

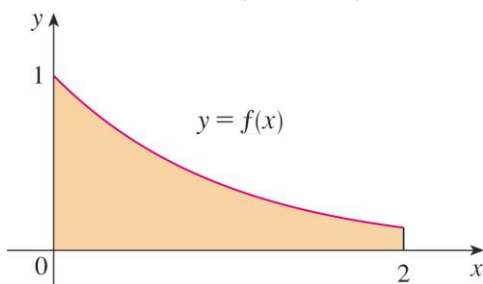


Trapezoidal Rule

1. Given the definite integral $\int_0^8 x^2 dx$,
- use the Trapezoidal Rule with four equal subintervals to approximate its value. Do not use your calculator!
 - is your answer to part (a) an overestimate or an underestimate? Justify your answer.
 - use your graphing calculator to find the exact value of $\int_0^8 x^2 dx$. Does your value agree with your answers to parts (a) and (b)?
-

2. Given the definite integral $\int_{-1}^2 (20 - x^4) dx$,
- use the Trapezoidal Rule with three equal subintervals to approximate its value. Do not use your calculator!
 - is your answer to part (a) an overestimate or an underestimate? Justify your answer.
 - use your graphing calculator to find the exact value of $\int_{-1}^2 (20 - x^4) dx$. Does your value agree with your answers to parts (a) and (b)?
-

3. The trapezoidal rule and the left and right Riemann sums were used to estimate $\int_0^2 f(x) dx$, where f is the function whose graph is shown below. The same number of subintervals were used to produce each approximation. The estimates were 0.7811, 0.8675, and 0.9543.



- Which rule produced each estimate? Justify your answer.
 - Between which two approximations does the exact value of $\int_0^2 f(x) dx$ lie? Justify your answer.
-

4.

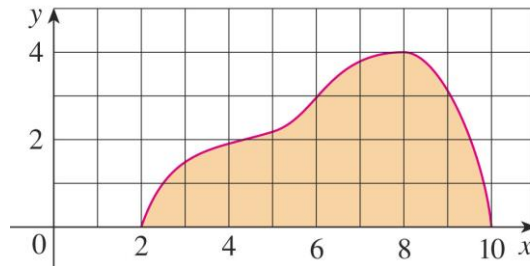
x	-5	-3	0	1	5
$f(x)$	10	7	5	8	11

Given the values for $f(x)$ on the table above, approximate the area under the graph of $f(x)$ from $x = -5$ to $x = 5$ using four subintervals and a **Trapezoidal approximation**.

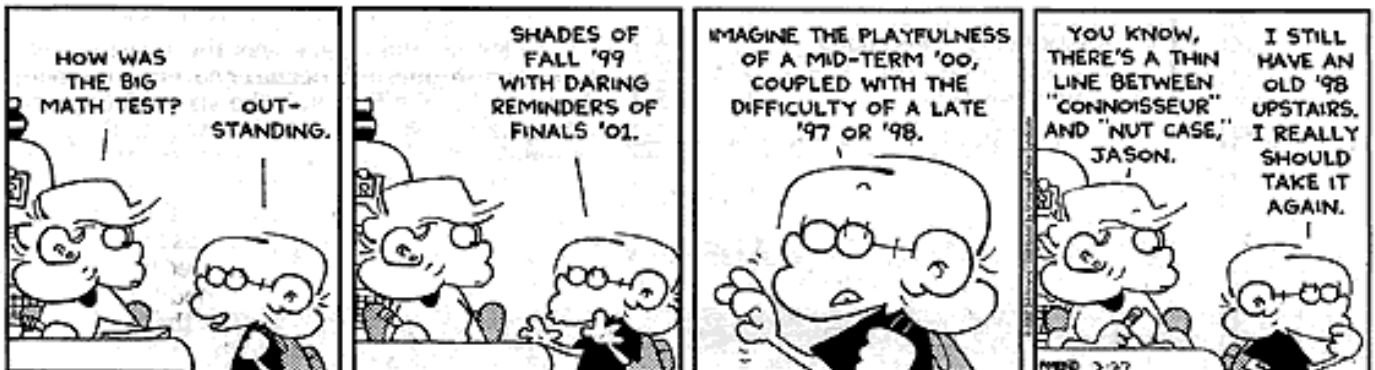
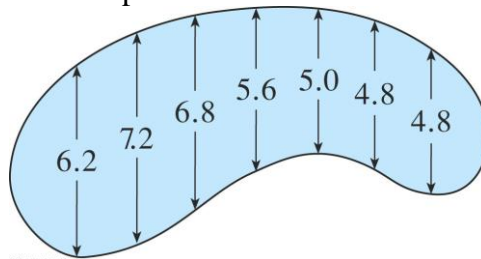
5. A radar gun was used to record the speed of a runner during the first 5 seconds of a race (see table below.) Use the Trapezoidal rule with five equal subintervals to estimate the distance covered by the runner during those 5 seconds.

t (seconds)	0	1	2	3	4	5
v (meters/second)	0	7.34	9.73	10.51	10.76	10.81

6. Estimate the area under the graph in the figure below by using the Trapezoidal rule with eight equal subintervals



7. The widths (in meters) of a kidney-shape swimming pool were measured at 2-meter intervals as indicated in the figure below. Use a trapezoidal rule to estimate the area of the pool.





Trapezoidal Rule

1.

a) Using a trapezoidal rule $\int_0^8 x^2 dx \approx \frac{8-0}{2 \cdot 4} (0 + 2(4 + 16 + 36) + 64) = 176$

b) The answer from (a) is an overestimate because the graph of $y = x^2$ is concave up (notice that $y'' = 2 > 0$.) Therefore the trapezoids yield “extra” area.

c) $\int_0^8 x^2 dx = 170 \frac{2}{3}$. The exact area is smaller than the Trapezoid approximation, as it should be.

2.

a) Using a trapezoidal rule $\int_{-1}^2 (20 - x^4) dx \approx \frac{2 - (-1)}{2 \cdot 3} (19 + 2(20 + 19) + 4) = 50.5$

b) The answer from (a) is an underestimate because the graph of $y = 20 - x^4$ is concave down (notice that $y'' = -12x^2 \leq 0$.) Therefore the trapezoids do not yield “enough” area.

c) $\int_{-1}^2 (20 - x^4) dx = 53.4$. The exact area is larger than the Trapezoid approximation, as it should be.

3.

a) Since the graph is decreasing the right Riemann sum will be a smaller approximation than the left Riemann sum. Since the graph is concave down, the trapezoidal rule will yield an overestimate, but smaller than the left Riemann sum. Therefore, 0.7811 is the right Riemann sum, 0.8675 is the Trapezoidal rule, and 0.9543 is the left Riemann sum.

b) The exact answer for $\int_0^2 f(x) dx$ must be between 0.7811, the right Riemann sum, and 0.8675, the trapezoidal rule. This is because the right Riemann sum will yield an underestimate whereas the left Riemann sum and the trapezoidal rule will yield overestimates. Of these last two, the trapezoidal rule is a closer approximation.

4.

x	-5	-3	0	1	5
$f(x)$	10	7	5	8	11

$$\int_{-5}^5 f(x) dx \approx \frac{1}{2} \cdot 2(10 + 7) + \frac{1}{2} \cdot 3(7 + 5) + \frac{1}{2} \cdot 1(5 + 8) + \frac{1}{2} \cdot 4(8 + 11) = 79.5$$

5.

t (seconds)	0	1	2	3	4	5
v (meters/second)	0	7.34	9.73	10.51	10.76	10.81

$$\int_0^5 v(t) dt \approx \frac{5-0}{2 \cdot 5} (0 + 2(7.34 + 9.73 + 10.51 + 10.76) + 10.81) = 43.745$$

6. Estimate the area under the graph in the figure below by using the Trapezoidal rule with eight equal subintervals

$$\int_2^{10} f(x) dx \approx \frac{10-2}{2 \cdot 8} (0 + 2(1.5 + 2 + 2.25 + 3 + 3.75 + 4 + 3) + 0) = 19$$

7. The widths (in meters) of a kidney-shape swimming pool were measured at 2-meter intervals as indicated in the figure below. Use a trapezoidal rule to estimate the area of the pool.

$$\text{Area} \approx \frac{1}{2} \cdot 2(0 + 2(6.2 + 7.2 + 6.8 + 5.6 + 5.0 + 4.8 + 4.8) + 0) = 80.8 \text{ square meters}$$



Trapezoidal Rule

1.

a) $\int_0^8 x^2 dx \approx 176$

b) An overestimate because the graph of $y = x^2$ is concave up (notice that $y'' = 2 > 0$.) Therefore the trapezoids yield “extra” area.

c) $\int_0^8 x^2 dx = 170\frac{2}{3}$. The exact area is smaller than the Trapezoid approximation, as it should be.

2.

a) $\int_{-1}^2 (20 - x^4) dx \approx 50.5$

b) An underestimate because the graph of $y = 20 - x^4$ is concave down (notice that $y'' = -12x^2 \leq 0$.) Therefore the trapezoids do not yield “enough” area.

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$$\int_{-5}^5 f(x) dx \approx 79.5$$

5.

t (seconds)	0	1	2	3	4	5
v (meters/second)	0	7.34	9.73	10.51	10.76	10.81

$$\int_0^5 v(t) dt \approx 43.745$$

6. Estimate the area under the graph in the figure below by using the Trapezoidal rule with eight equal subintervals

$$\int_2^{10} f(x) dx \approx 19$$

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$$\text{Area} \approx 80.8 \text{ square meters}$$
