UNIT 6 – Testing Series for Convergence Assessment

Name Exemplar PER______ DATE__________

CALC 23.0	CALC 24.0

Computation

4	3	2	1
Response has no recall errors, *minimal* procedural errors* and no conceptual errors**	**Response has no recall errors, minimal procedural errors and *minimal* conceptual errors**	**Response has no recall errors, but has several procedural errors <u>OR</u> several conceptual errors**	**Recall errors exist <u>OR</u> Steps taken are not related to problem <u>OR</u> Response left blank**

Written Responses

4	3	2	1
Response is written in a complete sentence and uses appropriate academic vocab	**Response is written in a complete sentence, and minimal errors exist in use of academic vocab**	**Response is not written in a complete sentence <u>OR</u> no academic vocab**	**Concept of response is not related to problem <u>OR</u> Response is left blank**

***Procedural errors** are mistakes made in the math
****Conceptual errors** are mistakes made in the steps one take

1. **(CALC 23.0) Show your work and circle the correct answer CLEARLY. Note: You must test at least three options.**

Determine which of the following series converge.

(a) $\sum_{n=1}^{\infty} \frac{1}{n}$ (b) $\sum_{n=0}^{\infty} 3\left(\frac{4}{3}\right)^n$ (c) $\sum_{n=0}^{\infty} \frac{(n+1)!}{2^n}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n^{3/2}}$ (e) None of these

(a) p series

$p=1$

This diverges

(b) Geometric

$r = \frac{4}{3} > 1$

This diverges

(d) p series

$p = 3/2 > 1$

This converges

2. **(CALC 23.0) Show your work and circle the correct answer CLEARLY. Note: You must test at least three options.**

. Determine which of the following series diverge.

(a) $\sum_{n=0}^{\infty} \frac{1^n}{2^n}$ (b) $\sum_{n=1}^{\infty} (4+(-1)^n)$ (c) $\sum_{n=0}^{\infty} \frac{(-1)^n}{(n+1)!}$ (d) $\sum_{n=1}^{\infty} \frac{1}{n^2}$ (e) None of these

(a) $\downarrow$

$\left(\frac{1}{2}\right)^n$

Geometric

$r = \frac{1}{2} < 1$

This converges!

(b)

n	Σ
1	3
2	5
3	3
4	5
$\vdots$	

This diverges!

(d) p-series

$p = 2 > 1$

This converges!

3. **(CALC 23.0) Show your work and circle the correct answer CLEARLY.**

Determine if $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2}$ ← Alternator is convergent or divergent. If convergent, classify the series as absolutely convergent or conditionally convergent.

(a) Divergent (b) Conditionally Convergent (c) Absolutely Convergent (d) None of these

① $\lim_{n\to\infty} \frac{1}{n^2} = 0$ ✓

② $\frac{1}{n^2} > \frac{1}{(n+1)^2}$ so $\frac{1}{1^2} > \frac{1}{2^2} > \frac{1}{3^2} \dots$

Terms are decreasing so converges by AST

$\sum_{n=1}^{\infty} \frac{1}{n^2}$ p series

$p = 2 > 1$

Converges!

4. **(CALC 24.0) Show your work and box your final answer.**

Determine the interval, including possible endpoints where the following series converges, if it does.

$$\sum_{n=1}^{\infty} \frac{3^{n-1}(x-3)^n}{n \cdot 2^n}$$

$$\lim_{n\to\infty} \left| \frac{3^{\cancel{n+1-1}\,1}(x-3)^{\cancel{n}+1}}{(n+1)2^{\cancel{n}+1}} \cdot \frac{n\cancel{2^n}}{\cancel{3^{n-1}}\cancel{(x-3)^n}} \right|$$

$$= \lim_{n\to\infty} \left| \frac{3(x-3)n}{(n+1)2} \right| \text{ or } \lim_{n\to\infty} \left| \frac{3(x-3)}{2} \right| \left| \frac{n}{(n+1)} \right| = \frac{3(x-3)}{2}$$

$$-1 < \frac{3(x-3)}{2} < 1$$

$$-2 < 3(x-3) < 2$$

$$-\frac{2}{3} < x-3 < \frac{2}{3} \rightarrow \boxed{\frac{7}{3} \leq x < \frac{11}{3}}$$

$x = \frac{11}{3}$

$$\sum^{\infty} \frac{3^{n-1}\left(\frac{2}{3}\right)^n}{n2^n} \rightarrow \frac{\cancel{3^n}\,3^{-1}\,\cancel{2^n}}{n\cancel{2^n}\cancel{3^n}} = \frac{1}{3n}$$

diverges by p-series!

$x = \frac{7}{3}$

$$\sum^{\infty} \frac{3^{n-1}\left(\frac{-2}{3}\right)^n}{n2^n} \rightarrow \frac{\cancel{3^n} \cdot 3^{-1}\,\overset{(-1)^n}{\cancel{(-2)^n}}}{n \cdot \cancel{2^n}\,\cancel{3^n}} = \frac{(-1)^n}{3n}$$

Converges by AST!

5. (CALC 24.0) Show your work and box your final answer.

Find the interval of convergence including endpoints for the following:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n+1}(x-5)^n}{n \cdot 5^n}$$

$$\lim_{n\to\infty} \left| \frac{-1^{n+1+1}(x-\cancel{5})^{n+1}}{(n+1)\, 5^{n+1}} \cdot \frac{n\cancel{5^n}}{\cancel{-1^{n+1}}(x-\cancel{5})^n} \right|$$

$$= \lim_{n\to\infty} \left| \frac{-1\,(x-5)n}{5\,(n+1)} \right| \Rightarrow \left| \frac{(x-5)}{5} \right| \left| \frac{n}{n+1} \right|$$

$$= \frac{x-5}{5}$$

$$-1 < \frac{x-5}{5} < 1$$

$$-5 < x-5 < 5$$

$$\boxed{0 < x \leq 10}$$

$x=0$

$$\sum_{n=1}^{\infty} \frac{-1^{n+1}(-\cancel{5})^n}{n\cancel{5^n}} \Rightarrow \frac{-1^{n+1}(-1)^n}{n} \Rightarrow \frac{1}{n}$$

diverges by p-series

$x=10$

$$\sum_{n=1}^{\infty} \frac{-1^{n+1}(\cancel{5})^n}{n\cancel{5^n}}$$

converges by AST!