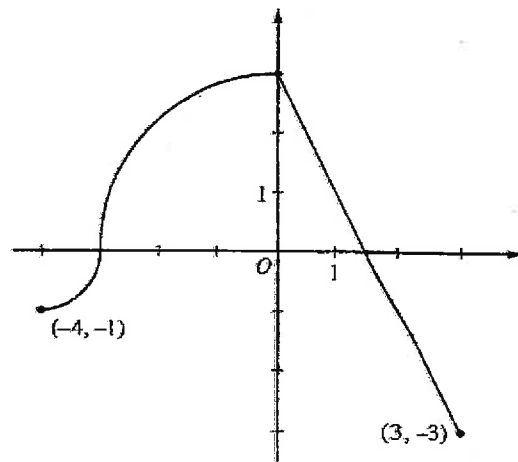


**AP[®] CALCULUS AB
2011 SCORING GUIDELINES**

Question 4

The continuous function f is defined on the interval $-4 \leq x \leq 3$. The graph of f consists of two quarter circles and one line segment, as shown in the figure above.



Graph of f

Let $g(x) = 2x + \int_0^x f(t) dt$.

- Find $g(-3)$. Find $g'(x)$ and evaluate $g'(-3)$.
- Determine the x -coordinate of the point at which g has an absolute maximum on the interval $-4 \leq x \leq 3$. Justify your answer.
- Find all values of x on the interval $-4 < x < 3$ for which the graph of g has a point of inflection. Give a reason for your answer.
- Find the average rate of change of f on the interval $-4 \leq x \leq 3$. There is no point c , $-4 < c < 3$, for which $f'(c)$ is equal to that average rate of change. Explain why this statement does not contradict the Mean Value Theorem.

(a) $g(-3) = 2(-3) + \int_0^{-3} f(t) dt = -6 - \frac{9\pi}{4}$
 $g'(x) = 2 + f(x)$
 $g'(-3) = 2 + f(-3) = 2$

3 : $\begin{cases} 1 : g(-3) \\ 1 : g'(x) \\ 1 : g'(-3) \end{cases}$

(b) $g'(x) = 0$ when $f(x) = -2$. This occurs at $x = \frac{5}{2}$.
 $g'(x) > 0$ for $-4 < x < \frac{5}{2}$ and $g'(x) < 0$ for $\frac{5}{2} < x < 3$.

3 : $\begin{cases} 1 : \text{considers } g'(x) = 0 \\ 1 : \text{identifies interior candidate} \\ 1 : \text{answer with justification} \end{cases}$

Therefore g has an absolute maximum at $x = \frac{5}{2}$.

(c) $g''(x) = f'(x)$ changes sign only at $x = 0$. Thus the graph of g has a point of inflection at $x = 0$.

1 : answer with reason

(d) The average rate of change of f on the interval $-4 \leq x \leq 3$ is $\frac{f(3) - f(-4)}{3 - (-4)} = -\frac{2}{7}$.

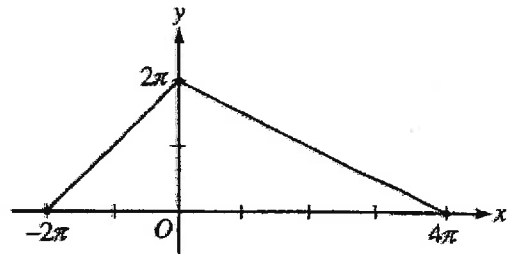
2 : $\begin{cases} 1 : \text{average rate of change} \\ 1 : \text{explanation} \end{cases}$

To apply the Mean Value Theorem, f must be differentiable at each point in the interval $-4 < x < 3$. However, f is not differentiable at $x = -3$ and $x = 0$.

AP[®] CALCULUS AB
2011 SCORING GUIDELINES (Form B)

Question 6

Let g be the piecewise-linear function defined on $[-2\pi, 4\pi]$ whose graph is given above, and let $f(x) = g(x) - \cos\left(\frac{x}{2}\right)$.



Graph of g

- (a) Find $\int_{-2\pi}^{4\pi} f(x) dx$. Show the computations that lead to your answer.
- (b) Find all x -values in the open interval $(-2\pi, 4\pi)$ for which f has a critical point.
- (c) Let $h(x) = \int_0^{3x} g(t) dt$. Find $h\left(-\frac{\pi}{3}\right)$.

$$\begin{aligned} \text{(a)} \quad \int_{-2\pi}^{4\pi} f(x) dx &= \int_{-2\pi}^{4\pi} \left(g(x) - \cos\left(\frac{x}{2}\right) \right) dx \\ &= 6\pi^2 - \left[2\sin\left(\frac{x}{2}\right) \right]_{x=-2\pi}^{x=4\pi} \\ &= 6\pi^2 \end{aligned}$$

2: $\begin{cases} 1: \text{antiderivative} \\ 1: \text{answer} \end{cases}$

$$\text{(b)} \quad f'(x) = g'(x) + \frac{1}{2}\sin\left(\frac{x}{2}\right) = \begin{cases} 1 + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } -2\pi < x < 0 \\ -\frac{1}{2} + \frac{1}{2}\sin\left(\frac{x}{2}\right) & \text{for } 0 < x < 4\pi \end{cases}$$

4: $\begin{cases} 1: \frac{d}{dx}\left(\cos\left(\frac{x}{2}\right)\right) \\ 1: g'(x) \\ 1: x = 0 \\ 1: x = \pi \end{cases}$

$f'(x)$ does not exist at $x = 0$.

For $-2\pi < x < 0$, $f'(x) \neq 0$.

For $0 < x < 4\pi$, $f'(x) = 0$ when $x = \pi$.

f has critical points at $x = 0$ and $x = \pi$.

$$\begin{aligned} \text{(c)} \quad h(x) &= g(3x) \cdot 3 \\ h\left(-\frac{\pi}{3}\right) &= 3g(-\pi) = 3\pi \end{aligned}$$

3: $\begin{cases} 2: h(x) \\ 1: \text{answer} \end{cases}$