Tajima HS Solis PreCalc

Summer Day 15 – 5.2.1 PROMPTS

Name ______________________________ PER________ DATE______________

SECOND 'DO NOW'

5-127.

This problem is a checkpoint for operations with rational expressions and complex fractions. Simplify each of the following expressions. Be sure to state any necessary restrictions of the variable(s). Homework Help ✎

1 . $\frac{x^2-1}{x^2-6x-7} \div \frac{x^3+x^2-2x}{x-7}$

2.

$$\frac{\frac{1}{a}+\frac{2}{b}}{\frac{2}{a}+\frac{1}{b}}$$

TPS-C

5-25. In a previous CPM course you may have seen the ancient puzzle mathematicians first created in India in the second century BCE. Below are some clues to help you figure out how the puzzle works. Use the clues in the puzzle to evaluate each logarithmic expression below. Homework Help ✎

$\log_2(16) = 4 \quad \log_9(81) = 2 \quad \log_5(25) = 2 \quad \log(1000) = 3$

a. $\log_3(81)$

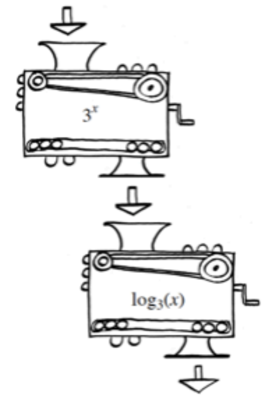
b. $\log_4(64)$

c. $\log_6\left(\frac{1}{36}\right)$

d. $\log(100)$

Today's lesson will review a very important function family, the logarithmic family (commonly called "log"). Both exponential and logarithmic functions are widely used to model situations in biology, chemistry, business, economics, and engineering. Logarithms are useful when dealing with very large values. Logarithms (the word is derived from "logical arithmetic") were developed in the late sixteenth century by the Scottish mathematician John Napier (1550 - 1617). By shortening the process of computation, logarithms served for many years to speed up the work of navigators, astronomers, and engineers.

5-37. Leoma has hooked up two function machines as shown in the diagram at right.



a. When Leoma puts 4 into the first machine, 81 comes out. Then the 81 goes into the second machine. What comes out?

b. Next Leoma puts a 2 into the first machine. What number comes out of this machine and goes into the second machine? What then comes out of the second machine?

c. If x goes into the first machine, what comes out of the second machine?

d. How are functions that are inside the function machines related?

e. Suppose the machines are reversed. If y goes into the "$\log_3(x)$" machine, a comes out. Write this as an equation.

f. If this same a goes into the "3^x" machine, what comes out? Write this as an equation.

g. How are your answers to parts (e) and (f) related?

5-38. You may recall that a **logarithm** (called a "log" for short) represents the power to which a fixed number (a base) must be raised to produce a given number. For example, $\log_2(16) = 4$ because $2^4 = 16$. The **common logarithm** is the logarithm base 10. It is expressed as $\log_{10}(x)$, but more often as $\log(x)$.

Without a calculator, evaluate each of the following logarithmic expressions. Look for and record any patterns or interesting results.

a. $\log_3(9)$

b. $\log\sqrt{10}$

c. $\log_4\left(\frac{1}{16}\right)$

d. $\log(1)$

e. $\log_7(7^5)$

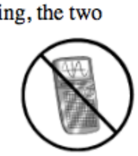
f. $2^{\log_2(16)}$

g. $\log_{0.2}(5)$

h. $10^{\log(n)}$

5-39. Another useful logarithm is the **natural logarithm**, or the logarithm base e. It is expressed as $\log_e(x)$, but more often as $\ln(x)$. When speaking, the two letters are stated separately as *"el en x"*.

Without a calculator, evaluate each of the following expressions involving the natural logarithm.



a. $\ln(1)$

b. $\ln(e)$

c. $\ln\sqrt{e}$

d. $e^{\ln(x)}$

5-40. Can a logarithm have any base? Can you take the logarithm of any number? With your team, investigate the possible values of n, m, and b in the equation below. Record your conclusions and be prepared to share your findings with the class.

$\log_b(n) = m$

5-41. Solve each of the following equations.

a. $\ln\left(\frac{3}{2}x+9\right)=1$

b. $\log(-3)=x$

c. $\log_{3x}(2187)=5x$

d. $\log_7(17)=\log_{49}(x)$

e. $6^{\log_{1/\sqrt{6}}(3)}$

TAKE NOTES!

Complete the following problems in your PRACTICE NOTEBOOK titled "Day 14 5.2.1 Practice"

5-43. Each logarithmic equation can be rewritten as an equivalent exponential equation, and vice versa. Copy each equation shown below, then rewrite it in the other form. Homework Help ✎

a. $y=7^x$

b. $\log_4(x)=y$

c. $e^y=x$

d. $W^K=B$

e. $K=\ln(B)$

f. $\log_{1/3}(P)=Q$

5-44. Without a calculator, solve each of the following logarithmic equations. Write your answers in exact form. Homework Help ✎

a. $\log_9(x)=-2$

b. $\ln(-12)=x$

c. $\ln(x)=6$

d. $\log_x(8)=4$

Summer Day 15 – 5.2.2 PROMPTS

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There are three properties of logarithms, each corresponding (not surprisingly) to a property of exponents. When logarithms were first invented, tedious multi-digit multiplication steps were replaced by table look-ups and simple addition because of the properties of logarithms. Although calculators have made logarithm tables obsolete, logarithms continue to be one of the most useful functions in mathematics.

5-51. Use graphing technology to generate the table below. Then work with your team to identify patterns within the table. Be sure to extend your table far enough to confirm your patterns. You do not need to copy the entire table by hand, but recording useful values may be helpful to identify patterns. Which values are helpful? How many patterns can you find?

x	1	2	3	4	5	6	7	8	…
$\log_2(x)$									…

5-52. Now use the table you created and/or the patterns you noticed to rewrite each expression as a single logarithm.

a. $\log_2(2) + \log_2(3)$

b. $\log_2(3) + \log_2(5)$

c. $\log_2(12) - \log_2(6)$

d. $\log_2(15) - \log_2(3)$

5-53. With your team, summarize the patterns you identified. Then explain why the patterns work. Be prepared to share your findings with the class.

5-54. Do the patterns you identified in problem 5-53 work for logarithms in any base? Answer this question by having each member of your team choose a different base and completing the following table. Choose appropriate input values for your base.

x									
$\log_b(x)$									

5-55. Now for a final property of logarithms! What is it? Add a row to each table you made, as shown below.

x	1	2	3	4	5	6	7	8	…
$\log_2(x)$									…
$\log_2(x^n)$									

x									
$\log_b(x)$									
$\log_b(x^n)$									

Each member of your team should choose a different value of n. Be sure to use a variety of types of numbers (i.e. positive, negative, and fractions).

Identify any patterns you notice. Summarize the pattern(s) and explain why the pattern(s) work(s). Be prepared to share your findings with the class.

5-56. Now use the tables you created and/or the patterns you noticed in problem 5-55 to write at least two equivalent expressions for each given expression.

a. $\log_7(x) + \log_7(x)$

b. $\ln(a) + \ln(a) + \ln(a) + \ln(a)$

c. $5\log(m)$

d. $\log_b(n) + \log_b(n) + \log_b(n)$

5-57. Mr. Cooper decides to hold a contest with his students. He gives teams the following expression and tells them they have one minute to write as many equivalent expressions as they can.

$$\log_2(8) + \log_2(8) + \log_2(8)$$

After 59 seconds Maddie's and David's teams each have six expressions, so when David quickly adds a 9 to his list Mr. Cooper declares, *"David's team wins!"* *"Mr. Cooper,"* Maddie exclaims, as she rolls her eyes, *"You didn't even check to see if all of the expressions were correct."*

Maddie's Team	David's Team
$3\log_2(8)$	$\log_2(24)$
$\log_2(512)$	$\frac{\log_2(1024)}{\log_2(2)}$
$\log_2(8^3)$	$\log_2(2^9)$
$\log_2\left(\frac{1024}{2}\right)$	$9\log_2(2)$
$\log_2(1024) - \log_2(2)$	$3\log_2(2^3)$
$3 + 3 + 3$	$\log_2(1024) - \log_2(512)$
	9

a. Whose team wins? Why?

b. Write three equivalent expressions neither team used.

c. Choose one incorrect expression and explain what misconception the students might have had.

5-58. PROPERTIES OF LOGARITHMS, PROVED!

Today you investigated three properties of logarithms.

The Product Property: $\log_b(xy) = \log_b(x) + \log_b(y)$

The Quotient Property: $\log_b\left(\frac{x}{y}\right) = \log_b(x) - \log_b(y)$

The Power Property: $\log_b(x^n) = n\log_b(x)$

a. Obtain a set of Product Property Proof cards. Each card contains a step in the proof of the Product Property of Logarithms. Organize the cards into a proof and give a reason for each step.

b. Using your proof of the Product Property as a guide, write a proof for the Quotient Property of Logarithms.

c. Prove the Power Property of Logarithms.

5-59. Can you apply the properties of logarithms to the following expression? Explain why or why not.

$$\log_3(88) - \log_4(22) + \log_5(3)$$

5-60. Use your properties of logarithms to rewrite each of the following expressions. Use a calculator to verify your answers.

a. $\log(5) + \log(7)$

b. $\ln(50) + \ln(100) - \ln(25)$

c. $\log(17^3)$

d. $\ln(4^{27}) + \ln(4)$

e. $\log_2(M) + 2\log_2(N)$

f. $\ln(a) - \ln(b) + \ln(c)$

5-61. Show that $\log_4(x) = \frac{1}{2}\log_2(x)$. Start with $y = \log_4(x)$ and rewrite the equation in exponential form.

Complete the following problems in your PRACTICE NOTEBOOK titled "Day 14 5.2.2 Practice"

5-63. Give a specific example to show that the following equation is *false*. Homework Help ✎

$$\log(M) - \log(N) = \frac{\log(M)}{\log(N)}$$

5-64. Evaluate each the following expressions without a calculator. Homework Help ✎

a. $\log(1)$

b. $\ln(1)$

c. $\log(10^3)$

d. $\ln(e^3)$

5-65. THE MEANING OF DECIMAL EXPONENTS Homework Help ✎

a. Express 0.7 as a fraction, and rewrite $10^{0.7}$ using this fraction.

b. The power property of exponents can be used to break up this fraction into two factors. Write the value of c so that $10^{0.7} = (10^c)^7$.

c. Rewrite your answer to part (b) as a root of 10 raised to a certain power by copying and filling in the blanks of $(\sqrt[\square]{10})^{\square}$.

d. Why is it generally better to take the root first, especially when you are working without a calculator?

e. Use a calculator to evaluate your expression from part (c).

f. Now calculate $10^{0.7}$. How does this answer compare with the previous one?

g. Reshma notices that the answer for $10^{0.7}$ is close to 5. Kahlil knows she can get a value closer to 5 by using more decimal places in the exponent. Use guess and check to determine p (to the nearest 0.001) so that 10^p is as close to 5 as possible.

h. In a flash of brilliance, Reshma suddenly knows how to get several more decimal places instantly. What keys can she press on her calculator to do this?

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Summer Day 15 – 5.2.3 PROMPTS

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How can the relationship between logarithms and exponents be use to solve equations? This lesson will give you practice with solving equations with exponents and logarithms.

5-79. Eduardo thinks he can solve $6^x = 20$ in just *one* step! He says the solution is $x = \log_6(20)$.

a. Is Eduardo correct? Explain why or why not.

b. Is Eduardo's solution practical? Explain.

c. Anita says she knows how to solve $6^x = 20$ using logarithms in another way. She tells Eduardo to start by taking the log (base 10) of both sides and then apply the Power Property of Logarithms. Use Anita's method to solve for x.

d. Eduardo's older brother Lemuel, who is taking Calculus, looks at Eduardo's equation and says, *"Just take the natural log of both sides."* Try Lemuel's method. Does it work?

e. If you have not done so already, use a calculator to get a numerical answer for each solution. Are all of your solutions equivalent?

5-80. Solve each of the following equations using a method of your choice. Begin by estimating a solution. Then solve the equation and give both an exact answer and an approximate answer. Be ready to share your strategies with the class.

a. $1.05^x = 2$

b. $15(3)^x = -6$

c. $-12(10)^x + 3 = -3$

5-81. Consider Anita's method from the problem 5-79.

a. Use her method to evaluate $\log_3(11)$. Start by letting $\log_3(11) = x$, then rewriting the equation in exponential form. Continue by using log base 10 to solve the equation.

b. Use your steps from part (a) to write an equivalent expression for $\log_b(a)$. This is the **change of base formula**.

c. The change of base formula allows you to rewrite any logarithm in any base. This is useful if your calculator only allows base 10 or the natural logarithm. Rewrite $\log_7(204)$ in two ways. Half of your team should use log base 10 while the other half of your team uses the natural logarithm. Evaluate your expressions using a calculator. Do your answers agree with your teammates' answers?

5-82. SOLVING LOGARITHMIC EQUATIONS

Use what you have learned about the relationship between exponents and logarithms and the properties of logarithms to solve each of the following equations. Give exact solutions.

a. $\log_7(x^2) = \log_7(8x - 15)$

b. $\log_2(x^3) + \log_2(x) - \log_2(2x) = 6$

c. $\log_7(x - 4) + \log_7(x + 2) = 1$

d. $3\ln(x) = \ln(e^5) - 2$

e. $-9\ln(x + 1) = -8$

f. $\ln(x + 8) - \ln(x - 4) = 10$