## SETS, SET NOTATION, AND OPERATIONS WITH SETS

## SETS AND SET NOTATION

A set is a collection of objects called elements or members. The members of a set are either written using roster notation, which lists all the members of the set, or set-builder notation, which tells how the set is created. The elements of a set are displayed between brackets-- \{ \} and sets are named using capital letters.

For the multiples of five larger than 20,
Roster notation is $\{25,30,35,40,45, \ldots\}$. The three dots indicate to continue the pattern. Set-builder notation is $\{x \mid x>20$ and $x$ is a multiple of 5$\}$. This is read "the set of all $x$ such that $x$ is greater than 20 and a multiple of five."

Some very commonly used sets are:
$\mathbb{Z}$--the set of integers
$\mathbb{Q}$--the set of rational numbers, and
$\mathbb{N}$--the set of natural numbers
$\mathbb{R}$--the set of real numbers

## Examples

Write in roster and set-builder notation.

M , the set of whole numbers less than ten.
roster: $\quad \mathrm{M}=\{0,1,2,3,4,5,6,7,8,9\}$
set-builder: $\quad \mathrm{M}=\{x \mid x$ is whole and $x<10\}$

B, the set of even, negative numbers.
roster: $\quad \mathrm{B}=\{\ldots-8,-6,-4,-2\}$
set-builder: $\mathrm{B}=\{x \mid x$ is even and $x<0\}$

## Problems

Write in roster and set-builder notation.

1. A , the set of integers greater than or equal to -5 .
2. B. the integers divisible by three.
3. C, the integers between -2 and 2 inclusive.
4. D , the prime numbers less than 20.
5. E, the perfect squares between 10 and 100.
6. F, the odd integers less than 10.

## UNION OF SETS

The union of two sets A and B , written $A \cup B$, is found by combining all of the elements that are in A , in B , or in both. When two mathematical equations or inequalities are joined by the word "or," it means to find the union of the two individual solutions.

## Examples

Find the union of each pair of sets.
If $\mathrm{A}=\{1,3,6,10\}$ and $\mathrm{B}=\{3,6,9,12,15\}$ then $A \cup B=\{1,3,6,9,10,12,15\}$
If $S=\{2,4,6,8,10\}$ and $T=\{4,6,10\}$ then $S \bigcup T=\{2,4,6,8,10\}=\mathrm{S}$
Graph $\{x \mid x \geq-2\}$ or $\{x \mid x \geq 3\}$. The solution is the union of the two graphs.


## Problems

Find the union of the two given sets.
7. $A=\{-3,-2,-1,0,1\}$ and $\mathrm{B}=\{0,1,2,3\}$
8. $\mathrm{C}=\{0,1,2,3,4,5\}$ and $\mathrm{D}=\{0,5,10,15\}$
9. $\mathrm{E}=\{\ldots-4,-2,0,2,4, \ldots\}$ and $\mathrm{F}=\{\ldots-3,-1,1,3, \ldots\}$
10. $\mathrm{G}=\{0,2,4,6,8, \ldots\}$ and $\mathrm{H}=\{0,4,8,12, \ldots\}$

Graph the solution.
11. $\{x \mid x \leq 3\}$ or $\{x \mid x \leq 5\}$
12. $\{x \mid x \geq 2\}$ or $\{x \mid \leq-3\}$
13. $\{x \mid x \geq 2\}$ or $\{x \mid \leq 3\}$

## INTERSECTION OF SETS

The intersection of two sets A and B , written $A \cap B$, is found by listing all the elements that are common to both A and B . When two mathematical equations or inequalities are joined by the word "and," it means to find the intersection of the two individual solutions.

## Examples

Find the intersection of each pair of sets.
If $\mathrm{A}=\{1,3,6,10\}$ and $\mathrm{B}=\{3,6,9,12,15\}$ then $A \cap B=\{3,6\}$
If $S=\{2,4,6,8,10\}$ and $T=\{4,6,10\}$ then $S \cap T=\{4,6,10\}=T$
Graph $\{x \mid x \geq-2\}$ and $\{x \mid x \leq 3\}$. The solution is the intersection of the two graphs.


If two sets or two graphs have no common elements then their intersection is the empty set written $\}$ or $\varnothing$.

## Problems

Find the intersection of the two given sets.
14. $A=\{0,5,10,15,20\}$ and $B=\{0,10,20,30,40\}$
15. $\mathrm{C}=\{-3,-1,0,1,3,5,7\}$ and $\mathrm{D}=\{2,3,5,7,11,13\}$
16. $\mathrm{E}=\{0,2,4,6, \ldots\}$ and $\mathrm{F}=(0,4,8,12, \ldots\}$
17. $\mathrm{G}=\{1,3,5,7, \ldots\}$ and $\mathrm{H}=\{0,10,20,30, \ldots\}$

Graph the solution.
18. $\{x \mid x \geq 2\}$ and $\{x \mid \leq 5\}$
19. $\{x \mid x \leq-2\}$ and $\{x \mid \leq 5\}$
20. $\quad\{x \mid x \leq-2\}$ and $\{x \mid \geq 2\}$

## Answers

| $\text { 1. } \begin{aligned} \mathrm{A} & =\{-5,-4,-3,-2,-1,0, \ldots\} \\ \mathrm{A} & =\{x \mid x \text { is integer and } x \geq-5\} \end{aligned}$ | $\text { 2. } \begin{aligned} \mathrm{B} & =\{\ldots,-6,-3,0,3, \ldots\} \\ \mathrm{B} & =\{x \mid x \text { is divisible by } 3\} \end{aligned}$ |
| :---: | :---: |
| 3. $\begin{aligned} & \mathrm{C}=\{-2,-1,0,1,2\} \\ & \mathrm{C}=\{x \mid x \text { is integer and }-2 \leq x \leq 2\} \end{aligned}$ | 4. $\begin{aligned} & \mathrm{D}=\{2,3,5,7,11,13,17\} \\ & \mathrm{D}=\{x \mid x \text { is prime and } x<20\} \end{aligned}$ |
| $\text { 5. } \begin{aligned} \mathrm{E} & =\{16,25,36,49,64,81\} \\ \mathrm{E} & =\{x \mid x \text { is square and } 10<x<100\} \end{aligned}$ | 6. $\begin{aligned} \mathrm{F} & =\{\ldots,-1,1,3,5,7,9\} \\ \mathrm{F} & =\{x \mid x \text { is odd integer and } x<10\}\end{aligned}$ |
| 7. $\{-3,-2,-1,0,1,2,3\}$ | 8. $\{0,1,2,3,4,5,10,15\}$ |
| 9. $\{\ldots,-2,-1,0,1,2, \ldots\}$ or Z | 10. $\{0,2,4,6, \ldots\}$ |
| 11. | 12. |
| 13. all real numbers | 14. $\{0,10,20\}$ |
| 15. $\{3,5,7\}$ | 16. $\{0,4,8,12, \ldots\}$ |
| 17. $\varnothing$ | 18. |
| 19. | 20. $\varnothing$ |

