Tajima HS　　　　Solis　　　　Pre-Calc

Summer Day 19 – 8.1.1 PROMPTS

Name ______________________________ PER________ DATE______________

DO NOW

Find the exact solution set of each equation if $0° \leq \theta < 360°$.

1. $tan^2\theta - 3 = 0$

2. $2sin^2\theta + sin\theta - 1 = 0$

Up to this point you have graphed sine and cosine curves, which have been shifted and stretched both vertically and horizontally, as well as adjusted the period. You have not yet addressed the case where there is both a horizontal shift and a horizontal stretch (change in the period) in the same function. In many applications, both of these transformations need to be applied, but which one comes first, or does it matter?

8-1. TRANSFORMATIONS OF $y = \sin(x)$

Laurel and Hardy are trying to write the equation of a sine function that has a graph with a period of $\frac{2\pi}{3}$ and a horizontal shift of $\frac{\pi}{2}$ units to the right. Laurel thinks the equation is $L(x) = \sin\left(3(x - \frac{\pi}{2})\right)$, but Hardy thinks it is $H(x) = \sin\left(3x - \frac{\pi}{2}\right)$. Who is correct? What is the difference between these two equations? How do each of these equations transform the graph of $y = \sin(x)$?

8-2. To understand why Laurel's and Hardy's equations created different graphs, think about what it means to have a horizontal shift and the order of operations.

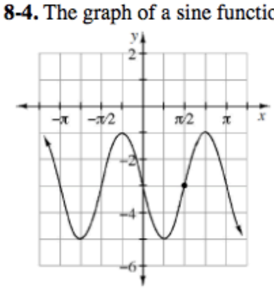
a. Examine Laurel's equation, $L(x) = \sin\left(3(x - \frac{\pi}{2})\right)$. For what value of x does $3\left(x - \frac{\pi}{2}\right) = 0$? What is the horizontal shift of this graph? Note that this is asking for when the input for sine is zero, not where sine equals zero.

b. Now consider Hardy's function, $H(x) = \sin\left(3x - \frac{\pi}{2}\right)$. For what value of x does $3x - \frac{\pi}{2} = 0$? What is the horizontal shift of this graph?

c. In Laurel's equation, the horizontal shift is easily visible. Modify Hardy's equation so that the horizontal shift is easily visible. (It will be in the same form as Laurel's.)

8-3. The graph of $y = \sin(x)$ is stretched vertically by a factor of 2, makes 3 complete cycles in an interval of 2π, is shifted up 4 units, and shifted to the right π units. Write the equation of the transformed function.

8-4. The graph of a sine function is shown below.



a. State the amplitude, horizontal shift, vertical shift, and the period.

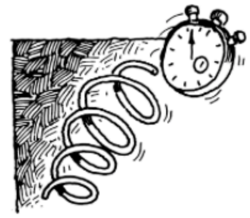
b. Write an equation using sine for this curve.

8-5. Sketch the graph of each of the following functions. Label key points with coordinates.

a. $y = 3\cos(\pi(x + 1)) - 2$

b. $y = 2\sin\left(\frac{1}{3}(x - \frac{\pi}{2})\right)$

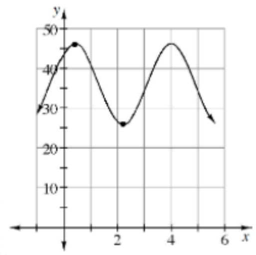
8-6. Jenny has a spring with a weight attached and she believes that she can model the motion of (ignoring damping) using a sine or cosine function. She starts a stopwatch and records the first high and low points of the weight. The first high point of 46 cm above the floor was reached when the stopwatch read 0.4 s. The next low point of 26 cm above the floor occurred when the stopwatch read 2.2 s.



a. Assume the graph is a stretched and shifted sinusoidal wave as shown. Copy the sketch and label the coordinates of the two known points on the graph.

b. State the period, amplitude, horizontal shift, and vertical shift needed to transform the graph of $y = \cos(x)$ into this one.

c. Write an equation expressing the height above the floor, h (cm), in terms of the time, t (s), on your stopwatch.

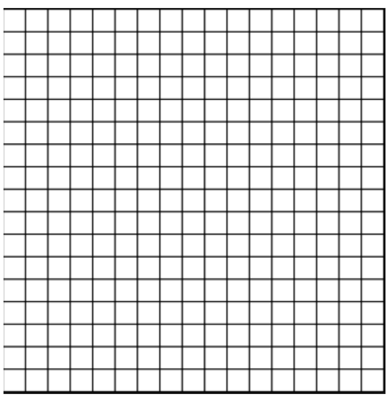


8-7. LEARNING LOG

In your Learning Log, write the general equations for transformations of $y = \sin(x)$ and $y = \cos(x)$ that include all four transformations. Include example graphs. Title this entry "Transformations of Sine and Cosine" and include today's date.

Independent Practice!

8-8. Sketch the graph of $y = 3\sin\left(\frac{\pi}{2}(x-2)\right) + 1$.

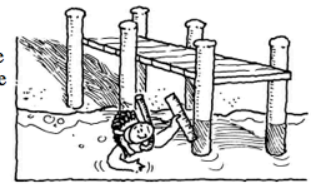


Summer Day 19 – 8.1.2 PROMPTS

Name ______________________________ PER________ DATE______________

Today you will apply what you have learned about transformations of $y = \sin(x)$ and $y = \cos(x)$ to model situations and solve problems.

8-16. Brielle and Elaine always travel to Sunset Beach together for their summer vacation. Every year they return to the same section of beach and spend their time around the pier. Elaine notices that the water rises and falls on the posts of the pier. She thinks the height of the water can be modeled using a sinusoidal function. Brielle is snorkeling and notices that the pier posts are slimy up to the lowest level of the tide, which is 2 feet for this particular post. Since Brielle and Elaine are always curious about the world around them on vacation, they have nothing better to do and find out there was a low point on the post at 8:00 a.m. and a high point 40 inches higher at 2:00 p.m.



a. Write an equation to model the height of the water relative to the post as a function of time where $t = 0$ represents 12 o'clock midnight and $h(t)$ represents the height of the water in inches.

b. At 11:30 a.m. the girls want to take a walk on the beach but decide to wait until high tide. What was the height of the water at 11:30 a.m.?

c. Brielle's little brother Sean is 2 feet 7 inches tall. When is the first time after 8:00 a.m. that the water will be "over his head" if he plays right next to the post?

8-17. Lamaj rides his bike over a piece of gum and continues riding his bike at a constant rate. At time = 1.25 seconds, the gum is at a maximum height above the ground and 1 second later the gum is on the ground again.

a. If the diameter of the wheel is 68 cm, write an equation that models the height of the gum in centimeters above the ground at any time, t, in seconds.

b. What is the height of the gum when Lamaj gets to the end of the block at $t = 15.6$ seconds?

c. When are the first and second times the gum reaches a height of 12 cm?

8-18. Amanda is watching her little brother Mike play on a swing set. She decides to model his distance above the ground using a sine or cosine function. She starts timing and finds that at $t = 2$ seconds Mike is at his highest point. He reaches his lowest point exactly 1.5 seconds later. Amanda estimates Mike's greatest height as 9 feet and his lowest height as 1 foot.

a. Write an equation that will calculate Mike's height at t seconds.

b. What is Mike's height when $t = 5.4$ seconds?

c. When are the first and second times that Mike reaches a height of 7.2 feet?

8-19. A pendulum hangs from a ceiling and swings back and forth towards a wall. Harry starts timing and at $t = 4$ seconds the pendulum is closest to the wall, 25 cm away. Three seconds later the pendulum is farthest from the wall, 83 cm away.

a. Write an equation to model the distance the pendulum is from the wall at any time t in seconds.

b. How far away from the wall is the when $t = 8$ seconds?

c. When is the first time the pendulum is 33 cm away from the wall?

8-20. A reflector on a bicycle wheel is 15 cm from the rim. The diameter of the wheel is 70 cm. At $t = 0.5$ seconds, the reflector is at its lowest point, 0.75 seconds later it returns to the same position. Assume the bicycle wheel is spinning at this constant rate.

a. Write an equation to model the height of the reflector above the ground at any time t in seconds.

b. What is the height of the reflector when $t = 5.2$ seconds?

c. When is the first time the reflector is at a height of 53 cm above the ground?

8-21. The temperature during the day in Mathamerica can be modeled by a sinusoidal function. At 4 a.m. the temperature is at a low of 65°F. At 4 p.m. the temperature hits a high of 103°F.

a. Write an equation which to model the temperature t hours after midnight.

b. What is the temperature at 11 a.m.?

c. When is the first time in the day when the temperature reaches 98°F?

8-22. The height of a piston in a cylinder can be modeled by a sinusoidal function. A piston is at its lowest point in a cylinder, 8 cm from the bottom, at $t = 3.2$ seconds. The piston is at its highest position, 39 cm from the bottom, at $t = 3.6$ seconds.

a. Write an equation to model the height of the piston, in centimeters, at any given time t in seconds.

b. What is the height of the piston 15 seconds after the engine has started?

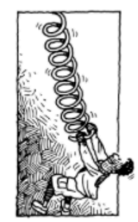
c. When is the first time the piston is 13 cm from the bottom?

8-23. Cooper Toy Company has designed a new toy that oscillates up and down and its position can be modeled by a sinusoidal curve. At time $t = 5$ seconds, the toy is at its maximum height, 18 cm above the ground. Four seconds later, the toy is at its minimum height, 6 cm above the ground.

a. Write an equation to model the height, in centimeters, of the toy at any time t, in seconds.

b. What is the height of the toy when $t = 26$ seconds?

c. When is the first time the height of the toy reaches 16 cm above the ground?

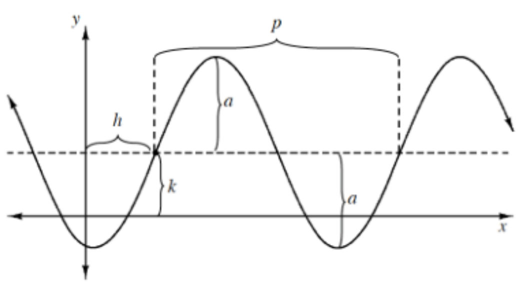


8-25. A spring is hanging from the ceiling of a room. It is pulled and released so that the distance from the floor with respect to time can be modeled using a sinusoidal function. At $t = 2.5$ seconds, the spring is at a minimum distance from the floor, 1.5 feet. At $t = 3.75$ seconds, the spring is at a maximum distance, 6 feet.

a. Write an equation to model the motion of the spring as a function of time (ignoring damping).

b. What is the distance of the spring from the floor at $t = 3$ seconds.

c. Determine the first two times when the spring is 2 feet from the floor.



The general equation for the **sine function** is ________________________

In the general equation:

A **cycle** of a periodic function is a connected piece of the graph of the function that is ____________________________

The **amplitude** (____) is ***a***.

The **period** is the _____________________________________. It is labeled ***p*** on the graph.

The number of periods in 2π is ***b***.

The ____________________ is h.

The ___________________ is ***k***. The __________________ is $y = k$.

Independent Practice!

8-26. Jade is having a wonderful time riding a carousel. Her distance, d (in feet), from the fence nearest the entrance is given by the equation $d = 25\sin\left(\frac{\pi t}{15}\right) + 35$, where t is the time in seconds since the carousel started to rotate. Homework Help ✎

a. How close can Jade be to the fence? How far?

b. How much time does it take for the carousel to complete one revolution?

c. When is Jade 60 feet from the fence for the second time?

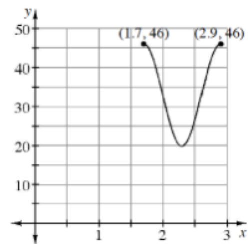
d. Sketch the graph of this situation. Label and scale the axes appropriately.

8-31. What is the average rate of change for the following functions over the given intervals? Homework Help ✎

a. $f(x) = \sin(x)$ $\frac{\pi}{2} \le x \le \pi$

b. $f(x) = \log(x)$ $1 \le x \le 10$

8-34. Lauren is playing with a yo-yo and keeping track of its height over time. She has plotted some key points on the graph at right. The y−values are in inches and the x−values are in seconds. Complete the following problems based on the graph. Homework Help ✎



a. How much time does it take to complete one cycle?

b. How long is the string?

c. Write an equation that models the height of the yo-yo over time.

d. Determine the first two times when the yo-yo is exactly 30 inches above the ground.

Summer Day 19 – 8.1.3 PROMPTS

Name ______________________________________ PER________ DATE_______________

Quick Check

8-40. Solve for x if $0 \le x < 2\pi$. Homework Help ✎

a. $4\sin^2(x) = 1$

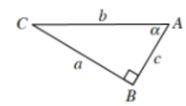
b. $3\tan^2(x) = 1$

8-41. Simplify each of the following rational expressions. Homework Help ✎

a. $\frac{4x^2-8x}{x^2-9} \cdot \frac{x^2-x-12}{2x^3-8x^2}$

b. $(x^2 - y^2) \div \frac{x^2+2xy+y^2}{4x^2+4xy}$

8-48. For ΔABC at the right, state the ratio in terms of the sides. Homework Help ✎



a. $\frac{1}{\cos(\alpha)} =$

b. $\frac{1}{\sin(\alpha)} =$

c. $\frac{1}{\tan(\alpha)} =$

So far in this course you have worked with three trigonometric functions: sine, cosine, and tangent. There are three more: **cosecant**, **secant**, and **cotangent**. They are useful for changing division to multiplication in complex expressions and in calculus to convert one trigonometric expression into another form that is easier to work with.

The three other trigonometric functions are defined as the reciprocals of the three you already know. In this lesson you will extend your understanding of graphing reciprocal functions to graphing the reciprocal trigonometric functions. The reciprocal trigonometric functions are:

$$\csc(x) = \frac{1}{\sin(x)} \qquad \sec(x) = \frac{1}{\cos(x)} \qquad \cot(x) = \frac{1}{\tan(x)}$$

8-54. Obtain a Lesson 8.2.1 Resource Page. Your team will investigate the graph of the reciprocal trigonometric function $y = \csc(x)$. Notice that the table on the resource page has three columns. Label the second column $\sin(x)$ and the third column $\csc(x)$.

a. Complete the $\sin(x)$ column of the table and make a careful graph of $y = \sin(x)$.

b. Now use the values in the $\sin(x)$ column of your table to calculate the values for $\csc(x)$. Then make a careful graph of $y = \csc(x)$.

c. Describe the graph of $y = \csc(x)$. What are its domain and range? What are the intercepts? Are there asymptotes? If so, what are their equations?

d. Describe the relationship between the graph of $y = \sin(x)$ and $y = \csc(x)$.

8-55. Use your graph or table to solve each of the following equations over the domain $0 \le x < 2\pi$. What do you notice about the solutions?

a. $\sin(x) = -\frac{1}{2}$

b. $\csc(x) = -2$

c. $\sin(x) = 1$

d. $\csc(x) = 1$

e. $\sin(x) = \frac{\sqrt{2}}{2}$

f. $\csc(x) = \sqrt{2}$

8-56. How can you use your calculator to solve $\csc(x) = 5$?

8-57. Obtain another copy of the Lesson 8.2.1 Resource Page. Use the same process as you did in problem 8−54 to create graph of $y = \sec(x)$ by first creating the table and graph of $y = \cos(x)$.

8-58. Obtain another copy of the Lesson 8.2.1 Resource Page. Use the same process as you did in problem 8−54 to create graph of $y = \cot(x)$ by first creating the table and graph of $y = \tan(x)$.

8-59. Make a list of the multiple tools you have for solving trigonometric equations. Then use an appropriate tool to solve each of the following equations over the domain $0 \le x < 2\pi$.

a. $\cot(x) = \sqrt{3}$

b. $\csc(x) = \frac{2}{\sqrt{3}}$

c. $\cot(x) = -2$

d. $\sec(x) = -3$

8-61. There are no buttons for secant, cosecant, or cotangent on a calculator. In order to evaluate these expressions, the definition of the reciprocal function needs to be utilized. Use a calculator to evaluate each of the expressions below. The first answer is given as a check. All angles are in radians. Homework Help ✎

a. $\csc(3) \approx 7.086$

b. $\cot\left(\frac{2}{7}\right)$

c. $\sec\left(\frac{4\pi}{5}\right)$