Tajima HS Solis AP CALC BC

UNIT 5 – Infinite Series Assessment

Name Exemplar PER_______ DATE_____________

23.0	25.0	26.0

Computation

4	3	2	1
Response has no recall errors, *minimal* procedural errors* and no conceptual errors**	**Response has no recall errors, minimal procedural errors and *minimal* conceptual errors**	**Response has no recall errors, but has several procedural errors OR several conceptual errors**	**Recall errors exist OR Steps taken are not related to problem OR Response left blank**

Written Responses

4	3	2	1
Response is written in a complete sentence and uses appropriate academic vocab	**Response is written in a complete sentence, and minimal errors exist in use of academic vocab**	**Response is not written in a complete sentence OR no academic vocab**	**Concept of response is not related to problem OR Response is left blank**

***Procedural errors** are mistakes made in the math
****Conceptual errors** are mistakes made in the steps one take

NOTE: Please put all notes, iPads and textbooks away. You're only allowed to use the hint sheet on the back.


Los Angeles Times

CALIFORNIA

Report on LAPD gang officers a mixed bag

Getting into college is hard. A degree? Harder

States' drought

1. **(CALC 23.0)** *Show your work in the space below and circle your answer.*

3. What is the value of $\sum_{n=1}^{\infty} \frac{(-3)^{n+1}}{5^n}$?

(A) $-\frac{15}{8}$ (B) $-\frac{9}{8}$ (C) $-\frac{3}{8}$ (D) $\frac{9}{8}$ (E) $\frac{15}{8}$

$\frac{(-3)^{n+1}}{5^n} = \frac{-3^1(-3)^n}{5^n} = -3\left(-\frac{3}{5}\right)^n$ Geometric! $\left|-\frac{3}{5}\right| < 1$ converges ✓

$S = \frac{a_1}{1-r} = \frac{-3\left(-\frac{3}{5}\right)^1}{1 - -\frac{3}{5}} = \frac{\frac{9}{5}}{1+\frac{3}{5}} = \frac{\frac{9}{5}}{\frac{8}{5}} = \frac{9}{\cancel{5}} \cdot \frac{\cancel{5}}{8} = \frac{9}{8}$

2. **(CALC 23.0)** *Show your work in the space below and circle your answer.*

If $f(x) = \sum_{k=1}^{\infty} (\sin^2 x)^k$, then $f(1)$ is

(A) 0.369 (B) 0.585 (C) 2.400 (D) 2.426 (E) 3.426

$f(1) = \sum_{k=1}^{\infty} (\sin^2(1))^k$

$= \sum_{k=1}^{\infty} (0.708)^k$ Geometric! $|0.708| < 1$ converges

$S = \frac{0.708}{1-0.708} = \frac{0.708}{0.292} = 2.425$

3. (CALC 25.0) *Show your work in the space below and circle your answer.*

The Taylor series for $\sin x$ about $x=0$ is $x-\frac{x^3}{3!}+\frac{x^5}{5!}-\dots$. If f is a function such that $f'(x)=\sin(x^2)$, then the coefficient of x^7 in the Taylor series for $f(x)$ about $x=0$ is

(A) $\frac{1}{7!}$ (B) $\frac{1}{7}$ (C) 0 (D) $-\frac{1}{42}$ (E) $-\frac{1}{7!}$

We're given $f'(x)$ and asked about $f(x)$

$\sin(x) = x - \frac{x^3}{3!} + \frac{x^5}{5!}$

$f'(x) = \sin(x^2) = (x^2) - \frac{(x^2)^3}{3!} + \frac{(x^2)^5}{5!} - \dots$

$f'(x) = x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots$

$\int f'(x) = \int x^2 - \frac{x^6}{3!} + \frac{x^{10}}{5!} - \dots$

$f(x) = \frac{x^3}{3} - \frac{x^7}{7\cdot 3!} + \frac{x^{11}}{11\cdot 5!} - \dots$

Coefficent is $\frac{-1}{7\cdot 3!} = \frac{-1}{42}$

4. (CALC 25.0) *Show your work in the space below and box your answers.*

Which of the following is the Maclaurin series for $\frac{1}{(1-x)^2}$?

(A) $1-x+x^2-x^3+\cdots$

(B) $1-2x+3x^2-4x^3+\cdots$

(C) $1+2x+3x^2+4x^3+\cdots$

(D) $1+x^2+x^4+x^6+\cdots$

(E) $x+\frac{x^2}{2}+\frac{x^3}{3}+\frac{x^4}{4}+\cdots$

Yes! Not D, again!

If $f(x) = \frac{1}{1-x}$,

$f'(x) = \frac{1}{(1-x)^2}$

Prof

$f'(x) = \frac{\cancel{(1-x)(0)} - (1)(-1)}{(1-x)^2}$

so $f(x) = 1 + x + x^2 + x^3 + \dots$

$f'(x) = 0 + 1 + 2x + 3x^2 + \dots$

4th derivative!

5. (CALC 26.0) *Show your work in the space below and circle your answer.*

3RD degree!

The function f has derivatives of all orders for all real numbers, and $f^{(4)}(x) = e^{\sin x}$. If the third-degree Taylor polynomial for f about $x = 0$ is used to approximate f on the interval $[0, 1]$, what is the Lagrange error bound for the maximum error on the interval $[0, 1]$?

(A) 0.019 (B) 0.097 (C) 0.113 (D) 0.399 (E) 0.417

$$R_3(x) = \left| \frac{|f^4(x)|(1-0)^4}{4!} \right|$$

$$e^{\sin 0} = e^0 = 1$$

$$e^{\sin 1} = 2.3198$$

$$< \frac{2.3198(1)^4}{24} = 0.097$$

6. (CALC 26.0) Show your work and box your answer(s).

The function f has derivatives of all orders for all real numbers x. Assume $f(2) = -3$, $f'(2) = 5$, $f''(2) = 3$, and $f'''(2) = -8$.

(a) Write the third-degree Taylor polynomial for f about $x = 2$ and use it to approximate $f(1.5)$.

$$f(x) = f(2) + \frac{f'(2)(x-2)}{1!} + \frac{f''(2)(x-2)^2}{2!} + \frac{f'''(2)(x-2)^3}{3!}$$

$$\boxed{f(x) = -3 + 5(x-2) + \frac{3}{2}(x-2)^2 - \frac{8}{6}(x-2)^3}$$

$$f(1.5) = -3 + 5(1.5-2) + \frac{3}{2}(1.5-2)^2 - \frac{8}{6}(1.5-2)^3 = -4.958$$

(b) The fourth derivative of f satisfies the inequality $|f^{(4)}(x)| \leq 3$ for all x in the closed interval $[1.5, 2]$. Use the Lagrange error bound on the approximation to $f(1.5)$ found in part (a) to explain why $f(1.5) \neq -5$.

$$R_3(x) = \left| \frac{|f^4(x)|(1.5-2)^4}{4!} \right| < \frac{3(-.5)^4}{24}$$

$$R_3(x) < 0.0078$$

The error is less than 0.0078, so if the estimate is incorrect, the least it can be is $-4.958 - 0.0078 = -4.9658$

Thus, it will always be more than -5.

Important Maclaurin Series

Function	Maclaurin Series
e^x	$\sum_{k=0}^{\infty}\frac{x^k}{k!}=1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\frac{x^4}{4!}+\cdots$
$\sin x$	$\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k+1}}{(2k+1)!}=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\cdots$
$\cos x$	$\sum_{k=0}^{\infty}(-1)^k\frac{x^{2k}}{(2k)!}=1-\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$
$\frac{1}{1-x}$	$\sum_{k=0}^{\infty}x^k=1+x+x^2+x^3+\cdots$ (if $-1<x<1$)
$\ln(1+x)$	$\sum_{k=1}^{\infty}(-1)^{k+1}\frac{x^k}{k}=x-\frac{x^2}{2}+\frac{x^3}{3}-\frac{x^4}{4}+\cdots$ (if $-1<x\le 1$)

Taylor's Theorem

If a function f is differentiable through order $n+1$ in an interval containing the center $x=c$, then for each $x=a$ in the interval, there exists a number $x=z$ between a and c such that

$$f(a)=f(c)+f'(c)(a-c)+\frac{f''(c)}{2!}(a-c)^2+\cdots+\frac{f^{(n)}(c)}{n!}(a-c)^n+R_n(a)$$

where the remainder $R_n(a)$ is given by $R_n(a)=\left|\frac{\left|f^{(n+1)}(z)\right|}{(n+1)!}(a-c)^{n+1}\right|$, called the **Lagrange Remainder (or Lagrange Error Bound).**