AP Calculus

REVIEW FOR 3rd QUARTER MIDTERM

Name_______________________________

Seat # ________ Date ____________________

Slope Fields, FTC, and L'Hopital's Rule

L'Hopital's Rule

Evaluate each limit:

1. $\lim_{x\to 3} \frac{2x-6}{x^2-9}$

2. $\lim_{x\to \infty} \frac{5x^2+3x-1}{3x^2-7}$

3. $\lim_{x\to 3} \frac{\sqrt{x+1}-2}{x-3}$

4. $\lim_{x\to 0} \frac{\sin 2x}{\sin 3x}$

5. $\lim_{x\to +\infty} \frac{e^x}{x^3}$

6. $\lim_{x\to -\infty} \frac{e^x}{x^3}$

Fundamental Theorem of Calculus, Part 2:

1. $\frac{d}{dx}\left(\int_3^x \ln t \cdot dt\right) =$

2. $\frac{d}{dx}\left(\int_{-2}^{4x^2} \tan^3 t \cdot dt\right) =$

3. $\frac{d}{dx}\left(\int_a^{f(x)} g(t) \cdot dt\right) =$

4. $\frac{d}{dx}\left(\int_{-x}^{6x} t\cos t \cdot dt\right) =$

5. $\frac{d}{dx}\left(\int_{x^2}^{x^3} \sqrt{t^4+1} \cdot dt\right) =$

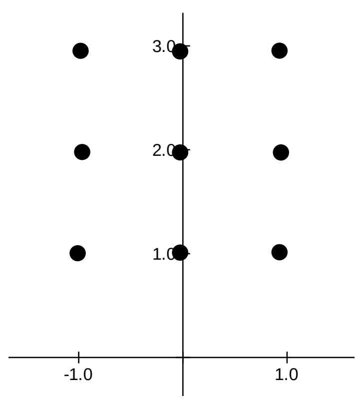
6. $\frac{d}{dx}\left(\int_0^{\frac{\pi}{3}} \sin t \cdot dt\right) =$

Slope Fields

ESSAY ADAPTED FROM 1998 BC EXAM: (GRAPHING CALCULATOR MAY BE USED)

1. Consider the differential equation given by $\frac{dy}{dx} = \frac{xy}{2}$.

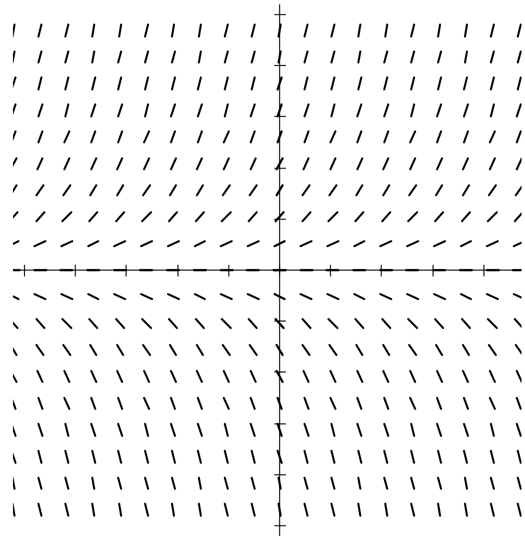
a) On the axes provided below, sketch a slope field for the given differential equation at the nine points indicated.



b) Let $y = f(x)$ be the particular solution to the given differential equation with the initial condition $f(0) = 3$. Sketch the graph of $f(x)$.

c) Find the particular solution $y = f(x)$ to the given differential equation with the initial condition $f(0) = 3$. Use your solution to find $f(0.2)$.

2. The slope field for a differential equation is shown at the right. Which statement is true for solutions of the differential equation?



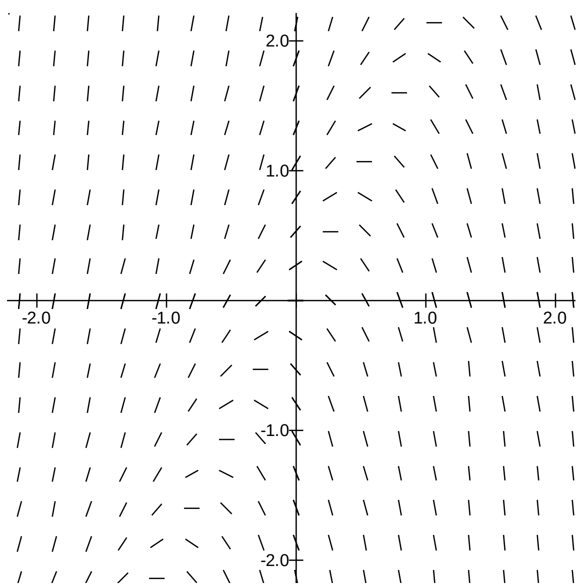
I. For $x < 0$ all solutions are decreasing.
II. All solutions level off near the x-axis.
III. For $y > 0$ all solutions are increasing.

a) I only
b) II only
c) III only
d) II and III only
e) I, II, and III

ESSAY ADAPTED FROM 2002 BC EXAM: (NO CALCULATOR ALLOWED)

3. Consider the differential equation $\frac{dy}{dx} = 2y - 4x$.

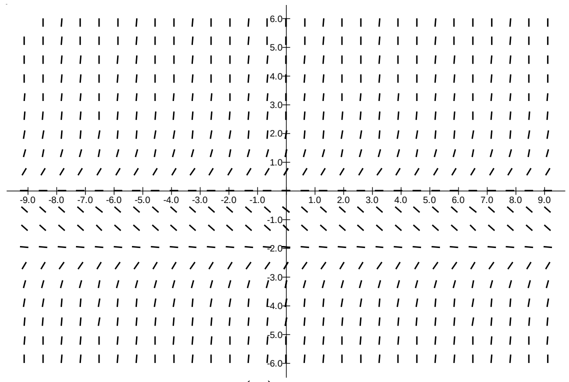
a) The slope field for the given differential equation is provided. Sketch the solution curve that passes through the point (0, 1) and sketch solution curve that passes through the point (0, –1).



b) Find the value of b for which $y = 2x + b$ is a solution to the given differential equation. Justify your answer.

c) Let g be the function that satisfies the given differential equation with the initial condition $g(0) = 0$. Does the graph of g have a local extremum at the point (0, 0)? If so, is the point a local maximum or a local minimum? Justify your answer.

4.



Which statement is true about the solutions, $y(x)$, of a differential equation whose slope field is shown above?

I. If $y(0) > 0$, then $\lim_{x\to\infty} y(x) \approx 0$

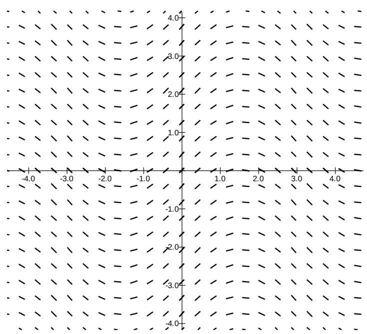
II. If $-2 < y(0) < 0$, then $\lim_{x\to\infty} y(x) \approx -2$

III. If $y(0) < -2$, then $\lim_{x\to\infty} y(x) \approx -2$

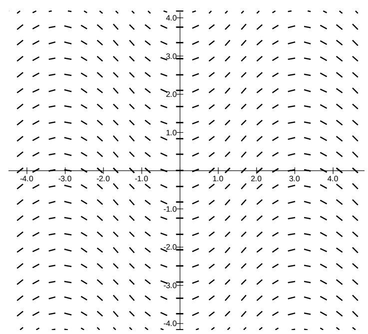
(A) I only (B) II only (C) III only
(D) II and III only (E) I, II, and III

5. Which choice represents the slope field for $\frac{dy}{dx} = \cos x$?

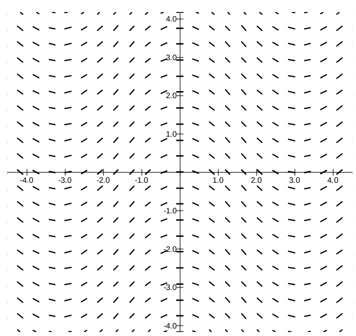
(A)



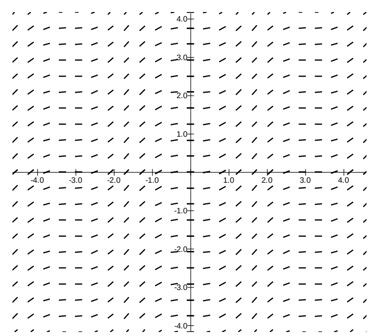
(B)



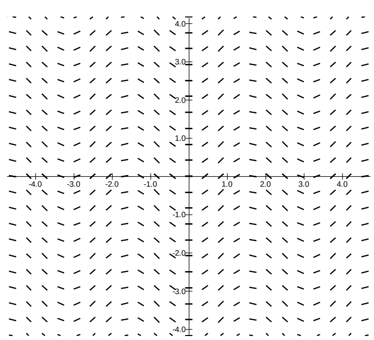
(C)



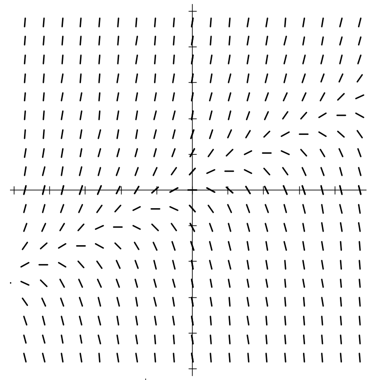
(D)



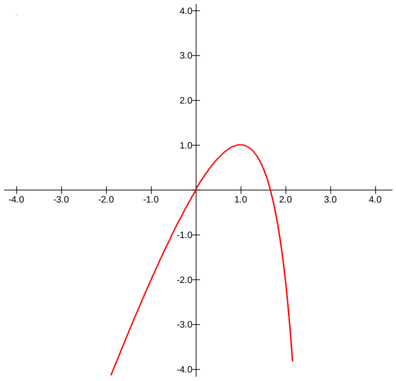
(E)



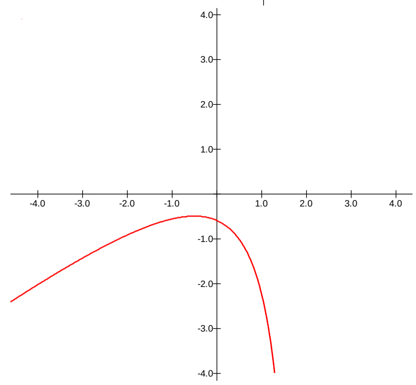
6. Which one of the following could be the graph of the solution of the differential equation whose slope field is shown to the right?



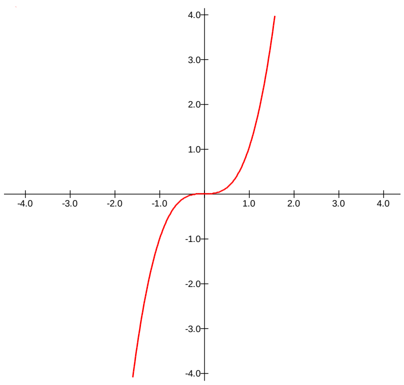
(A)



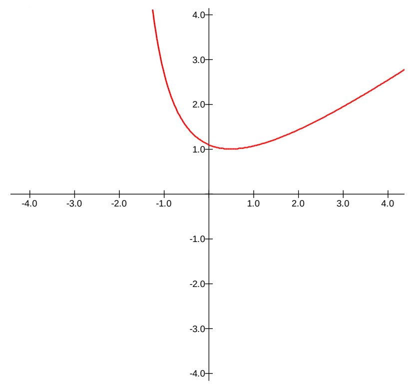
(B)



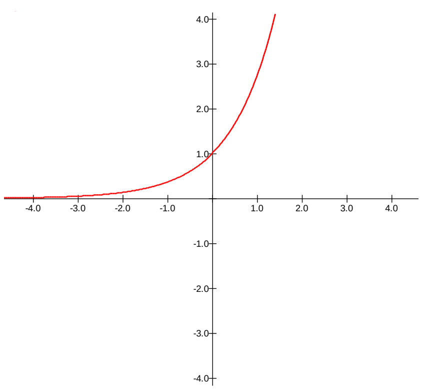
(C)



(D)



(E)



7. The slope field for the differential equation $\dfrac{dy}{dx} = \dfrac{x^2 y + y^2}{4x + 2y}$ will have vertical segments when

(A) $y = 2x$, only (B) $y = -2x$, only (C) $y = -x^2$, only

(D) $y = 0$, only (E) $y = 0$ or $y = -x^2$,

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ANSWER KEY

Slope Fields, FTC, and L'Hopital's Rule

L'Hopital's Rule

Evaluate each limit:

1. $\lim_{x\to 3}\frac{2x-6}{x^2-9}=\frac{1}{3}$

2. $\lim_{x\to\infty}\frac{5x^2+3x-1}{3x^2-7}=\frac{5}{3}$

3. $\lim_{x\to 3}\frac{\sqrt{x+1}-2}{x-3}=\frac{1}{4}$

4. $\lim_{x\to 0}\frac{\sin 2x}{\sin 3x}=\frac{2}{3}$

5. $\lim_{x\to +\infty}\frac{e^x}{x^3}=+\infty$

6. $\lim_{x\to -\infty}\frac{e^x}{x^3}=0$

Fundamental Theorem of Calculus, Part 2:

1. $\ln x$

2. $8x\tan^3\left(4x^2\right)$

3. $g\left(f(x)\right)\cdot f'(x)$

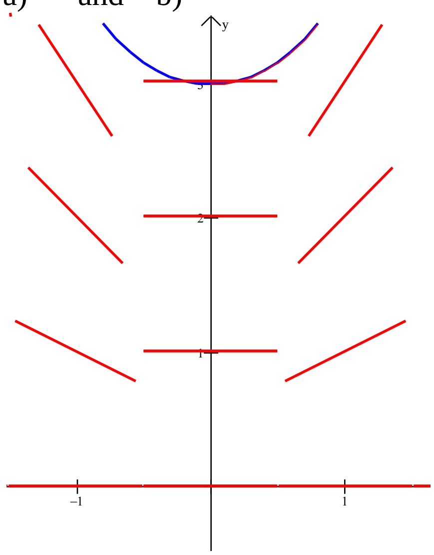
4. $36x\cos 6x - x\cos(-x)$
   or
   $36x\cos 6x - x\cos x$

5. $3x^2\sqrt{x^{12}+1}-2x\sqrt{x^8+1}$

6. 0

Slope Fields

1. a) and b)



c) $\frac{dy}{dx}=\frac{xy}{2}\Rightarrow\frac{1}{y}dy=\frac{x}{2}dx\Rightarrow\int\frac{1}{y}dy=\int\frac{x}{2}dx\Rightarrow\ln|y|=\frac{x^2}{4}+C$

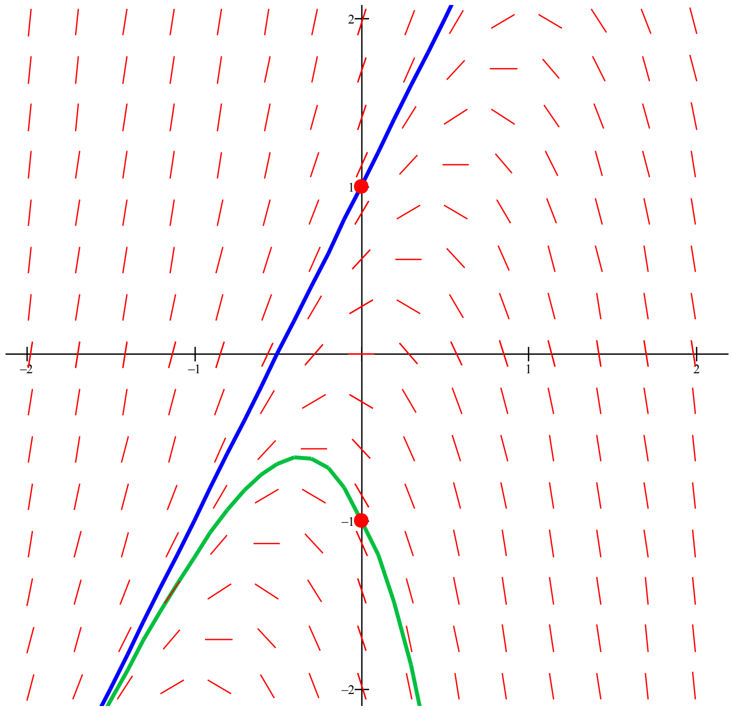
$y=Ae^{x^2/4}\Rightarrow$ at $(0,3)\Rightarrow 3=Ae^0=A\Rightarrow y=3e^{x^2/4}$

Therefore: $f(0.2)=3e^{0.04/4}=3.030$

2. I. False
   II. True
   III. True
   (D)

3.

a)



b) Substitute $y = 2x + b$ and $y' = 2$ into the differential equation:

$$\frac{dy}{dx} = 2y - 4x \Rightarrow 2 = 2\cdot(2x + b) - 4x \Rightarrow 2 = 2b \Rightarrow b = 1$$

c) At (0, 0) we have $g' = \frac{dy}{dx} = 2\cdot(0) - 4\cdot(0) = 0 \Rightarrow$ possible local extrema

Find $g'' \Rightarrow g'' = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2y - 4x) = 2\frac{dy}{dx} - 4$

At (0, 0) we have $g'' = 2\cdot(0) - 4 = -4 \Rightarrow g$ is concave down at (0, 0)

Therefore, there is a local maximum at (0, 0).

4. I. False
II. True
III. True
(D)

5. (A)

6. (B)

7. (B)