
Precalculus midterm 2 practice problems: One-to-one functions and inverses

1. Determine whether the following functions are one-to-one. Justify your answer with the technique of your choice.

a) $f(x) = x^2 - x$

c) $f(x) = 5x - 2$

b) $g(x) = -\sqrt{2-x}$

d) $g(x) = \frac{x}{x^2 - 9}$

2. Calculate the inverses of the following one-to-one functions.

a) $f(x) = \frac{x+5}{2x-5}$

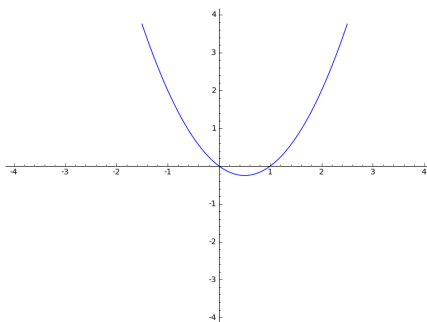
c) $f(x) = -5\sqrt{x+1}$

b) $g(x) = \sqrt[3]{x} + 1$

d) $g(x) = 6x - 10$

Answers

1. a) Ans: No
Reason 1: It fails the horizontal line test.

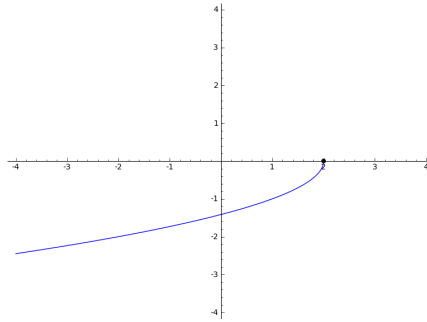


Reason 2: Since $x^2 - x = x(x - 1)$ we see that f has two roots at $x = 0$ and $x = 1$, so it can't be one-to-one.

Reason 3: Even degree polynomials are never one-to-one.

b) Ans: Yes

Reason 1: It passes the horizontal line test.

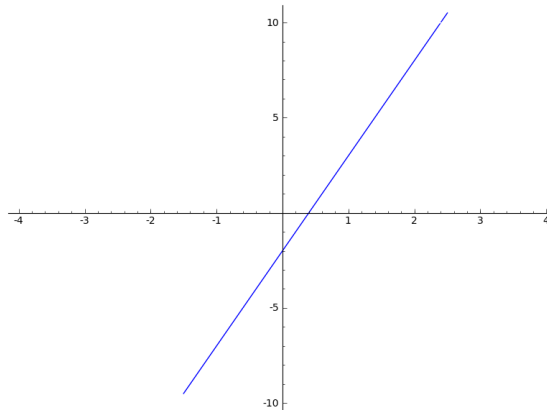


Reason 2: If you solve $y = -\sqrt{2-x}$ for x , you get only one solution $x = 2 - y^2$.

c) Ans: Yes

Reason 1: Non-horizontal lines are always one-to-one.

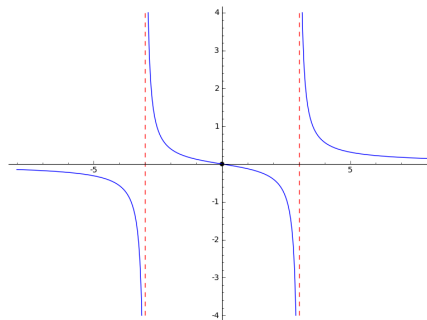
Reason 2: It passes the horizontal line test.



Reason 3: If you solve $y = 5x - 2$ for x , you get only one solution $x = \frac{1}{5}y + \frac{2}{5}$.

d) Ans: No

Reason 1: It fails the horizontal line test.



2. a) $f^{-1}(x) = \frac{5x + 5}{2x - 1}$
b) $g^{-1}(x) = (x - 1)^3$
c) $f^{-1}(x) = \frac{1}{25}x^2 - 1$
d) $g^{-1}(x) = \frac{1}{6}x + \frac{5}{3}$