Unit 2: APPLICATION OF DERIVATIVES REVIEW – Assessment

Name Answer Key **PER** ________ **DATE** ____________

4.0	5.0	9.0	12.0

Computation

4	3	2	1
Response has no recall errors, *minimal* procedural errors* and no conceptual errors**	**Response has no recall errors, minimal procedural errors and *minimal* conceptual errors**	**Response has no recall errors, but has several procedural errors <u>OR</u> several conceptual errors**	**Recall errors exist <u>OR</u> Steps taken are not related to problem <u>OR</u> Response left blank**

Written Responses

4	3	2	1
Response is written in a complete sentence and uses appropriate academic vocab	**Response is written in a complete sentence, and minimal errors exist in use of academic vocab**	**Response is not written in a complete sentence <u>OR</u> no academic vocab**	**Concept of response is not related to problem <u>OR</u> Response is left blank**

***Procedural errors** are mistakes made in the math
****Conceptual errors** are mistakes made in the steps one take

BOX YOUR ANSWERS!!!

(4.0) For Q.1 and 2, show your work and circle the best possible answer.

1.

Let f be the function given by $f(x) = (2x-1)^5(x+1)$. Which of the following is an equation for the line tangent to the graph of f at the point where $x = 1$?

(A) $y = 21x + 2$

(B) $y = 21x - 19$

(C) $y = 11x - 9$

(D) $y = 10x + 2$

(E) $y = 10x - 8$

$f'(x) = (2x-1)^5 \cdot 1 + 5(2x-1)^4 \cdot 2\,(x+1)$ (u, v', u', v)

$f'(1) = (2(1)-1)^5 + 5(2(1)-1)^4 \cdot 2(1+1)$

plug in $x=1$

$= (1)^5 + 5(1)^4 \cdot 2(2)$

$= 1 + 5(4)$

slope → $f'(1) = 21$

$f(1) = (2(1)-1)^5(1+1)$

$= (1)^5(2)$

$= 2$

$x_1 = 1$, $y_1 = 2$

$y - y_1 = m(x - x_1)$

$y - 2 = 21(x-1)$

$y - 2 = 21x - 21$

$y = 21x - 19$

2.

For the function f, $f'(x) = 2x + 1$ and $f(1) = 4$. What is the approximation for $f(1.2)$ found by using the line tangent to the graph of f at $x = 1$? (x_1, y_1)

(A) 0.6 (B) 3.4 (C) 4.2 (D) 4.6 (E) 4.64

$f'(1) = 2(1) + 1$

$= 3$

$y - y_1 = m(x - x_1)$

$y - 4 = 3(x-1)$

$y - 4 = 3x - 3$

$y = 3x + 1$

$y = 3(1.2) + 1$

$= 3.6 + 1$

$= 4.6$

(5.0) For Q.3 and 4, show your work and circle the best possible answer.

3.

If $f(x) = \cos^3(4x)$, then $f'(x) =$ $3\cos^2(4x)\cdot(-\sin(4x))\cdot 4$

$= -12\cos^2(4x)\sin(4x)$

(A) $3\cos^2(4x)$

(B) $-12\cos^2(4x)\sin(4x)$

(C) $-3\cos^2(4x)\sin(4x)$

(D) $12\cos^2(4x)\sin(4x)$

(E) $-4\sin^3(4x)$

4.

If $f(x) = ae^{-ax}$ for $a > 0$, then $f'(x) =$ $ae^{-ax}\cdot -a$ (the derivative of "what's inside")

$= -a^2e^{-ax}$

(A) e^{-ax}

(B) ae^{-ax}

(C) a^2e^{-ax}

(D) $-ae^{-ax}$

(E) $-a^2e^{-ax}$

(9.0) For Q. 5 and 6, show your work and circle the best possible answer.

5. $f'(x) < 0$ $f''(x) > 0$

Let f be the function defined by $f(x) = 2x^3 - 3x^2 - 12x + 18$. On which of the following intervals is the graph of f both decreasing and concave up?

(A) $(-\infty, -1)$ (B) $\left(-1, \frac{1}{2}\right)$ (C) $(-1, 2)$ (D) $\left(\frac{1}{2}, 2\right)$ (E) $(2, \infty)$

$f'(x) = 6x^2 - 6x - 12$

$0 = 6(x^2 - x - 2)$

$= 6(x-2)(x+1)$

$x = 2$ $x = -1$

$f''(x) = 12x - 6$

$0 = 12x - 6$

$6 = 12x$

$\frac{1}{2} = x$

	-2 (−1)	0 ($\frac{1}{2}$)	1 (2)	3
$f'(x)$	+	−	−	+
$f''(x)$	−	−	+	+

↑

6. already derived!

Let f be the function with first derivative given by $f'(x) = (3 - 2x - x^2)\sin(2x - 3)$. How many relative extrema does f have on the open interval $-4 < x < 2$?

(A) Two (B) Three (C) Four (D) Five (E) Six

$0 = (3 - 2x - x^2)\sin(2x - 3)$

$0 = (3 + x)(1 - x)\sin(2x - 3)$

$x = -3$ ✓ $x = 1$ ✓ $2x - 3 = 0, \pi$ or 2π

$2x - 3 = 0$

$x = 3/2$ ✓

$2x - 3 = \pi$

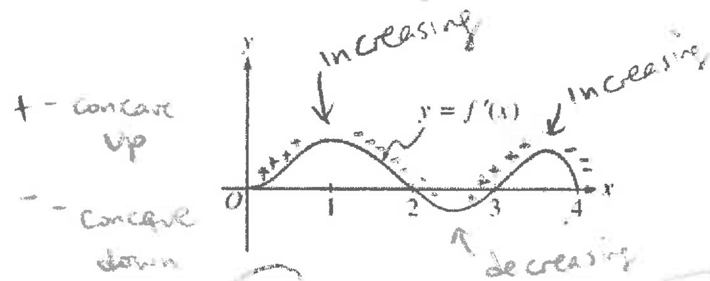
$x = \frac{\pi + 3}{2} \approx 3$ X

$2x - 3 = 2\pi$

$x = \frac{2\pi - 3}{2} \approx 1.5$ ✓

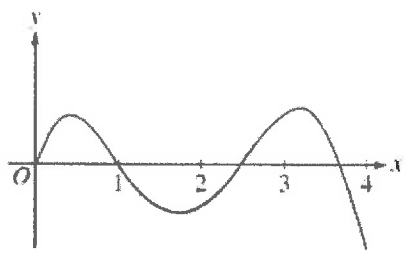
(9.0) For Q. 7, explain your answer choice in the space provided.

7.

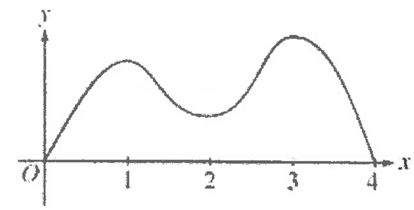


The figure above shows the graph of f', the derivative of the function f. If $f(0) = 0$, which of the following could be the graph of f?

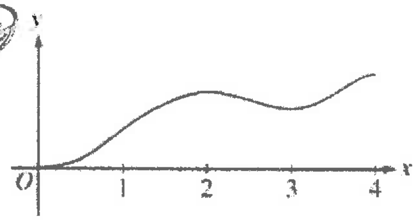
(A)



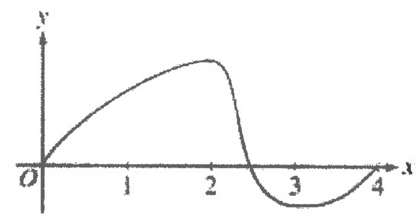
(B)



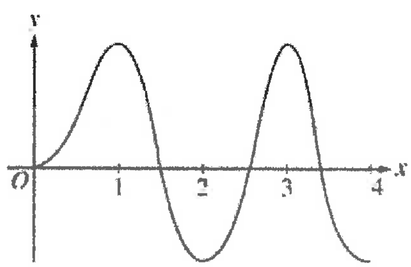
(C)



(D)



(E)

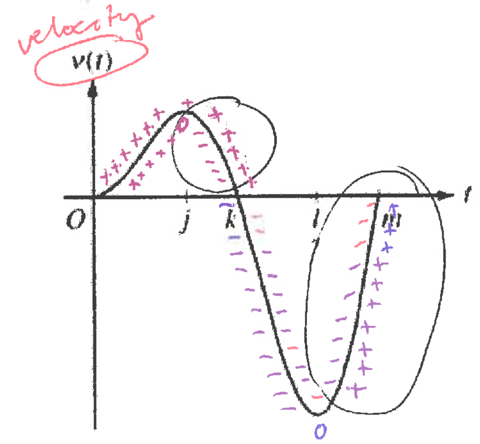


The graph of f must be C since according to the graph of f'(x), f must be increasing and concave up from $0 \le x \le 1$ and increasing and concave down from about $3.5 \le x \le 4$

(12.0) For Q. 8, explain your answer choice in the space provided.

8.

acceleration ($v'(t)$)



$v(t) = 0$

A particle moves along a straight line. The graph of the particle's velocity $v(t)$ at time t is shown above for $0 \le t \le m$, where j, k, l, and m are constants. The graph intersects the horizontal axis at $t = 0$, $t = k$, and $t = m$ and has horizontal tangents at $t = j$ and $t = l$. For what values of t is the speed of the particle decreasing?

$v'(t) = 0$

(A) $j \le t \le l$

(B) $k \le t \le m$

(C) $j \le t \le k$ and $l \le t \le m$

(D) $0 \le t \le j$ and $k \le t \le l$

(E) $0 \le t \le j$ and $l \le t \le m$

The particle is decreasing its speed from $j \le t \le k$ and $l \le t \le m$. This is true since the velocity and acceleration do not match signs for those intervals. In the first interval, the velocity is positive (above the x-axis) and the acceleration is negative (the slope of the graph. In the second interval, the velocity is negative and the acceleration is positive.

(12.0) For Q. 9, show your work and circle the best possible answer.

9.

$v(t)=0$

A particle moves on the x-axis so that at any time t, $0 \leq t \leq 1$, its position is given by $x(t) = \sin(2\pi t) + 2\pi t$. For what value of t is the particle at rest?

(A) 0 (B) $\frac{1}{8}$ (C) $\frac{1}{4}$ (D) $\frac{1}{2}$ (E) 1

$x(t) = \sin(2\pi t) + 2\pi t$

$v(t) = \cos(2\pi t) \cdot 2\pi + 2\pi$

$0 = 2\pi\cos(2\pi t) + 2\pi$

$-2\pi = 2\pi\cos(2\pi t)$

$-1 = \cos(2\pi t)$

$\frac{2\pi t}{2\pi} = \frac{\pi}{2\pi}$ ... since $\cos(\pi) = -1$

$t = \frac{1}{2}$