ANSWER KEY AND EXPLANATIONS

Section I, Part A

1. D	7. D	13. C	19. D	24. D
2. B	8. D	14. D	20. C	25. E
3. E	9. E	15. D	21. B	26. D
4. C	10. B	16. E	22. C	27. C
5. A	11. B	17. C	23. C	28. E
6. E	12. D	18. D		

1. **The correct answer is (D).** Solve this integral using u-substitution. Let $u = 2x$, so $du = 2\ dx$.

   $\int_0^1 e^{2x}dx$ becomes $\frac{1}{2}\int_0^2 e^u du$, which yields $\frac{1}{2}\left(e^2 - 1\right)$.

2. **The correct answer is (B).** This is a rather complicated Chain Rule application. The derivative of tan u is $\sec^2 u\ du$, but we mustn't forget to also take the derivative of $e^{\sin x}$. Since

   $\frac{d}{dx}\left(e^{\sin x}\right) = \cos x\ e^{\sin x}$,

   $\frac{d}{dx}\tan\left(e^{\sin x}\right) = \sec^2\left(e^{\sin x}\right)\cos x\ e^{\sin x}$

3. **The correct answer is (E).** Be careful here. Although it resembles a Fundamental Theorem Part Two problem, it is not. The problem asks for $F(2)$, not $F'(2)$! So,

   $F(2) = \int_2^4 t^2\ dt = \frac{56}{3}$.

4. **The correct answer is (C).** Straight-forward evaluation of a derivative at a point problem: $f'(x) = 2\tan x\sec^2 x + \cos x$. So,

   $f'\left(\frac{\pi}{4}\right) = 4 + \frac{\sqrt{2}}{2}$, which is $\frac{8+\sqrt{2}}{2}$.

5. **The correct answer is (A).** We need both the first and second derivatives to be negative for this function to be decreasing and concave down. $f'(x) = 4x^3 - 6x^2 - 4x$ and $f''(x) = 12x^2 - 12x - 4$. By using the wiggle graph below,

we can easily see that choices (B) and (C) can be eliminated, so we must check out the values of $f''(-2)$, $f''(-1)$, and $f''(1)$. $f''(1) = -4 < 0$.

6. **The correct answer is (E).** Here, we should take the derivative of each I, II, and III and see what we get.

$$\frac{d}{dx}\left(\frac{-\cos^4 x}{4}\right) = \frac{-4\cos^3 x}{4}(-\sin x) = \cos^3 x \sin x$$

$$\frac{d}{dx}\left(\frac{\sin^2 x}{2} - \frac{\sin^4 x}{4}\right) = \sin x\cos x - \frac{4\sin^3 x\cos x}{4}$$

$$= \sin x\cos x - \sin^3 x\cos x = \sin x\cos x(1-\sin^2 x) = \sin x\cos x(\cos^2 x) = \cos^3 x\sin x$$

$$\frac{d}{dx}\left(\frac{1-\cos^4 x}{4}\right) = \frac{-4\cos^3 x(-\sin x)}{4} = \cos^3 x\sin x$$

7. **The correct answer is (D).** Another lengthy Chain Rule—just don't forget to take the derivative of $2x$, the argument of the argument. Since the derivative of e^u is $e^u\ du$ and the

derivative of sin$u$ is cos$u$ du, then $\frac{d}{dx}e^{\sin 2x} = 2\cos 2x e^{\sin 2x}$.

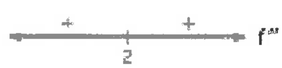
8. **The correct answer is (D).** Remember, speed is the absolute value of velocity. Since $|-15|>|10|$, the maximum speed is reached at $t = 5$ seconds.

9. **The correct answer is (E).** Acceleration can be thought of as the absolute value or slope of velocity. The slope of the velocity curve is steepest on [5,6].

10. **The correct answer is (B).** In order to write the equation for a line, we need its slope and a point on that line. We already have the point, $\left(\frac{\pi}{2},1\right)$, so the big problem is determining its slope, which is the derivative of the curve when $x=\frac{\pi}{2}$. $y' =$ cos$x$ − 2sin$x$, so $y'\left(\frac{\pi}{2}\right) = -2$. Using point ≠ slope form, the equation for the line could be written as $y-1=-2\left(x-\frac{\pi}{2}\right)$. Since this is not a choice, we must change this to standard form, $2x + y = \pi + 1$.

11. **The correct answer is (B).** Since the f is decreasing over $(-\infty,-1)$, its derivative, f', must be negative over this same interval. This eliminates choices (C), (D), and (E). Examining the interval $(-1,0)$, the graph of f is decreasing here; thus, the graph of f' must be negative. Only choice (B) meets this requirement.

12. **The correct answer is (D).** By examining the second-derivative wiggle graph below, we can see that the second derivative is positive at, before, and after 2. Therefore, there is no point of inflection at $x = 2$.



13. **The correct answer is (C).** Pretty simple problem—we determine the derivative, set it equal to zero, and use a wiggle graph.

$$f'(x) = 3e^{\sin x}\cos x = 0$$

Since $3e^{\sin x}$ will never be 0, we set cos$x$ = 0 and solve.

cos$x$ = 0 when $x=\frac{\pi}{2}+n\pi$ for any integer n. By examining the wiggle graph below, we can see that the derivative is negative over $[\frac{\pi}{2}, \frac{3\pi}{2}]$.

14. **The correct answer is (D).** This one seems tricky, but it actually works out quite quickly. If $h(x) = f(g'(x))$, then by using the chain rule, $h'(x) = f'(g'(x))g''(x)$. The problem tells us that $f'(x)$ will always be positive. Since $h'(3)$ somehow becomes negative, $g''(3)$ must be negative. Therefore, g must be concave down at $x = 3$.

15. **The correct answer is (D).** Do we understand the definitions of continuity and differentiability?

(A) Does the limit exist as x approaches 3? Yes.

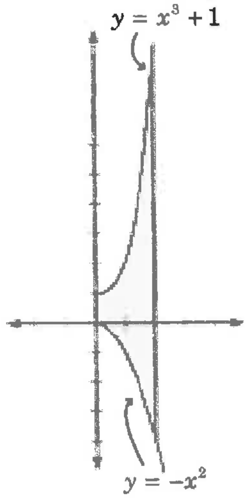
(B) Is f continuous at $x = 5$? Yes.

(C) Does $f'(2) = 0$? Yes. (It has a horizontal tangent line.)

(D) Does $f'(4)$ exist? No.

(E) Is $f(2.5) > f'(2.5)$? Yes. ($f(2.5) > 0$, and since f is decreasing at $x = 2.5$, $f'(2.5) < 0$.)

16. The correct answer is (E). We should always sketch the region.



As we can see, $y = x^3 + 1$ is above $y = -x^2$ over the entire interval. So,

$$A = \int_0^2 \left(x^3 + 1 - \left(-x^2\right) \right) dx$$
$$= \frac{x^4}{4} + x + \frac{x^3}{3} \Big|_0^2$$
$$= 4 + 2 + \frac{8}{3} = \frac{26}{3}$$

17. The correct answer is (C). Some implicit differentiation: Remember, everything here is differentiated with respect to x. Don't forget the product rule for $3xy$.

$$3x^2 + 3y^2 \frac{dy}{dx} = 3x \frac{dy}{dx} + 3y$$
$$\left(3y^2 - 3x\right)\frac{dy}{dx} = 3y - 3x^2$$
$$\frac{dy}{dx} = \frac{3y - 3x^2}{3y^2 - 3x}$$
$$\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$$

18. The correct answer is (D). This is a very simple u-substitution integral. Let $u = 2x$, so $du = 2dx$.

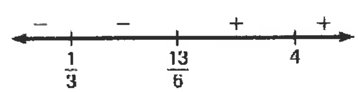
It follows that $\int_0^{\pi/4} \sin 2x \, dx$ becomes

$$\frac{1}{2}\int_0^{\pi/2} \sin u \, du = -\frac{1}{2} \cos u \Big|_0^{\pi/2}$$
$$= -\frac{1}{2}(0 - 1) = \frac{1}{2}.$$

19. The correct answer is (D). Again, we will rely on the magic of the wiggle graph to supply us with the solution to this differentiation problem.

$$f'(x) = 3(x - 4)^2(3x - 1)^2[3(x - 4) + (3x - 1)]$$
$$= 3(x - 4)^2(3x - 1)^2(6x - 13)$$

By setting $f'(x) = 0$ and solving, we quickly discover that the zeros of the derivative are $\frac{1}{3}$, $\frac{13}{6}$, and 4. By examining the wiggle graph below, we can see that the only value where the derivative changes from negative to positive is at $x = \frac{13}{6}$.



20. The correct answer is (C). Whenever the problem asks for the "average value of the function," we should immediately think of the mean value theorem for integration. We are looking for $f(c)$ in this formula:

$f(c) = \frac{1}{b-a}\int_a^b f(x) \, dx$. Applying the MVT for Integration here yields the following equation:

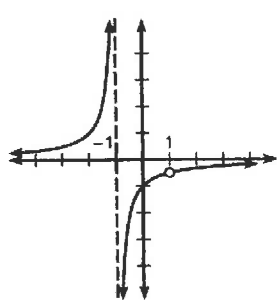
$$f(c) = \frac{1}{\frac{\pi}{3} - \frac{\pi}{4}} 4\int_{\pi/4}^{\pi/3} \sin 2x \, dx$$
$$= \frac{1}{\frac{\pi}{12}} \cdot \frac{1}{2}\int_{\pi/2}^{2\pi/3} \sin u \, du$$
$$= \frac{6}{\pi}\left(-\cos u\right)\Big|_{\pi/2}^{2\pi/3}$$
$$= -\frac{6}{\pi}\left(-\frac{1}{2} - 0\right)$$
$$= \frac{3}{\pi}$$

21. The correct answer is (B). Remember, limits at infinity are like horizontal asymptotes. If the top degree is greater than the bottom degree, the limit does not exist. If the bottom degree is greater than the top degree, the limit is zero. If, as in this case, the degrees are equal, then the limit is the ratio of the leading coefficient of the numerator over that of the denominator. Here, that ratio is $-\frac{5}{11}$.

22. The correct answer is (C). Sketch the curve. We may simplify the expression first by using cancellation. However, we must remember that by canceling out a term involving *x*, we are removing a discontinuity. So, the graph of the original function will have a point discontinuity.

$$f(x) = \frac{1-x}{x^2-1} = \frac{1-x}{(x-1)(x+1)}$$
$$= -\frac{1}{x+1}$$

with a point discontinuity at $x = 1$.



By examining the graph above, we can see that it is concave down to the right of the asymptote. These are actually two intervals, because the function is not continuous at $x = 1$. They are $(-1,1)$ and $(1,\infty)$.

23. The correct answer is (C). The two curves, $y = \sqrt{x}$ and $y = x^2$ intersect at $x = 0$ and $x = 1$. So, these will be our limits of integration. For x values between 0 and 1, $\sqrt{x} > x^2$. This being the case, when we determine the area of the region, we should subtract $\sqrt{x} - x^2$. Therefore, $A = \int_0^1 \left(\sqrt{x} - x^2\right) dx$.

24. The correct answer is (D). This is just the derivative of $f(x) = \tan 2x$ evaluated at $x = \frac{\pi}{8}$. $f'(x) = 2\sec^2 2x$, so $f'\left(\frac{\pi}{8}\right) = 2\sec^2\frac{\pi}{4} = 4$.

25. The correct answer is (E).

A little tricky u-substitution integral. Let $u = \ln x$, then $du = \frac{dx}{x}$.

It follows that

$$\int_1^{e^2} \left(\frac{\ln^2 x}{x}\right) dx = \int_0^2 u^2 du$$
$$= \frac{u^3}{3}\Big|_0^2 = \frac{8}{3} - 0 = \frac{8}{3}$$

26. The correct answer is (D). Since the question asks for average velocity and we are given the position equation, we should determine the slope of the secant line:

$$\text{Average velocity} = \frac{s(2\pi) - s(0)}{2\pi - 0}$$
$$= \frac{0 + 2 + 2 + 2 - (2 + 2)}{2\pi}$$
$$= \frac{2}{2\pi} = \frac{1}{\pi}$$

27. The correct answer is (C).

Since $\lim_{x\to(\ln 2)^-} f(x) = \lim_{x\to(\ln 2)^+} f(x) = e^{\ln 2} = 2$, then $\lim_{x\to \ln 2} f(x)$ exists and is equal to 2 as well.

28. The correct answer is (E). This is a rather challenging related-rates problem. We are looking for $\frac{dA}{dt}$ when $h = 24$. First, we need a primary equation. This will be the formula for the area of a triangle:

$$A = \frac{1}{2}bh$$

Since the base is a constant, 10, this becomes

$$A = \frac{1}{2}h(10) = 5h$$

Differentiating with respect to t yields

$$\frac{dA}{dt} = 5\frac{dh}{dt}$$

How do we determine $\frac{dh}{dt}$? We must find a relationship other than $A = \frac{1}{2}bh$ involving h. How about using the tangent equation?

$$\tan\theta = \frac{h}{10} \text{ or } h = 10\tan\theta$$

Differentiating this with respect to t yields

$$\frac{dh}{dt} = 10\sec^2\theta\frac{d\theta}{dt}$$

Since we know $\frac{d\theta}{dt} = \frac{15}{26}$, this equation becomes

$$\frac{dh}{dt} = 10\sec^2\theta\frac{15}{26}$$

The last unknown to identify is $10\sec^2\theta$. We can use the Pythagorean theorem to help here. Since $h = 24$ and $b = 10$, the hypotenuse must be 26. So,

$$10\sec^2\theta = 10\left(\frac{26}{10}\right)^2$$

Plugging this expression in for $\frac{dh}{dt}$ gives us

$$\frac{dA}{dt} = (5)(10)\left(\frac{26}{10}\right)^2\left(\frac{15}{26}\right)$$
$$= 50\left(\frac{13}{5}\right)\left(\frac{3}{2}\right) = 195$$

answers practice test 1

Section I, Part B

29. A	33. C	37. C	40. C	43. D
30. E	34. B	38. E	41. B	44. E
31. D	35. D	39. E	42. C	45. D
32. E	36. A			

29. The correct answer is (A). We must use the quotient rule to evaluate the following derivative:

$$f'(x)=\frac{3e^{3x}\sin x^2-2xe^{3x}\cos x^2}{\sin^2 x^2}$$
$$=e^{3x}\left(\frac{3\sin x^2-2x\cos x^2}{\sin^2 x^2}\right)$$

30. The correct answer is (E). To write the equation for a line, we need the slope of the line and a point on the line. We already have its slope, since $f'(x) = 10$; the slope of the tangent line is 10 as well. To find the point on the line, we must set the derivative of f equal to 10 and solve for x; then, substitute this x-value into f to determine the corresponding y-value.

$$f'(x) = 2e^{2x} = 10$$
$$e^{2x} = 5$$
$$2x = \ln 5$$
$$x = \frac{\ln 5}{2}$$

Now, we substitute this value into f and the result is

$$f\left(\frac{\ln 5}{2}\right)= e^{\ln 5} = 5$$

Our problem has now been reduced to determining the equation for a line that passes through $\left(\frac{\ln 5}{2},5\right)$ and has slope 10. Point-slope form of this equation is

$$y-5=10\left(x-\frac{\ln 5}{2}\right)$$

Converting to slope-intercept form and using the calculator to evaluate the value of $\frac{\ln 5}{2}$, we get

$$y = 10x - 3.047$$

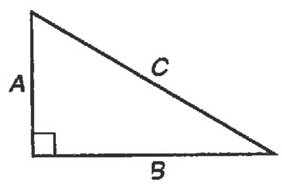
31. The correct answer is (D). This is an area accumulation problem. The function decreases from $x = 0$ to $x = 1$ by an amount equivalent to the area between the graph of f' and the x-axis, which is -2 units squared. This was determined by finding the area of the triangle. The function then increases from $x = 1$ to $x = 2$ by $\frac{3}{2}$ units squared. From $x = 2$ to $x = 4$, it increases $3 + \frac{3}{2}$ or $\frac{9}{2}$ units squared. From $x = 4$ to $x = 6$, the function decreases 2 units squared. Putting all of this together, we can see that the function's value is greatest at $x = 4$, followed by at $x = 6$, then at $x = 0$, and least at $x = 2$.

32. The correct answer is (E). This is the limit of the difference quotient that is the definition of the derivative. All this means is that $f'(5) = 3$.

For I., does the function's value necessarily equal the derivative's value? No, so I. is out. Since the derivative exists at $x = 5$, the function is differentiable there. Remember that differentiability implies continuity, so the function must be continuous at $x = 5$ as well.

33. The correct answer is (C). Use the calculator to graph the derivative given. Where the graph changes from positive to negative will be the local maximum. This occurs at $x = 0.511$.

34. The correct answer is (B). Related rates—oh boy! First, let's draw the following diagram:



We are looking for $\frac{dC}{dt}$. We know that $\frac{dA}{dt} = -40$ and $\frac{dB}{dt} = -30$. Now, we need an equation to relate A, B, and C. Since this is a right triangle, we can certainly use the Pythagorean theorem:

$$A^2 + B^2 = C^2$$

Differentiating with respect to t, which we do in every related-rates problem, yields

$$(2)\ 2A\frac{dA}{dt} + 2B\frac{dB}{dt} = 2C\frac{dC}{dt}$$

Solving this equation for $\frac{dC}{dt}$ gives us

$$\frac{A\frac{dA}{dt} + B\frac{dB}{dt}}{C} = \frac{dC}{dt}$$

What are the values of A, B, and C? To answer this, we use the facts that the cars each started 100 miles from Millville and have been traveling for 90 minutes or $\frac{3}{2}$ hours. Car A has traveled 60 miles, so $A = 40$. Car B has traveled 45 miles, so $B = 55$. By the Pythagorean theorem, $C = \sqrt{40^2 + 55^2}$. Substituting these values into the equation yields

$$\frac{dC}{dt} = \frac{40(-40) + 55(-30)}{\sqrt{40^2 + 55^2}} = -47.79$$

35. The correct answer is (D). This problem is best answered using the graphing calculator. If we examine the graph, we can see that it is continuous at $x = -1$. The graph has a jump discontinuity at $x = 1$; it is not differentiable there. Since the graph is increasing before and decreasing after $x = -1$, there is a local maximum at $x = -1$.

36. The correct answer is (A). This is an optimization problem. Let's first express the product x^2y only as a function of x:

$$p(x) = x^2(3x - 7) = 3x^3 - 7x^2$$

Next, we differentiate and set the derivative equal to zero and solve for x to determine our critical values:

$$p'(x) = 9x^2 - 14x = 0$$
$$x(9x - 14) = 0$$
$$x = 0 \text{ or } x = \frac{14}{9}.$$

By examining the wiggle graph below, we can see that the function's minimum occurs at $x = \frac{14}{9}$.

The problem asks for the minimum product. So, we must substitute $\frac{14}{9}$ back into $p(x)$, and we get -5.646.

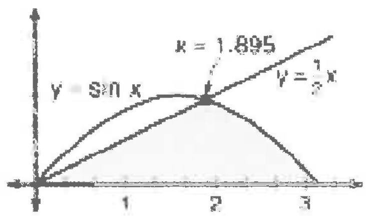
37. The correct answer is (C). Use your calculator to determine where these two curves intersect. This intersection point will give us a limit of integration. Since the graph of $y = \sin x$ is above the graph of $y = \frac{1}{4}x - 1$, we integrate to find the area:

$$A = \int_0^{3.314}\left(\sin x - \left(\frac{1}{4}x - 1\right)\right)dx$$

Our calculator will then do all the work and give us

$$A = 3.926$$

38. The correct answer is (E). Be careful here—make sure you have the right region, as shown in the diagram below:



The best method to use, since we are rotating about the y-axis, would be shells. However, we will need to break it up into two regions.

$$V = 2\pi\left(\int_0^{1.895}\left(x\left(\frac{1}{2}x\right)\right)dx + \int_{1.895}^{\pi}\left(x\left(\sin x\right)\right)dx\right)$$

Using our calculator,

$$V = 17.117$$

39. The correct answer is (E). Set the derivative of f equal to -1.024, and solve. By the quotient rule, we have

$$f'(x) = \frac{e^x 9x^2 - e^x 3x^3}{e^{2x}} = -1.024.$$

Graphing and determining the intercept yields $x = 4.797$.

40. The correct answer is (C). Since g is an antiderivative of f, then f is the derivative of g. The graph of the derivative of g changes from negative to positive at $x = c$, so g has a minimum there.

41. The correct answer is (B). Use your calculator for this one. First, determine the second derivative of f. Graph it, and find the x-intercept.

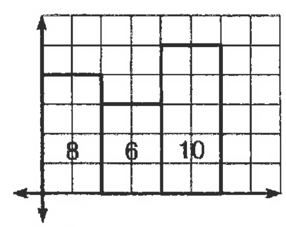
$$f'(x) = 5x^4 - 2x + \cos x$$

$$f''(x) = 20x^3 - 2 - \sin x$$

Our calculator shows us that the x-intercept of this second derivative is 0.4985.

42. The correct answer is (C). We have the graph of the derivative, but we are looking for the graph of the function. Since the derivative's graph is continuous, there are no discontinuities, cusps, or vertical tangents on the function's graph. This eliminates choices (A), (D), and (E). Since the derivative is positive and increasing from $x = 0$ to $x = a$, the function must be increasing and concave up over this same interval. Between the two choices remaining, only (C) meets this requirement.

43. The correct answer is (D). This problem requires a little drawing. We should plot the seven points, draw the rectangles, find the area of each one, and add them up.



$$8 + 6 + 10 = 24$$

44. The correct answer is (E). Let's examine each statement: Since there is an obvious cusp at $x = a$, (A) is a true statement. (B) says that the left-hand derivative does not equal the right-hand derivative at $x = a$, which means that there must be a cusp there—which there is. So, this is true also. The hole at $x = c$ would indicate that $f(c)$ is undefined, so (C) is true. (D) is true because there is a horizontal tangent at $x = b$. Since the discontinuity at $x = c$ is removable, the $\lim_{x\to c} f(x)$ exists,

$\lim_{x\to c^-}(f(x)) \neq \lim_{x\to c^+}(f(x))$ is false.

45. The correct answer is (D). In this problem, the question is for what value of k will the area under $\int_0^k\left(\sqrt{x}\,dx\right)$ equal $\int_0^2\left(x^2\,dx\right)$? Setting these two integrals equal to each other and solving for x yields