^3-1$
$\frac{du}{dx} = 3x^2$
$\frac{1}{3}du = x^2\,dx$
$\frac{1}{3}\int u^9\,du$
$\frac{1}{3}\left(\frac{u^{10}}{10}\right)+C$
$\frac{1}{30}(x^3-1)^{10}+C$

2) $\int x\sqrt{5x^2-3}\,dx =$

(A) $\frac{1}{15}(5x^2-3)^{\frac{3}{2}}+c$

B) $\frac{x^2(5x^2-3)^{\frac{3}{2}}}{3}+c$

C) $\frac{1}{10}(5x^2-3)^{\frac{3}{2}}+c$

D) $\frac{2}{3}(5x^2-3)^{\frac{3}{2}}+c$

E) $10(5x^2-3)^{\frac{3}{2}}+c$

$u = 5x^2-3$
$\frac{du}{dx} = 10x$
$\frac{1}{10}du = x\,dx$
$\frac{1}{10}\int \sqrt{u}\,du$
$\frac{1}{10}\int u^{\frac{1}{2}}\,du$
$\frac{1}{10}\left(\frac{2}{3}u^{\frac{3}{2}}\right)+C$
$\frac{1}{15}\left(u^{\frac{3}{2}}\right)+C$
$\frac{1}{15}(5x^2-3)^{3/2}+C$

3) $\int \frac{2x^2}{\sqrt{x^3+3}}\,dx =$

(A) $\frac{4}{3}\sqrt{x^3+3}+c$

B) $\frac{2}{3}\sqrt{x^3+3}+c$

C) $\frac{1}{3}\sqrt{x^3+3}+c$

D) $\frac{4}{3\sqrt{x^3+3}}+c$

E) $\frac{3}{4}\sqrt{x^3+3}+c$

$u = x^3+3$
$\frac{du}{dx} = 3x^2$
$\frac{1}{3}du = x^2\,dx$
$\frac{1}{3}\int \frac{2}{\sqrt{u}}\,du$
$\frac{2}{3}\int u^{-\frac{1}{2}}\,du$
$\frac{2}{3}\left(2u^{\frac{1}{2}}\right)+C$
$\frac{4}{3}u^{\frac{1}{2}}+C$
$\frac{4}{3}(x^3+3)^{\frac{1}{2}}+C$
$\frac{4}{3}\sqrt{x^3+3}+C$

4) $\int \frac{dx}{(5x+3)^7} =$

A) $\frac{1}{8(5x+3)^8}+c$

(B) $-\frac{1}{30(5x+3)^6}+c$

C) $-\frac{1}{40(5x+3)^8}+c$

D) $-\frac{1}{6(5x+3)^6}+c$

E) $\frac{1}{30(5x+3)^6}+c$

$u = 5x+3$
$\frac{du}{dx} = 5$
$\frac{1}{5}du = dx$
$\frac{1}{5}\int \frac{du}{u^7}$
$\frac{1}{5}\int u^{-7}\,du$
$\frac{1}{5}\left(\frac{u^{-6}}{-6}\right)+C$
$-\frac{1}{30}\left(\frac{1}{u^6}\right)+C$
$\frac{-1}{30(5x+3)^6}+C$

5) $\int \frac{(\sqrt{x}-1)^5}{\sqrt{x}}\,dx =$

A) $\frac{(\sqrt{x}-1)^6}{12}+c$

B) $\frac{(x\sqrt{x}-x)^6}{6}+c$

C) $\frac{(x-\sqrt{x})^6}{6}+c$

(D) $\frac{(\sqrt{x}-1)^6}{3}+c$

E) $\frac{(\sqrt{x}-1)^6}{6}+c$

$u = \sqrt{x}-1$
$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$
$2du = \frac{1}{\sqrt{x}}dx$
$2\int u^5\,du$
$2\left(\frac{u^6}{6}\right)+C$
$\frac{1}{3}u^6+C$
$\frac{1}{3}(\sqrt{x}-1)^6+C$

6) $\int (x^2+1)(x^3+3x-7)^{\frac{3}{5}}\,dx =$

(A) $\frac{5}{24}(x^3+3x-7)^{\frac{8}{5}}+c$

B) $\frac{5}{8}(x^3+3x-7)^{\frac{8}{5}}+c$

C) $\frac{1}{3}(x^3+3x-7)^{-\frac{2}{5}}+c$

D) $\frac{5}{16}(x^2+1)^2(x^3+3x-7)^{\frac{8}{5}}+c$

E) $\frac{8}{15}(x^3+3x-7)^{\frac{8}{5}}+c$

$u = x^3+3x-7$
$\frac{du}{dx} = 3x^2+3$
$\frac{du}{dx} = 3(x^2+1)$
$\frac{1}{3}du = (x^2+1)\,dx$
$\frac{1}{3}\int u^{\frac{3}{5}}\,du$
$\frac{1}{3}\left(\frac{5}{8}u^{\frac{8}{5}}\right)+C$
$\frac{5}{24}(x^3+3x-7)^{\frac{8}{5}}+C$

7) $\int \frac{dt}{\sqrt{t}\left(1-\sqrt{t}\right)^2} =$

A) $-\frac{1}{2(1-\sqrt{t})^3} + c$

B) $-\frac{2}{3(1-\sqrt{t})^3} + c$

C) $\frac{2}{1-\sqrt{t}} + c$

D) $-\frac{2}{1-\sqrt{t}} + c$

E) $\frac{1}{2(1-\sqrt{t})} + c$

$u = 1-\sqrt{t}$

$\frac{du}{dt} = -\frac{1}{2\sqrt{t}}$

$-2\,du = \frac{1}{\sqrt{t}}\,dt$

$-2\int \frac{du}{u^2}$

$-2\int u^{-2}\,du$

$-2\left(\frac{u^{-1}}{-1}\right) + C$

$2\left(\frac{1}{u}\right) + C$

$\frac{2}{1-\sqrt{t}} + C$

8) $\int \frac{x+2}{\left(x^2+4x-1\right)^2}\,dx =$

A) $-\frac{3}{(x^2+4x-1)^3} + c$

B) $\frac{x^2+2x}{6(x^2+4x-1)^3} + c$

C) $\frac{3}{(x^2+4x-1)^3} + c$

D) $-\frac{1}{2x^2+8x-2} + c$

E) $\frac{1}{2x^2+8x-2} + c$

$u = x^2+4x-1$

$\frac{du}{dx} = 2x+4$

$\frac{du}{dx} = 2(x+2)$

$\frac{1}{2}\,du = (x+2)\,dx$

$\frac{1}{2}\int \frac{du}{u^2}$

$\frac{1}{2}\int u^{-2}\,du$

$\frac{1}{2}\left(\frac{u^{-1}}{-1}\right) + C$

$-\frac{1}{2}\left(\frac{1}{u}\right) + C$

$\frac{-1}{2(x^2+4x-1)} + C$

9) $\int \left(x^2+2x+1\right)^{10}\,dx =$

A) $\frac{(x+1)^{19}}{19} + c$

B) $\frac{(x+1)^{21}}{21} + c$

C) $\frac{(x+1)^{13}}{13} + c$

D) $\frac{1}{11}\left(\frac{x^3}{3}+x^2+x\right)^{11} + c$

E) $\frac{(x^2+2x+1)^{11}}{11} + c$

$= \int (x+1)^{20}\,dx$

$\int u^{20}\,du$

$\frac{u^{21}}{21} + C$

$u = x+1$

$\frac{du}{dx} = 1$

$du = dx$

$\frac{(x+1)^{21}}{21} + C$

10) If functions f and g are differentiable functions, then $\int g'(\underbrace{f(x)}_{u})\,\underbrace{f'(x)\,dx}_{du} =$

A) $g'(x) + c$

B) $g(x) + c$

C) $g(x)\,f(x) + c$

D) $g(f'(x)) + c$

E) $g(f(x)) + c$

$u = f(x)$

$\frac{du}{dx} = f'(x)$

$du = f'(x)\,dx$

$\int g'(u)\,du$

$g(u) + C$

$g(f(x)) + C$

11) $\int x\sqrt{x-1}\,dx =$

A) $\frac{2}{3}(x^2-x)^{\frac{3}{2}} + c$

B) $\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + c$

C) $\frac{5}{2}(x-1)^{\frac{5}{2}} + \frac{3}{2}(x-1)^{\frac{3}{2}} + c$

D) $\frac{1}{3}x^2(x-1)^{\frac{3}{2}} + c$

E) $\frac{1}{2}(x-1)^4 + c$

$u = x-1 \rightarrow x = u+1$

$\frac{du}{dx} = 1 \quad du = dx$

$\int (u+1)\sqrt{u}\,du$

$\int \left(u^{\frac{3}{2}} + u^{\frac{1}{2}}\right) du$

$\frac{2}{5}u^{\frac{5}{2}} + \frac{2}{3}u^{\frac{3}{2}} + C$

$\frac{2}{5}(x-1)^{\frac{5}{2}} + \frac{2}{3}(x-1)^{\frac{3}{2}} + C$

12) $\int x^3\cos(x^4)\,dx =$

A) $\frac{x^4}{4}\sin(x^4) + c$

B) $-\frac{1}{4}\sin(x^4) + c$

C) $-\frac{x^4}{4}\sin(x^4) + c$

D) $\frac{1}{4}\sin(x^4) + c$

E) $\frac{x^4}{4}\sin\left(\frac{x^5}{5}\right) + c$

$u = x^4$

$\frac{du}{dx} = 4x^3$

$\frac{1}{4}\,du = x^3\,dx$

$\frac{1}{4}\int \cos u\,du$

$\frac{1}{4}\sin u + C$

$\frac{1}{4}\sin(x^4) + C$

13) $\int \sin 5x\,dx =$

A) $\cos 5x + c$

B) $-5\cos 5x + c$

C) $-\frac{1}{5}\cos 5x + c$

D) $\frac{1}{5}\cos 5x + c$

E) $5\cos 5x + c$

$u = 5x$

$\frac{du}{dx} = 5$

$\frac{1}{5}\,du = dx$

$\frac{1}{5}\int \sin u\,du$

$\frac{1}{5}(-\cos u) + C$

$-\frac{1}{5}\cos(5x) + C$

14) $\int \left(\tan^3 x\right)\left(\sec^2 x\right)dx =$

A) $\frac{1}{4}\tan^4 x + c$

B) $\frac{1}{2}\sec^2 x + c$

C) $\frac{1}{2}\tan^2(x) + c$

D) $4\tan^4 x + c$

E) $\frac{\sec^3 x \tan^4 x}{12} + c$

$u = \tan x$

$\frac{du}{dx} = \sec^2 x$

$du = \sec^2 x\,dx$

$\int u^3\,du$

$\frac{u^4}{4} + C$

$\frac{\tan^4 x}{4} + C$

15) $\int \frac{\cos\sqrt{x}}{\sqrt{x}}\,dx =$

A) $\frac{\cos^2\sqrt{x}}{2x} + c$

B) $2\sin\sqrt{x} + c$

C) $\frac{1}{2}\sin\sqrt{x} + c$

D) $-\frac{1}{2}\sin\sqrt{x} + c$

E) $-2\sin\sqrt{x} + c$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2\,du = \frac{1}{\sqrt{x}}\,dx$

$2\int \cos u\,du$

$2\sin u + C$

$2\sin\sqrt{x} + C$

16) $\int \sin 2\theta \cos 2\theta\,d\theta =$

A) $\frac{1}{4}\sin^2 2\theta + c$

B) $\frac{1}{2}\sin^2 2\theta + c$

C) $-\frac{1}{4}\sin^2 2\theta + c$

D) $-\frac{1}{2}\sin^2 2\theta + c$

E) $\sin^2 2\theta + c$

$u = \sin 2\theta$

$\frac{du}{d\theta} = 2\cos 2\theta$

$\frac{1}{2}\,du = \cos 2\theta\,d\theta$

$\frac{1}{2}\int u\,du$

$\frac{1}{2}\left(\frac{u^2}{2}\right) + C$

$\frac{u^2}{4} + C$

$\frac{\sin^2 2\theta}{4} + C$

17) $\int \frac{d\theta}{\cos^2 2\theta} =$

A) $\frac{1}{2}\cot 2\theta + c$

B) $\frac{1}{2}\tan 2\theta + c$

C) $-\frac{2}{\cos 2\theta} + c$

D) $2\tan 2\theta + c$

E) $-\frac{1}{2}\cot 2\theta + c$

$= \int \sec^2 2\theta\,d\theta$

$u = 2\theta$

$\frac{du}{d\theta} = 2$

$\frac{1}{2}\,du = d\theta$

$\frac{1}{2}\int \sec^2 u\,du$

$\frac{1}{2}\tan u + C$

$\frac{1}{2}\tan(2\theta) + C$

18) $\int \sec^3 x \tan x\,dx =$

A) $\frac{\tan^2 x}{2} + c$

B) $\frac{\sec^2 x}{2} + c$

C) $\frac{\sec^4 x \tan^2 x}{8} + c$

D) $\frac{\sec^4 x}{4} + c$

E) $\frac{\sec^3 x}{3} + c$

$\int \sec^2 x \cdot \sec x \cdot \tan x\,dx$

$u = \sec x$

$\frac{du}{dx} = \sec x \tan x$

$du = \sec x \tan x\,dx$

$\int u^2\,du$

$\frac{u^3}{3} + C$

$\frac{\sec^3 x}{3} + C$

19) $\int \frac{\sin\frac{3}{\theta}}{\theta^2}\,d\theta =$

A) $\frac{3\cos^2\left(\frac{3}{\theta}\right)}{\theta^3} + c$

B) $-\frac{1}{3}\cos\frac{3}{\theta} + c$

C) $\frac{\sin^2\left(\frac{3}{\theta}\right)}{6\theta^3} + c$

D) $-3\cos\frac{3}{\theta} + c$

E) $\frac{1}{3}\cos\frac{3}{\theta} + c$

$u = \frac{3}{\theta} = 3\theta^{-1}$

$\frac{du}{d\theta} = -3\theta^{-2}$

$\frac{du}{d\theta} = \frac{-3}{\theta^2}$

$-\frac{1}{3}\,du = \frac{1}{\theta^2}\,d\theta$

$-\frac{1}{3}\int \sin u\,du$

$-\frac{1}{3}(-\cos u) + C$

$\frac{1}{3}\cos\left(\frac{3}{\theta}\right) + C$

20) $\int \cos(\cos x)\sin x\,dx =$

A) $-\sin(\sin x) + c$

B) $-\sin(\cos x) + c$

C) $\cos(\cos x) + c$

D) $\sin(\cos x) + c$

E) $-\sin x + c$

$u = \cos x$

$\frac{du}{dx} = -\sin x$

$-du = \sin x\,dx$

$-\int \cos u\,du$

$-\sin u + C$

$-\sin(\cos x) + C$

21) $\int \frac{\cos 2\theta}{\sin^2 2\theta}\,d\theta =$

A) $-\frac{1}{3\sin^3 2\theta} + c$

B) $-\frac{1}{6\sin^3 2\theta} + c$

C) $2\sin 2\theta + c$

D) $\frac{1}{2\sin 2\theta} + c$

E) $-\frac{1}{2\sin 2\theta} + c$

$u = \sin 2\theta$

$\frac{du}{d\theta} = 2\cos 2\theta$

$\frac{1}{2}\,du = \cos 2\theta$

$\frac{1}{2}\int \frac{du}{u^2}$

$\frac{1}{2}\int u^{-2}\,du$

$\frac{1}{2}\left(\frac{u^{-1}}{-1}\right) + C$

$-\frac{1}{2}\left(\frac{1}{u}\right) + C$

$\frac{-1}{2\sin 2\theta} + C$