

## Unit 1: LIMITS AND DERIVATIVES REVIEW - Assessment

Name Exemplar PER \_\_\_\_\_ DATE \_\_\_\_\_

1.0	2.0	4.0

*Computation*

4	3	2	1
Response has no recall errors, <i>minimal</i> procedural errors* and no conceptual errors**	Response has no recall errors, minimal procedural errors and <i>minimal</i> conceptual errors	Response has no recall errors, but has several procedural errors <u>OR</u> several conceptual errors	Recall errors exist <u>OR</u> Steps taken are not related to problem <u>OR</u> Response left blank

*Written Responses*

4	3	2	1
Response is written in a complete sentence and uses appropriate academic vocab	Response is written in a complete sentence, and minimal errors exist in use of academic vocab	Response is not written in a complete sentence <u>OR</u> no academic vocab	Concept of response is not related to problem <u>OR</u> Response is left blank

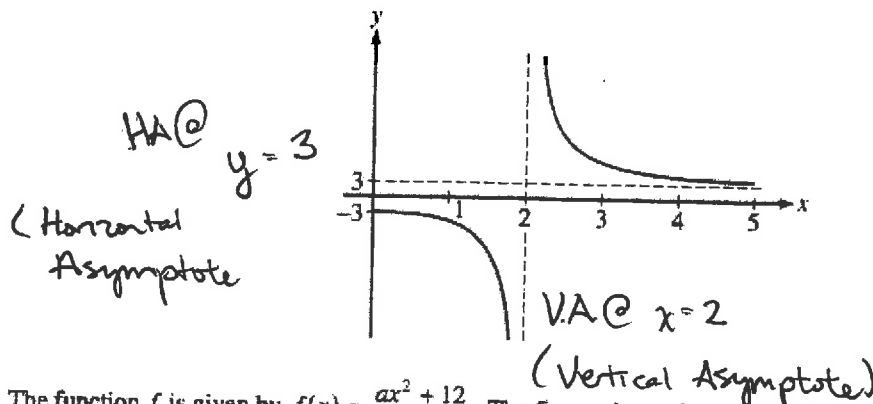
\*Procedural errors are mistakes made in the math

\*\*Conceptual errors are mistakes made in the steps one take

<b>BOX YOUR ANSWERS!!!</b>
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3. (1.0) Show your work and circle the best possible answer.



The function  $f$  is given by  $f(x) = \frac{ax^2 + 12}{x^2 + b}$ . The figure above shows a portion of the graph of  $f$ . Which of the following could be the values of the constants  $a$  and  $b$ ?

(A)  $a = -3, b = 2$

~~(B)  $a = 2, b = -3$~~

(C)  $a = 2, b = -2$

**(D)  $a = 3, b = -4$**

(E)  $a = 3, b = 4$

VA

$$\lim_{x \rightarrow 2} f(x) = \infty$$

$$\lim_{x \rightarrow 2} \frac{ax^2 + 12}{x^2 + b} = \infty \rightarrow x^2 + b = 0$$

$$(2)^2 + b = 0$$

$$4 + b = 0$$

$$b = -4$$

HA  $\lim_{x \rightarrow \infty} f(x) = 3$

$$\lim_{x \rightarrow \infty} \frac{ax^2 + 12}{x^2 - 4} = 3$$

↓

$$\frac{ax^2}{x^2} = 3$$

Degree of  $x^2 = 2$   
 $a = 3$

4. (2.0) Show your work and circle the best possible answer.

Let  $f$  be the function given by  $f(x) = \frac{(x-2)^2(x+3)}{(x-2)(x+1)}$ . For which of the following values of  $x$  is  $f$  not continuous?

(A)  $-3$  and  $-1$  only

(B)  $-3, -1,$  and  $2$

(C)  $-1$  only

**(D)  $-1$  and  $2$  only**

(E)  $2$  only

Hole

$$x - 2 = 0$$

$$x = 2$$

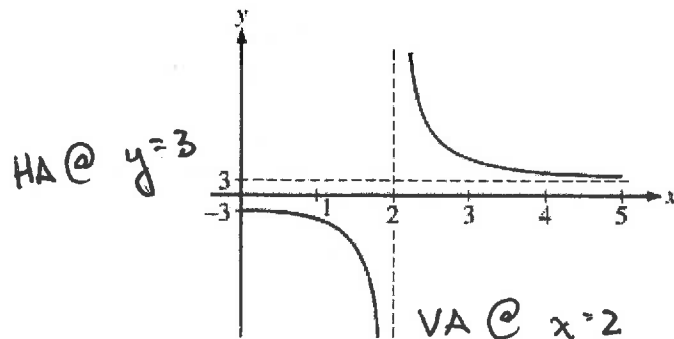
VA

$$x + 1 = 0$$

$$x = -1$$



3. (1.0) Show your work and circle the best possible answer.



The function  $f$  is given by  $f(x) = \frac{ax^2 + 12}{x^2 + b}$ . The figure above shows a portion of the graph of  $f$ . Which of the following could be the values of the constants  $a$  and  $b$ ?

- (A)  $a = -3$ ,  $b = 2$   
 (B)  $a = 2$ ,  $b = -3$   
 (C)  $a = 2$ ,  $b = -2$   
 (D)  $a = 3$ ,  $b = -4$   
 (E)  $a = 3$ ,  $b = 4$

method 1

$$x^2 + b = 0$$

$$2^2 + b = 0$$

$$4 + b = 0$$

$$b = -4$$

method 2

$$\lim_{x \rightarrow 2} f(x) = \infty$$

$\therefore$  b must equal

4. (2.0) Show your work and circle the best possible answer.

Let  $f$  be the function given by  $f(x) = \frac{(x-2)^2(x+3)}{(x-2)(x+1)}$ . For which of the following values of  $x$  is  $f$  not continuous?

- (A)  $-3$  and  $-1$  only  
 (B)  $-3$ ,  $-1$ , and  $2$   
 (C)  $-1$  only  
 (D)  $-1$  and  $2$  only  
 (E)  $2$  only

5. (2.0) Show your work and circle the best possible answer.

$$f(x) = \begin{cases} x^2 - 3x + 9 & \text{for } x \leq 2 \\ kx + 1 & \text{for } x > 2 \end{cases}$$

The function  $f$  is defined above. For what value of  $k$ , if any, is  $f$  continuous at  $x = 2$ ?

(A) 1

(B) 2

(C) 3

(D) 7

(E) No value of  $k$  will make  $f$  continuous at  $x = 2$ .

$$\lim_{x \rightarrow 2^-} x^2 - 3x + 9 = 2^2 - 3(2) + 9 = 7$$

$$\lim_{x \rightarrow 2^+} kx + 1 = 2k + 1$$

$$\text{So } 2k + 1 = 7$$

$$2k = 6$$

$$\boxed{k = 3}$$

6. (4.0) In the space provided, explain why your answer choice is correct in complete sentences.

$$\lim_{h \rightarrow 0} \frac{e^{(2+h)} - e^2}{h} = \frac{d}{dx} (e^x) \text{ evaluated @ } x=2$$

(A) 0

(B) 1

(C)  $2e$

(D)  $e^2$

(E)  $2e^2$

The limit above describes the derivative of

$f(x) = e^x$  evaluated at  $x=2$

7. (4.0) Show your work and circle the best possible answer.

If  $f(x) = \frac{3x-2}{2x+3}$ , then  $f'(x) = \frac{vu' - uv'}{v^2} \rightarrow \frac{(2x+3)(3) - (3x-2)(2)}{(2x+3)^2}$

(A)  $\frac{13}{(2x+3)^2}$

(B)  $\frac{3}{(2x+3)^2}$

(C)  $\frac{5}{(2x+3)^2}$

(D)  $\frac{13}{(2x+3)^2}$

(E)  $\frac{12x+5}{(2x+3)^2}$

$$\frac{(2x+3)(3) - (3x-2)(2)}{(2x+3)^2}$$

$$= \frac{6x+9 - (6x-4)}{(2x+3)^2}$$

$$= \frac{6x+9 - 6x+4}{(2x+3)^2}$$

$$= \frac{13}{(2x+3)^2}$$

8. (4.0) Show your work and circle the best possible answer

$$f(x) = \begin{cases} x+b & \text{if } x \leq 1 \\ ax^2 & \text{if } x > 1 \end{cases}$$

Let  $f$  be the function given above. What are all values of  $a$  and  $b$  for which  $f$  is differentiable at  $x = 1$ ?

(A)  $a = \frac{1}{2}$  and  $b = -\frac{1}{2}$

(B)  $a = \frac{1}{2}$  and  $b = \frac{3}{2}$

(C)  $a = \frac{1}{2}$  and  $b$  is any real number

(D)  $a = b + 1$ , where  $b$  is any real number

(E) There are no such values of  $a$  and  $b$ .

$$\lim_{x \rightarrow 1^-} x + b = 1 + b$$

so  $1 + b = a$  (1)

$$\lim_{x \rightarrow 1^+} ax^2 = a$$

One-sided  
Derivatives

$$\lim_{x \rightarrow 1^-} 1 = 1$$

so  $1 = 2a$

$$\lim_{x \rightarrow 1^+} 2ax = 2a$$

$\frac{1}{2} = a$  (2)

$\frac{d}{dx}(b) = 0$

(1)  $1 + b = \frac{1}{2}$  (2)

$b = -\frac{1}{2}$

