Domain and Range Worksheet

Given a function $y = f(x)$, the **Domain** of the function is the set of permissible inputs and the **Range** is the set of resulting outputs. Domains can be found algebraically; ranges are often found algebraically and graphically. Domains and Ranges are sets. Therefore, you must use proper set notation.

When finding the domain of a function, ask yourself what values can't be used. Your domain is everything else. There are simple basic rules to consider:

- The domain of all polynomial functions is the Real numbers **R**.
- Square root functions can not contain a negative underneath the radical. Set the expression under the radical greater than or equal to zero and solve for the variable. This will be your domain.
- Rational functions can not have zeros in the denominator. Determine which values of the input cause the denominator to equal zero, and set your domain to be everything else.

Examples: Consider $f(x) = x^3 - 6x^2 + 5x - 11, \quad g(t) = \sqrt{2 - 3t}, \quad h(p) = \dfrac{p-1}{p^2 - 4}$.

Answers:

- Since $f(x)$ is a polynomial, the domain of $f(x)$ is **R**.
- Since $g(t)$ is a square root, set the expression under the radical to greater than or equal to zero: $2 - 3t \geq 0 \rightarrow 2 \geq 3t \rightarrow 2/3 \geq t$. Therefore, the domain of $g(t) = \{\, t \mid t \leq 2/3 \,\}$. Confirm by graphing: you will see that the graph "lives" to the left of 2/3 on the horizontal axis.
- Since $h(p)$ is a rational function, the bottom can not equal zero. Set $p^2 - 4 = 0$ and solve: $p^2 - 4 = 0 \rightarrow (p + 2)(p - 2) = 0 \rightarrow p = -2$ or $p = 2$. These two p values need to be avoided, so the domain of $h(p)$ is $\{\, p \mid$ all **R** except $p = -2$ or 2 $\,\}$.

Comment on interval notation:

- The set of all reals can be abbreviated **R**, but not {**R**}. It can also be written $\{(-\infty, \infty)\}$ but not $\{[-\infty, \infty]\}$. It can also be written $\{\, -\infty < x < \infty \,\}$.
- For $h(p)$, the domain could be written any of the following ways: $\{(-\infty, -2) \cup (-2, 2) \cup (2, \infty)\}$ or $\{\mathbf{R} \backslash \{-2, 2\}\}$. The backslash \ is read as "except".

Whatever method you use, be consistent and correct.

Domain Practice:

Algebraically determine the following domains. Use correct set notation.

1. $d(y) = y + 3$
2. $g(k) = 2k^2 + 4k - 6$
3. $b(n) = \sqrt{2n - 8}$
4. $m(t) = \sqrt{9 - 3t}$
5. $u(x) = \dfrac{x - 5}{2x + 4}$
6. $a(r) = r + \dfrac{1}{r - 1}$
7. $y(c) = \dfrac{2}{c^2 + 3c}$
8. $q(w) = \dfrac{w + 4}{w^2 + 1}$

Answers at end.

Challenging domain problems: these contain combinations of functions.

9. $f(x) = \dfrac{x}{\sqrt{x + 3}}$

10. $t(v) = \sqrt{v^2 + 2v - 8}$

11. $n(t) = \sqrt{\dfrac{t}{1 + t}}$

12. $x(y) = y^4 + 2y + \sqrt{y} + \dfrac{1}{y}$

Range practice: Use your calculator to graph and determine the ranges of the functions numbered 1-5.

Answers:

1.   **R**, since the function is a polynomial (line).
2.   **R**, since the function is a polynomial (parabola).
3. $\{\, n \mid n \geq 4 \,\}$
4. $\{\, t \mid t \leq 3 \,\}$
5. $\{\, x \mid \mathbf{R} \text{ except } x = -2 \,\}$ or $\{\, x \mid \mathbf{R}\backslash\{-2\}\}$
6. $\{\, r \mid \mathbf{R} \text{ except } r = 1 \,\}$ or $\{\, r \mid \mathbf{R}\backslash\{1\}\}$
7. $\{\, c \mid \mathbf{R} \text{ except } c = 0 \text{ or } c = -3 \,\}$ or $\{\, c \mid \mathbf{R}\backslash\{0,-3\}\}$
8.   **R**. The denominator can not be solve for zero. No value of w causes the denominator to equal zero.
9. $\{\, x \mid x > -3 \,\}$. In this case, the radical can not contain negatives, while the denominator can not contain zero (a zero under the radical is acceptable, but it makes the bottom zero, which is not acceptable).
10. $\{\, v \mid v \leq -4 \text{ or } v \geq 2 \,\}$. The expression under the radical is a quadratic: it needs to be set greater than or equal to zero. Factor it and plot the points -4 and 2 (this is where the expression = 0, which is okay). Use a sign chart to determine when the expression is greater than zero.
11. $\{\, t \mid t < -1 \text{ or } t \geq 0 \,\}$. Set the expression $\frac{t}{1+t} \geq 0$. Do NOT cross multiply!!!! Determine where the top = 0 (top = 0 at $t = 0$) and where bottom = 0 (bottom = 0 at $t = -1$). Use a sign chart to determine when the expression is 0 or greater. Notice that it's okay for $t = 0$ but not okay for $t = -1$. Why?
12. $\{\, y \mid y > 0 \,\}$. For the first two terms, all y is acceptable. For the third term that has the radical, $y \geq 0$. But in the fourth term, $y \neq 0$, so we have to exclude the 0. The only set of numbers for which all four terms are defined is $y > 0$.

Ranges:

1.   **R**. It's a line.
2. $\{\, g(k) \mid g(k) \geq -8 \,\}$. It's a parabola. Find the vertex.
3. $\{\, b(n) \mid b(n) \geq 0 \,\}$.
4. $\{\, m(t) \mid m(t) \geq 0 \,\}$.
5. $\{\, u(x) \mid \mathbf{R} \text{ except } u(x) = 1/2 \,\}$. Graph it and you'll see the graph level off horizontally along the line $u(x) = 1/2$. This is a horizontal asymptote (Section 2.4).