1992 AB6

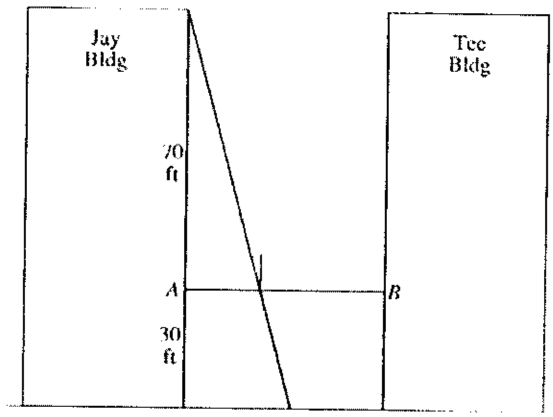
At time t, $t \geq 0$, the volume of a sphere is increasing at a rate proportional to the reciprocal of its radius. At $t = 0$, the radius of the sphere is 1 and at $t = 15$, the radius is 2. (The volume V of a sphere with a radius r is $V = \frac{4}{3}\pi r^3$.)

(a) Find the radius of the sphere as a function of t.

(b) At what time t will the volume of the sphere be 27 times its volume at $t = 0$?

1991 AB6

A tight rope is stretched 30 feet above the ground between the Jay and the Tee buildings, which are 50 feet apart. A tightrope walker, walking at a constant rate of 2 feet per second from point A to point B, is illuminated by a spotlight 70 feet above point A, as shown in the diagram.



(a) How fast is the shadow of the tightrope walker's feet moving along the ground when she is midway between the buildings? (Indicate units of measure.)

(b) How far from point A is the tightrope walker when the shadow of her feet reaches the base of the Tee Building? (Indicate units of measure.)

(c) How fast is the shadow of the tightrope walker's feet moving up the wall of the Tee Building when she is 10 feet from point B? (Indicate units of measure.)

1990 AB4

The radius r of a sphere is increasing at a constant rate of 0.04 centimeters per second. (Note: The volume of a sphere with radius r is $V = \frac{4}{3}\pi r^3$.)

(a) At the time when the radius of the sphere is 10 centimeters, what is the rate of increase of its volume?

(b) At the time when the volume of the sphere is 36π cubic centimeters, what is the rate of increase of the area of a cross section through the center of the sphere?

(c) At the time when the volume and the radius of the sphere are increasing at the same numerical rate, what is the radius?