Unit 3: Application of Derivatives

Name ______________________ **PER** ________ **DATE** ____________

11.0	4.11	12.0	4.4
LT 3A	**LT 3B**	**LT 3C**	**LT 3D**

Computation

4	3	2	1
Response has no recall errors, *minimal* procedural errors* and no conceptual errors**	**Response has no recall errors, minimal procedural errors and *minimal* conceptual errors**	**Response has no recall errors, but has several procedural errors <u>OR</u> several conceptual errors**	**Recall errors exist <u>OR</u> Steps taken are not related to problem <u>OR</u> Response left blank**

Written Responses

4	3	2	1
Response is written in a complete sentence and uses appropriate academic vocab	**Response is written in a complete sentence, and minimal errors exist in use of academic vocab**	**Response is not written in a complete sentence <u>OR</u> no academic vocab**	**Concept of response is not related to problem <u>OR</u> Response is left blank**

***Procedural errors** are mistakes made in the math
****Conceptual errors** are mistakes made in the steps one take

BOX YOUR ANSWERS and WRITE IN A COMPLETE SETENCE!!!

SCORES	LT 3A
Your predicted score: ______ Mr. Solis' score: ______	1. Find two positive numbers whose product is 750 and for which the sum of one and 10 times the other is a minimum.

U

P

$x \cdot y = 750 \rightarrow y = \frac{750}{x}$

$x + 10y = M$

$x + 10\left(\frac{750}{x}\right) = M$

$x + 7500x^{-1} = M$

$1 - 7500x^{-2} = M'$

$1 - 7500x^{-2} = M'$

S

$\left(1 - \frac{7500}{x^2} = 0\right)x^2$

$x^2 - 7500 = 0$

$x^2 = 7500$

$\boxed{x = 86.602}$ or $50\sqrt{3}$

$y = \frac{750}{86.602} = \boxed{8.66 = y}$ or $5\sqrt{3}$

2.
Which points on the graph of $y = 8 - x^2$ are closest to the point $(0, 4)$?

P

$D = \sqrt{(y_2 - y_1)^2 + (x_2 - x_1)^2}$

$D = \sqrt{(8 - x^2 - 4)^2 + (x - 0)^2}$

$D = \sqrt{(4 - x^2)^2 + x^2}$

$D = \sqrt{16 - 8x^2 + x^4 + x^2}$

$D = \sqrt{16 - 7x^2 + x^4}$

S

$D' = \frac{1}{2}(16 - 7x^2 + x^4)^{-\frac{1}{2}} \cdot (-14x + 4x^3)$

$D' = \frac{-14x + 4x^3}{2\sqrt{16 - 7x^2 + x^4}}$

$\left(0 = \frac{-14x + 4x^3}{2\sqrt{16 - 7x^2 + x^4}}\right) 2\sqrt{16 - 7x^2 + x^4}$

$0 = -14x + 4x^3$

$0 = 2x(-7 + 2x^2)$

$x = 0$ or $x = \pm\sqrt{\frac{7}{2}}$

✓

$y = 8 - \left(\sqrt{\frac{7}{2}}\right)^2 = 8 - \frac{7}{2}$

$= \frac{16 - 7}{2}$

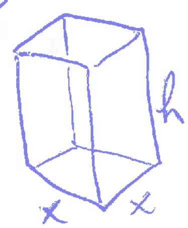
$= \frac{9}{2}$

$\boxed{\left(\sqrt{\frac{7}{2}}, \frac{9}{2}\right) \quad \left(-\sqrt{\frac{7}{2}}, \frac{9}{2}\right)}$

There are two points closest to $(0, 4)$

Your predicted score:	**3.** **Engineers are designing a box-shaped aquarium with a square bottom and an open top. The aquarium must hold 256 ft³ of water. What dimensions should they use to create an acceptable aquarium with the least amount of glass?**
Mr. Solis' score:	

V



B

$V = x^2 h$

$x^2 h = 256$

$h = \frac{256}{x^2}$

$SA = 4xh + x^2$

$SA = 4x\left(\frac{256}{x^2}\right) + x^2$

$SA = \frac{1024}{x} + x^2$

S

$SA' = -\frac{1024}{x^2} + 2x$

$\left(0 = -\frac{1024}{x^2} + 2x\right)x^2$

$0 = -1024 + 2x^3$

$1024 = 2x^3$

$512 = x^3$

$8 = x$

$h = \frac{256}{8^2} = 4$

$\boxed{\text{The engineers should make the box } 8\text{ft} \times 8\text{ft} \times 4\text{ft}}$

SCORES	LT 3B
Your predicted score:	
Mr. Solis' score:	

For 4 – 6, use implicit differentiation to solve for dy/dx.

4. $4x^2 = -2xy - xy^2 + 5$

$8x = \left[-2x\frac{dy}{dx} + -2y\right] - \left[x\,2y\frac{dy}{dx} + 1y^2\right] + 0$

$8x = -2x\frac{dy}{dx} - 2y - 2xy\frac{dy}{dx} - y^2$

$8x + 2y + y^2 = -2x\frac{dy}{dx} - 2xy\frac{dy}{dx} \Rightarrow 8x + 2y + y^2 = \frac{dy}{dx}(-2x - 2xy)$

$\boxed{\frac{8x + 2y + y^2}{-2x - 2xy} = \frac{dy}{dx}}$

5. $2 = 3x^2 + 3x^2y^2 + y^2$

$0 = 6x + \left[3x^2 \cdot 2y\frac{dy}{dx} + 6x \cdot y^2\right] + 2y \cdot \frac{dy}{dx}$

$0 = 6x + 3x^2 2y\frac{dy}{dx} + 6xy^2 + 2y\frac{dy}{dx}$

$-6x - 6xy^2 = 3x^2 2y\frac{dy}{dx} + 2y\frac{dy}{dx}$

$-6x - 6xy^2 = \frac{dy}{dx}(6x^2y + 2y)$

$\frac{-6x - 6xy^2}{6x^2y + 2y} = \frac{dy}{dx}$

$\boxed{\frac{-3x - 3xy^2}{3x^2y + y} = \frac{dy}{dx}}$

6. $x^3 + \cos y - 4x^2 = 10$

$3x^2 - \sin y\frac{dy}{dx} - 8x = 0$

$-\sin y\frac{dy}{dx} = 8x - 3x^2$

$\boxed{\frac{dy}{dx} = \frac{-8x + 3x^2}{\sin y}}$

SCORES	LT 3C
Your predicted score: **Mr. Solis' score:**	7. The sides of a square are increasing at a rate of 10 cm/sec. How fast is the area enclosed by the square increasing when the area is 150 cm².

U ds/dt

$A = 150 \text{ cm}^2$

P

$A = s^2$

$\frac{dA}{dt} = 2s \cdot \frac{ds}{dt}$ what's s? $150 = s^2$ $\sqrt{150} = s$ $12.247 = s$

S $\frac{dA}{dt} = 2(12.247)(10)$

$\boxed{\frac{dA}{dt} = 244.949 \frac{cm^2}{sec}}$

The square's area is increasing at a rate of $244.949 \frac{cm^2}{sec}$

8. A spherical balloon is being filled in such a way that the surface area is increasing at a rate of 20 cm²/sec when the radius is 2 meters. At what rate is air being pumped in the balloon when the radius is 2 meters.

U

P

$V = \frac{4}{3}\pi r^3$ $SA = 4\pi r^2$

$\frac{dSA}{dt} = 2(4)\pi r \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \cdot \frac{dr}{dt}$

$20 = 8\pi(2)\frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ what's $\frac{dr}{dt}$? $\Rightarrow \frac{20}{16\pi} = \frac{dr}{dt}$

See last page

$\frac{dV}{dt} = 4\pi(2)^2\left(\frac{20}{16\pi}\right)$

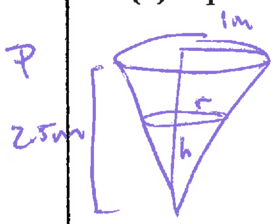
$\boxed{\frac{dV}{dt} = 20 \frac{cm^3}{sec}}$

The volume of the balloon is changing at a rate of 20 cm³/sec

U

9. A tank of water is in the shape of a cone (assume the "point" of the cone is pointing downwards) and is leaking water at a rate of 35 cm^3/sec. The base radius of the tank is 1 meter and the height of the tank is 2.5 meters. When the depth of the water is 1.25 meters at what rate is the (a) depth changing and (b) the radius of the top of the water changing?

P



$\frac{dV}{dt} = -35\,\frac{cm^3}{sec}$

$V = \frac{1}{3}\pi r^2 h$

$h = 1.25$

(a) $\frac{dh}{dt} = ?$

(b) $\frac{dr}{dt} = ?$

Ratio: $\frac{1}{2.5} = \frac{r}{h}$

$h = 2.5r$

$\frac{h}{2.5} = r$

Plug in r

$V = \frac{1}{3}\pi\left(\frac{h}{2.5}\right)^2 h$

$V = \frac{1}{3}\pi \frac{h^2}{6.25} h$

$V = \frac{h^3\pi}{18.75}$

Derivative

$\frac{dV}{dt} = \frac{3h^2\pi}{18.75}\cdot\frac{dh}{dt}$

Solve

~~$\frac{3h^2\pi}{18.75}\frac{dh}{dt}$~~

follow arrow

$-35 = \frac{3(1.25)^2\pi}{18.75}\frac{dh}{dt}$

$-35\left(\frac{18.75}{3(1.25)^2\pi}\right) = \frac{dh}{dt}$

$\frac{-656.25}{14.719} = \frac{dh}{dt}$

(a) $\boxed{44.585\,\frac{cm}{sec} = \frac{dh}{dt}}$

(b) $\frac{h}{2.5} = r$

$\frac{1}{2.5}\frac{dh}{dt} = \frac{dr}{dt}$

$\frac{1}{2.5}(44.585) = \frac{dr}{dt}$

$\boxed{17.834\,\frac{cm}{sec} = \frac{dr}{dt}}$

SCORES	LT 3D
Your predicted score:	For each problem, find a linear approximation of the given quantity.
Mr. Solis' score:	

10) $\cos 89°$

$89 \times \frac{\pi}{180} = \frac{89\pi}{180}$ $\frac{90\pi}{180}$

$x = \frac{89\pi}{180}$

$a = \frac{90\pi}{180}$

$f(x) = \cos x$

$f'(x) = -\sin x$

$y \cong f(a) + f'(a)(x-a)$

$y \cong \cos(\frac{\pi}{2}) - \sin(\frac{\pi}{2})(\frac{89\pi}{180} - \frac{90\pi}{180})$

$y \cong 0 - (1)(\frac{-\pi}{180})$

$\boxed{y \cong \frac{\pi}{180}}$

11) $\sqrt{16.2}$

$x = 16.2$

$a = 16$

$f(x) = \sqrt{x}$

$f'(x) = \frac{1}{2\sqrt{x}}$

$y \cong f(a) + f'(a)(x-a)$

$y \cong \sqrt{16} + \frac{1}{2\sqrt{16}}(16.2-16)$

$y \cong 4 + \frac{1}{2(4)}(.2) = \boxed{4.02}$

12. The sides of a square are measured to be 5 ft, with a possible error of $\pm\frac{1}{5}$ ft. Estimate the possible propagated error in the calculated area.

$A = s^2$

$\frac{dA}{dt} = 2s\frac{ds}{dt}$

$\frac{dA}{dt} = 2(5)(\frac{1}{5})$

$\frac{dA}{dt} = \pm 2 ft^2$

8. (After Mr Solis realized that both 'cm' and 'm' are being used 😑)

U $\frac{dSA}{dt} = \frac{20\,cm^2}{sec}$ when $r = \cancel{2m}$ 200 cm

$\frac{dV}{dt} = ?$

P

$V = \frac{4}{3}\pi r^3$

$\frac{dV}{dt} = 3\left(\frac{4}{3}\right)\pi r^2 \cdot \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi r^2 \frac{dr}{dt}$ what's $\frac{dr}{dt} = ?$

$SA = 4\pi r^2$

$\frac{dSA}{dt} = 8\pi r \cdot \frac{dr}{dt}$

$20 = 8\pi(200)\frac{dr}{dt}$

$\frac{20}{8\pi(200)} = \frac{dr}{dt}$

$.004\,\frac{cm}{sec} = \frac{dr}{dt}$

$\frac{dV}{dt} = 4\pi(200cm)^2\left(.004\frac{cm}{sec}\right)$

$\frac{dV}{dt} = \cancel{4\pi}(200\,cm)^{\cancel{2}}\left(\frac{20}{2\cdot\cancel{8\pi}(\cancel{200})}\frac{cm}{sec}\right)$

$\frac{dV}{dt} = \frac{4000}{2}\frac{cm^3}{sec}$

$\boxed{\frac{dV}{dt} = 2000\frac{cm^3}{sec}}$

The volume of the balloon is changing at a rate of $2000\frac{cm^3}{sec}$

7. ii