UNIT 7 – Parametric, Vector and Polar Functions

Name Exempler PER____ DATE______________

CALC 31.0	CALC 32.0

Computation

4	3	2	1
Response has no recall errors, *minimal* procedural errors* and no conceptual errors**	**Response has no recall errors, minimal procedural errors and *minimal* conceptual errors**	**Response has no recall errors, but has several procedural errors OR several conceptual errors**	**Recall errors exist OR Steps taken are not related to problem OR Response left blank**

Written Responses

4	3	2	1
Response is written in a complete sentence and uses appropriate academic vocab	**Response is written in a complete sentence, and minimal errors exist in use of academic vocab**	**Response is not written in a complete sentence OR no academic vocab**	**Concept of response is not related to problem OR Response is left blank**

***Procedural errors** are mistakes made in the math
****Conceptual errors** are mistakes made in the steps one take

1. **(CALC 31.0)** *Show your work neatly and circle the correct solution.*

The position of a particle moving in the xy-plane is given by the parametric equations $x(t) = t^3 - 3t^2$ and $y(t) = 12t - 3t^2$. At which of the following points (x, y) is the particle at rest? ← $dy/dx = 0$ → $\frac{dy}{dt} = 0$

(A) $(-4, 12)$ (B) $(-3, 6)$ (C) $(-2, 9)$ (D) $(0, 0)$ (E) $(3, 4)$

$\frac{dy}{dt} = 12 - 6t$

$0 = 12 - 6t$

$2 = t$

$x(2) = 2^3 - 3(2)^2$
$= 8 - 12$
$= -4$

$y(2) = 12(2) - 3(2)^2 = 24 - 12 = 12$

2. **(CALC 31.0)** *Show your work neatly and circle the correct solution.*

Which of the following integrals gives the length of the curve $y = \ln x$ from $x = 1$ to $x = 2$? boundaries

(A) $\int_1^2 \sqrt{1 + \frac{1}{x^2}}\, dx$

no square root → ~~(B)~~ $\int_1^2 \left(1 + \frac{1}{x^2}\right) dx$

~~(C)~~ $\int_1^2 \sqrt{1 + e^{2x}}\, dx$

didn't derive y ~~(D)~~ $\int_1^2 \sqrt{1 + (\ln x)^2}\, dx$

~~(E)~~ $\int_1^2 \left(1 + (\ln x)^2\right) dx$

***HINT**: Let x = t*

$x = t$ $\quad y = \ln t$

$\frac{dx}{dt} = 1$ $\quad \frac{dy}{dt} = \frac{1}{t}$

ARC length $= \int_1^2 \sqrt{(1)^2 + \left(\frac{1}{t}\right)^2}\, dt$

3. **(32.0)** *Show your work neatly and circle the correct solution.*

$r' = 2\cos\theta$

What is the slope of the line tangent to the polar curve $r = 1 + 2\sin\theta$ at $\theta = 0$?

(A) 2 (B) $\frac{1}{2}$ (C) 0 (D) $-\frac{1}{2}$ (E) -2

product rule twice

$$\frac{dy}{dx} = \frac{y'_\theta}{x'_\theta} = \frac{(r\sin\theta)'}{(r\cos\theta)'} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}$$

$$= \frac{2\cos\theta\sin\theta + (1+2\sin\theta)\cos\theta}{2\cos\theta\cos\theta - (1+2\sin\theta)\sin\theta}$$

plug in $\theta = 0$

$$\frac{2\cos 0\cancel{\sin 0} - (1+2\sin 0)\cos 0}{2\cos 0\cos 0 - (1+\cancel{2\sin 0})\sin 0} = \frac{1\cdot 1}{2\cdot 1\cdot 1}$$

$\boxed{\frac{1}{2}}$

4. **(CALC 32.0)** *Show your work neatly and circle the correct solution.*

Find the slope of the polar curve at the indicated point.

$r = 3 + 6\cos\theta, \theta = \frac{\pi}{2}$

A) -2 B) $\frac{1}{2}$ C) 2 D) $-\frac{1}{2}$

as seen above

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta} = \frac{-6\sin\theta(\sin\theta) + (3+6\cos\theta)\cos\theta}{-6\sin\theta(\cos\theta) - (3+6\cos\theta)\sin\theta}$$

$r' = -6\sin\theta$

plug in $\theta = \frac{\pi}{2}$

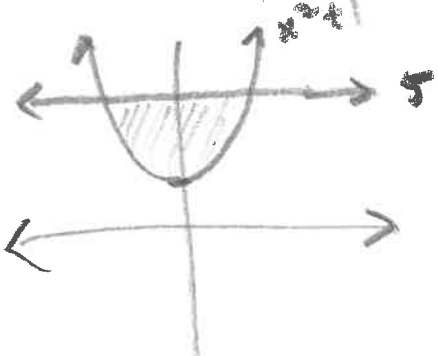
$$\frac{-6\sin\frac{\pi}{2}(\sin\frac{\pi}{2}) + (3+6\cancel{\cos\frac{\pi}{2}})\cos\frac{\pi}{2}}{-6\sin\frac{\pi}{2}\cancel{(\cos\frac{\pi}{2})} - (3+6\cos\frac{\pi}{2})\sin\frac{\pi}{2}}$$

$$= \frac{-6(1)(1)}{-(3+0)\cdot 1} = \frac{-6}{-3} = \boxed{2}$$

5. **AB Review (CALC 16.0)** *Show your work neatly and circle the correct solution.*

The area of the region enclosed by the graph of $y = x^2 + 1$ and the line $y = 5$ is

(A) $\frac{14}{3}$ (B) $\frac{16}{3}$ (C) $\frac{28}{3}$ (D) $\frac{32}{3}$ (E) 8π



① $5 = x^2 + 1$
$-5 \quad -5$
$0 = x^2 - 4$
$0 = (x-2)(x+2)$
$x = 2 \quad x = -2$

② $\int_{-2}^{2} 5 - x^2 - 1 \, dx$

③ $5x - \frac{x^3}{3} - x \Big|_{-2}^{2} =$

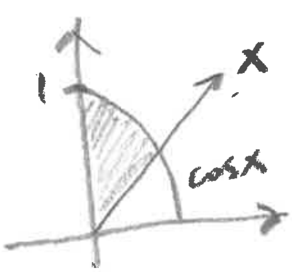
④ $\left(5(2) - \frac{(2)^3}{3} - (2)\right) -$
$\left(5(-2) - \frac{(-2)^3}{3} - (-2)\right) =$

⑤ $20 - 16/3 - 4 = 16 - \frac{16}{3} = \frac{32}{3}$

6. **AB Review (CALC 16.0)** *Show your work neatly and circle the correct solution.*

What is the area of the region in the first quadrant enclosed by the graphs of $y = \cos x$, $y = x$, and the y-axis?

(A) 0.127 (B) 0.385 (C) 0.400 (D) 0.600 (E) 0.947



$\cos x = x$
Use calculator to see intersection @ $x = 0.739$

$\int_0^{\boxed{0.739}} \cos x - x \, dx = 0.4004$

"MATH + 9"

7. **AB Review (CALC 6.0)** *Show your work neatly and circle the correct solution.*

Not parametric!

A particle moves along the x-axis so that its position at time t is given by $x(t) = t^2 - 6t + 5$. For what value of t is the velocity of the particle zero? $\rightarrow \frac{dx}{dt} = 0$

(A) 1 (B) 2 (C) 3 (D) 4 (E) 5

$\frac{dx}{dt} = 2t - 6$

$0 = 2t - 6$

$6 = 2t$

$3 = t$

8. **AB Review (CALC 6.0)** *Show your work neatly and circle the correct solution.*

The maximum acceleration attained on the interval $0 \le t \le 3$ by the particle whose velocity is given by $v(t) = t^3 - 3t^2 + 12t + 4$ is

(A) 9 (B) 12 (C) 14 (D) 21 (E) 40

$v(t) = t^3 - 3t^2 + 12t + 4$

$a(t) = 3t^2 - 6t + 12 \longrightarrow a'(t) = 6t - 6$

$0 = 6t - 6$

$6 = 6t$

$1 = t$

	.5	1	2
$a'(t)$	−		+

Need to set derivative of $a(t)$ equal to 0 and make wiggle chart!

$t = 1$ is a minimum so test endpoints!

$a(0) = 12$

$a(3) = 21$ ✓✓