ALLIANCE: Q2

AP CALCULUS AB FREE-RESPONSE QUESTIONS

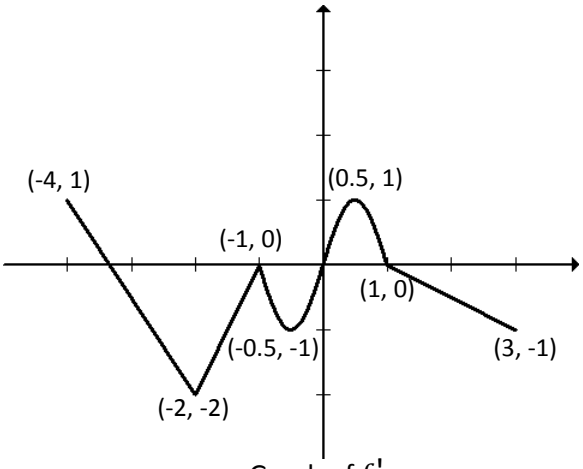
QUESTION 2

Let f' be the continuous function defined on [-3, 3] whose graph is shown above. The graph consists of three linear segments, with a sinusoidal curve on the interval [-1, 1].

(a) Find the values of $f'(-3)$ and $f''(-3)$.

(b) Find the x-coordinate of each point at which the graph of f has a horizontal tangent line. For each of these points, determine whether f has a relative minimum, relative maximum, or neither a minimum nor a maximum at the point. Justify your answers.

(c) For $-3 < x < 3$, find all values of x for which the graph of f has a point of inflection. Explain your reasoning.



Graph of f'

(a) $f'(-3) = -0.5$, $f''(-3) = -1.5$

2 : $\begin{cases} 1 : f'(-3) \\ 1 : f''(-3) \end{cases}$

(b) The graph of f has a horizontal tangent line where $f'(x) = 0$. This occurs at $x = -10/3$, $x = -1$, $x = 0$, and $x = 1$.

$f'(x)$ changes sign from positive to negative at $x = -10/3$ and at $x = 1$, so f has a relative maximum at $x = -10/3$ and at $x = 1$.

$f'(x)$ changes sign from negative to positive at $x = 0$, so f has a relative minimum at $x = 0$.

$f'(x)$ does not change sign at $x = -1$, so f has neither a minimum nor a maximum at $x = -1$.

4 : $\begin{cases} 2 : x = -10/3,\ x = -1,\ x = 0, \text{ and } x = 1 \\ 2 : \text{classification of relative extrema with justifications} \end{cases}$

(c) The graph of f has a point of inflection at each of $x = -2$, $x = -1$, $x = -0.5$, and $x = 0.5$, because f'' (or the slope of f') changes sign at each of these values.

3 : $\begin{cases} 2 : \text{answers} \\ 1 : \text{explanation} \end{cases}$