t (minutes)	0	4	8	12	16
$H(t)$ (°C)	65	68	73	80	90

3. The temperature, in degrees Celsius (°C), of an oven being heated is modeled by an increasing differentiable function H of time t, where t is measured in minutes. The table above gives the temperature as recorded every 4 minutes over a 16-minute period.

   (a) Use the data in the table to estimate the instantaneous rate at which the temperature of the oven is changing at time $t = 10$. Show the computations that lead to your answer. Indicate units of measure.

   (b) Write an integral expression in terms of H for the average temperature of the oven between time $t = 0$ and time $t = 16$. Estimate the average temperature of the oven using a left Riemann sum with four subintervals of equal length. Show the computations that lead to your answer.

   (c) Is your approximation in part (b) an underestimate or an overestimate of the average temperature? Give a reason for your answer.

   (d) Are the data in the table consistent with or do they contradict the claim that the temperature of the oven is increasing at an increasing rate? Give a reason for your answer.

(a) $H'(10) \approx \dfrac{H(12)-H(8)}{12-8} = \dfrac{80-73}{4} = \dfrac{7}{4}\,°\text{C/min}$	$2: \begin{cases} 1: \text{difference quotient} \\ 1: \text{answer with units} \end{cases}$
(b) Average temperature is $\frac{1}{16}\int_0^{16} H(t)\,dt$ $\int_0^{16} H(t)\,dt \approx 4\cdot(65+68+73+80)$ Average temperature $\approx \dfrac{4\cdot 286}{16} = 71.5°\text{C}$	$3: \begin{cases} 1: \frac{1}{16}\int_0^{16} H(t)\,dt \\ 1: \text{left Riemann sum} \\ 1: \text{answer} \end{cases}$
(c) The left Riemann sum approximation is an underestimate of the integral because the graph of H is increasing. Dividing by 16 will not change the inequality, so $71.5°\text{C}$ is an underestimate of the average temperature.	1 : answer with reason
(d) If a continuous function is increasing at an increasing rate, then the slopes of the secant lines of the graph of the function are increasing. The slopes of the secant lines for the four intervals in the table are $\frac{3}{4}$, $\frac{5}{4}$, $\frac{7}{4}$, and $\frac{10}{4}$, respectively. Since the slopes are increasing, the data are consistent with the claim. OR By the Mean Value Theorem, the slopes are also the values of $H'(c_k)$ for some times $c_1 < c_2 < c_3 < c_4$, respectively. Since these derivative values are positive and increasing, the data are consistent with the claim.	$3: \begin{cases} 1: \text{considers slopes of four secant lines} \\ 1: \text{explanation} \\ 1: \text{conclusion consistent with explanation} \end{cases}$

2013AP AB

t (days)	0	10	22	30
$W'(t)$ (GL per day)	0.6	0.7	1.0	0.5

3. The twice-differentiable function W models the volume of water in a reservoir at time t, where $W(t)$ is measured in gigaliters (GL) and t is measured in days. The table above gives values of $W'(t)$ sampled at various times during the time interval $0 \le t \le 30$ days. At time $t = 30$, the reservoir contains 125 gigaliters of water.

(a) Use the tangent line approximation to W at time $t = 30$ to predict the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 32$. Show the computations that lead to your answer.

(b) Use a left Riemann sum, with the three subintervals indicated by the data in the table, to approximate $\int_0^{30} W'(t)\,dt$. Use this approximation to estimate the volume of water $W(t)$, in gigaliters, in the reservoir at time $t = 0$. Show the computations that lead to your answer.

(c) Explain why there must be at least one time t, other than $t = 10$, such that $W'(t) = 0.7$ GL/day.

(d) The equation $A = 0.3W^{2/3}$ gives the relationship between the area A, in square kilometers, of the surface of the reservoir, and the volume of water $W(t)$, in gigaliters, in the reservoir. Find the instantaneous rate of change of A, in square kilometers per day, with respect to t when $t = 30$ days.

(a) An equation of the tangent line is $y = 0.5(t-30)+125$. $W(32) \approx 0.5(32-30)+125 = 126$	1 : answer
(b) $\int_0^{30} W'(t)\,dt \approx (10)(0.6)+(12)(0.7)+(8)(1.0) = 22.4$ $W(0) = W(30) - \int_0^{30} W'(t)\,dt = 125 - 22.4 = 102.6$	3 : $\begin{cases} 1 : \text{left Riemann sum} \\ 1 : \text{approximation} \\ 1 : \text{answer} \end{cases}$
(c) W' is differentiable $\Rightarrow$ W' is continuous. $W'(30) = 0.5 < 0.7 < 1.0 = W'(22)$ By the Intermediate Value Theorem, there must be at least one time t, $22 \le t \le 30$, such that $W'(t) = 0.7$.	2 : explanation
(d) $\frac{dA}{dt} = (0.3)\frac{2}{3}W^{-1/3}\cdot\frac{dW}{dt} = \frac{0.2}{\sqrt[3]{W}}\cdot\frac{dW}{dt}$ $\left.\frac{dA}{dt}\right\|_{t=30} = \frac{0.2}{\sqrt[3]{125}}\cdot 0.5 = 0.02$	3 : $\begin{cases} 2 : \frac{dA}{dt} \\ 1 : \text{answer} \end{cases}$

2012 AP AB

t (minutes)	0	4	9	15	20
$W(t)$ (degrees Fahrenheit)	55.0	57.1	61.8	67.9	71.0

1. The temperature of water in a tub at time t is modeled by a strictly increasing, twice-differentiable function W, where $W(t)$ is measured in degrees Fahrenheit and t is measured in minutes. At time $t = 0$, the temperature of the water is 55°F. The water is heated for 30 minutes, beginning at time $t = 0$. Values of $W(t)$ at selected times t for the first 20 minutes are given in the table above.

   (a) Use the data in the table to estimate $W'(12)$. Show the computations that lead to your answer. Using correct units, interpret the meaning of your answer in the context of this problem.

(b) Use the data in the table to evaluate $\int_0^{20} W'(t)\,dt$. Using correct units, interpret the meaning of $\int_0^{20} W'(t)\,dt$ in the context of this problem.

(c) For $0 \le t \le 20$, the average temperature of the water in the tub is $\frac{1}{20}\int_0^{20} W(t)\,dt$. Use a left Riemann sum with the four subintervals indicated by the data in the table to approximate $\frac{1}{20}\int_0^{20} W(t)\,dt$. Does this approximation overestimate or underestimate the average temperature of the water over these 20 minutes? Explain your reasoning.

(d) For $20 \le t \le 25$, the function W that models the water temperature has first derivative given by $W'(t) = 0.4\sqrt{t}\cos(0.06t)$. Based on the model, what is the temperature of the water at time $t = 25$?

(a) $W'(12) \approx \frac{W(15) - W(9)}{15 - 9} = \frac{67.9 - 61.8}{6}$ $= 1.017$ (or 1.016) The water temperature is increasing at a rate of approximately 1.017 °F per minute at time $t = 12$ minutes.	2 : { 1 : estimate 1 : interpretation with units
(b) $\int_0^{20} W'(t)\, dt = W(20) - W(0) = 71.0 - 55.0 = 16$ The water has warmed by 16 °F over the interval from $t = 0$ to $t = 20$ minutes.	2 : { 1 : value 1 : interpretation with units
(c) $\frac{1}{20}\int_0^{20} W(t)\, dt \approx \frac{1}{20}(4 \cdot W(0) + 5 \cdot W(4) + 6 \cdot W(9) + 5 \cdot W(15))$ $= \frac{1}{20}(4 \cdot 55.0 + 5 \cdot 57.1 + 6 \cdot 61.8 + 5 \cdot 67.9)$ $= \frac{1}{20} \cdot 1215.8 = 60.79$ This approximation is an underestimate, because a left Riemann sum is used and the function W is strictly increasing.	3 : { 1 : left Riemann sum 1 : approximation 1 : underestimate with reason
(d) $W(25) = 71.0 + \int_{20}^{25} W'(t)\, dt$ $= 71.0 + 2.043155 = 73.043$	2 : { 1 : integral 1 : answer