Tajima HS Solis AP CALCULUS

Unit 4: Integration Assessment

Name ______________________________ **PER** ________ **DATE** ____________

4B	4C	4D

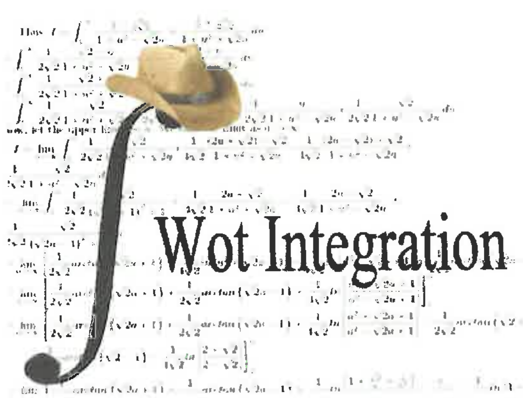
Computation

4	3	2	1
Response has no recall errors, *minimal* procedural errors* and no conceptual errors**	**Response has no recall errors, minimal procedural errors and *minimal* conceptual errors**	**Response has no recall errors, but has several procedural errors OR several conceptual errors**	**Recall errors exist OR Steps taken are not related to problem OR Response left blank**

Written Responses

4	3	2	1
Response is written in a complete sentence and uses appropriate academic vocab	**Response is written in a complete sentence, and minimal errors exist in use of academic vocab**	**Response is not written in a complete sentence OR no academic vocab**	**Concept of response is not related to problem OR Response is left blank**

***Procedural errors** are mistakes made in the math
****Conceptual errors** are mistakes made in the steps one take



BOX YOUR ANSWERS!!!

SCORES	LT 4B
Your predicted score:	1. Using academic language, what is an integral of a function? Provide an example.
	The integral of a function is its "antiderivative," or the answer to, "What is this derivative's original function?" Ex: $f(x) = 2x$, $\int f(x)\,dx = x^2 + C$
Mr. Solis' score:	2. What is the significance of the '+C' in an indefinite integral?
	When you find a derivative's original function, there are actually an infinite number of possibilities since the final constant is unknown. Thus, we include the placeholder "+C"
	3. Evaluate the integrals below
	a. $\int \frac{x^2+2x-3}{x^4}\,dx$
	$= \int \frac{x^2}{x^4} + \frac{2x}{x^4} - \frac{3}{x^4}\,dx$
	$= \int x^{-2} + 2x^{-3} - 3x^{-4}\,dx$
	$= \frac{x^{-1}}{-1} + \frac{2x^{-2}}{-2} - \frac{3x^{-3}}{-3} + C$
	$= \frac{-1}{x} - \frac{1}{x^2} + \frac{1}{x^3} + C$
	b. $\int -\sec x \cdot \tan x\,dx$
	$= -\int \sec x \tan x\,dx$
	$= -\sec x + C$

SCORES	LT 4C
Your predicted *average* score: ______ **Mr. Solis' score:** ______	*4. Evaluate the indefinite integral below.*

a. $\int 8x^3(2x^4-1)^4\,dx$

b. $\int -18x^2\cos(2x^3-3)\,dx$

a.
$u = 2x^4 - 1$
$\frac{du}{dx} = 8x^3$
$dx = \frac{du}{8x^3}$

$= \int 8x^3 (u)^4 \frac{du}{8x^3}$
$= \int u^4\,du$
$= \frac{u^5}{5} + C$
$= \boxed{\frac{(2x^4-1)^5}{5} + C}$

b.
$u = 2x^3 - 3$
$\frac{du}{dx} = 6x^2$
$dx = \frac{du}{6x^2}$

$= \int -18x^2 \cos(u)\frac{du}{6x^2}$
$= -3\int \cos u\,du$
$= -3(\sin u) +$
$= -3\sin u + C$
$= \boxed{-3\sin(2x^3+3)+C}$

5. Evaluate the indefinite integral below.

$\int \frac{15x^4}{3x^5\sqrt{9x^{10}-9}}\,dx = \int \frac{15x^4}{3x^5\sqrt{(3x^5)^2-3^2}}\,dx$

$u = 3x^5$
$\frac{du}{dx} = 15x^4$
$dx = \frac{du}{15x^4}$

$= \int \frac{15x^4}{u\sqrt{u^2-3^2}}\frac{du}{15x^4}$

$= \int \frac{1}{u\sqrt{u^2-3^2}}\,du = \frac{1}{3}\sec^{-1}\frac{|u|}{3} + C = \boxed{\frac{1}{3}\sec^{-1}\frac{|3x^5|}{3} +}$

*6. Explain your reasoning for choosing what **u** equals in Question 5.*

I recognized the ~~[illegible]~~ integral formula for $\sec^{-1}(u)$ and knew u must be '$3x^5$'

OR

I ~~[illegible]~~ knew using '$u = 3x^5$' would cancel $15x^4$ in the numerator after finding $\frac{du}{dx}$

SCORES	LT 4D
Your predicted *average* score: ______ Mr. Solis' score: ______	7. Solve the differential equation below

$$y' = 2xy, \quad y(0) = 1$$

$$\frac{dy}{dx} = 2xy$$

$$\int \frac{dy}{y} = \int 2x\, dx$$

$$e^{\ln y} = e^{x^2 + C}$$

$$y = e^{x^2 + C} = e^{x^2} e^{C} = Ce^{x^2}$$

$$\boxed{y = Ce^{x^2}}$$

plug in (0,1)

$$1 = Ce^{(0)^2}$$

$$1 = C(1)$$

$$1 = C$$

so

$$\boxed{y = e^{x^2}}$$

8. Solve the differential equation below.

$$\frac{ds}{dt} = \cos t + \sin t, \qquad s(\pi) = 1$$

$$\int ds = \int \cos t + \sin t\, dt$$

$$s = \sin t - \cos t + C$$

plug in $(\pi, 1)$

$$1 = \sin(\pi) - \cos(\pi) + C$$

$$1 = 0 - (-1) + C$$

$$1 = 1 + C$$

$$0 = C \quad \text{so} \quad \boxed{s = \sin t - \cos t}$$

9.
Given the velocity,

$$v = \frac{ds}{dt} = 32t - 2,$$

and the initial position of the body as s (1/2) = 4. Find the body's position at time t.

$$\frac{ds}{dt} = 32t - 2$$

$$\int ds = \int 32t - 2\, dt$$

$$s = \frac{32t^2}{2} - 2t + C$$

$$s = 16t^2 - 2t + C$$

$$(4) = 16\left(\tfrac{1}{2}\right)^2 - 2\left(\tfrac{1}{2}\right) + C$$

$$4 = 16\left(\tfrac{1}{4}\right) - 1 + C$$

$$4 = 4 - 1 + C$$

$$1 = C$$

so $\boxed{s = 16t^2 - 2t + 1}$

10.
Given the acceleration, a = d^2s/dt^2 = -4sin 2t, initial velocity v (0) = 2, and the initial position of the body as s (0) = -3. Find the body's position at time t.

$$\int -4\sin 2t\, dt$$

Find 'v'

$u = 2t$

$\frac{du}{dt} = 2$

$dt = \frac{du}{2}$

$$= \int -4 \sin u \frac{du}{2}$$

$$= -2\int \sin u\, du$$

$$= -2(-\cos u) + C$$

$$= 2\cos u + C$$

$$v = 2\cos 2t + C$$

$$2 = 2\cos 2(0) + C$$

$$2 = 2\cos 0 + C$$

$$2 = 2(1) + C$$

$0 = C$ so $v = 2\cos 2t$

Find 's'

$$\int 2\cos 2t\, dt$$

$u = 2t$

$\frac{du}{dt} = 2$

$dt = \frac{du}{2}$

$$\int 2\cos u \frac{du}{2}$$

$$= \sin u + C$$

$$= \sin 2t + C$$

$$-3 = \sin 2(0) + C$$

$$-3 = 0 + C$$

$$-3 = C$$

$\boxed{s = \sin(2t) - 3}$