

Particle Motion -- With your Graphing Calculator!

1. (2005-B) A particle moves along the x-axis so that its velocity v at time t , for $0 \leq t \leq 5$, is given by $v(t) = \ln(t^2 - 3t + 3)$.

- Find the acceleration of the particle at time $t=4$.
- Is the particle speeding up or slowing down at $t=4$? Explain.
- Find all times t in the open interval $0 < t < 5$ at which the particle changes direction. During which time intervals, for $0 \leq t \leq 5$, does the particle travel to the left?
- Find the average acceleration of the particle over the interval $0 \leq t \leq 2$.

A. $y_1 = \ln(t^2 - 3t + 3)$
 $y_2 = nDeriv(y_1, x, x)$
 $y_2(4) \leftarrow$ in Home Win.

$a(4) = v'(4) = 0.714$

B. $y_1(4) = v(4) = 1.946$

Since $v(4)$ is positive and $a(4)$ is positive, the particle is speeding up at $t=4$.

C. $y_1 = \ln(t^2 - 3t + 3)$
 $\begin{array}{ccccccc} + & 0 & - & 0 & + & & \\ \leftarrow & | & & | & & & \rightarrow \\ & 1 & & 2 & & & \end{array}$

Changes direction at $t=1, t=2$.
 Traveling left when $t \in (1, 2)$.

D. Avg Acc = $\frac{\Delta v}{\Delta t}$
 $= \frac{v(2) - v(0)}{2 - 0}$
 $= \frac{0 - 1.0986}{2}$
 $= -0.549$

2.

(2004) A particle moves along the y-axis so that its velocity v at time $t \geq 0$ is given by $v(t) = 1 - \tan^{-1}(e^t)$. At time $t=0$, the particle is at $y=-1$.

- Find the acceleration of the particle at time $t=2$.
- Is the speed of the particle increasing or decreasing at time $t=2$? Give a reason for your answer.
- Find the time $t \geq 0$ at which the particle reaches its highest point. Justify your answer.

A. $y_1 = 1 - \tan^{-1}(e^x)$
 $y_2 = nDeriv(y_1, x, x)$
 In H.W. $\rightarrow y_2(2)$

$a(2) = -0.133$

B. $v(2) = y_1(2) = -0.436$

Since $v(2)$ and $a(2)$ are both negative, the particle's speed is increasing.

C. Highest point \rightarrow maximum position

maximum position
 \uparrow
 1st deriv of \uparrow which is $v(t)$

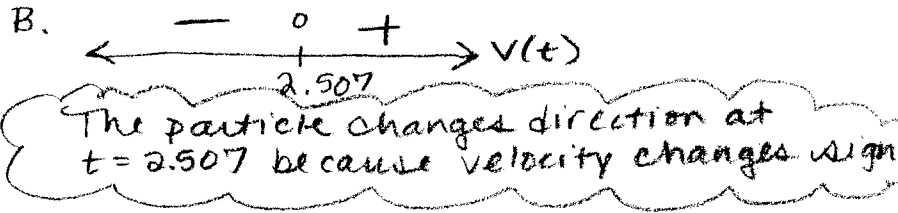
$\begin{array}{ccccccc} + & 0 & - & & & & \\ \leftarrow & | & & & & & \rightarrow \\ & 0.443 & & & & & \end{array}$

The particle reaches its highest point at $t=0.443$ because $v(t)$ changes from positive to negative at that time.

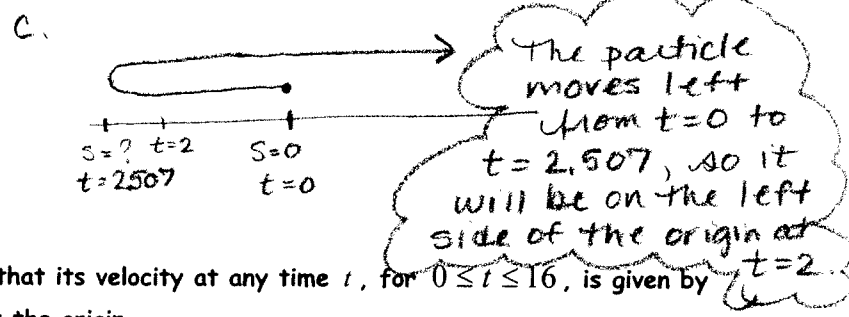
3. (2003) A particle moves along the x-axis so that its velocity at any time t is given by $v(t) = -(t+1)\sin\left(\frac{t^2}{2}\right)$.

- A. Find the acceleration of the particle at $t = 2$. Is the speed of the particle increasing at $t = 2$? Why or why not?
- B. Find all times in the interval $0 \leq t \leq 3$ when the particle changes direction. Justify your answer.
- C. If the particle starts at the origin at $t = 0$, on which side of the origin will the particle be at $t = 2$? Justify your answer.

A. $y_1 = -(t+1)\sin\left(\frac{t^2}{2}\right)$
 $y_2 = \text{ndDeriv}(y_1, x, x)$
 $a(2) = y_2(2) = 1.588$

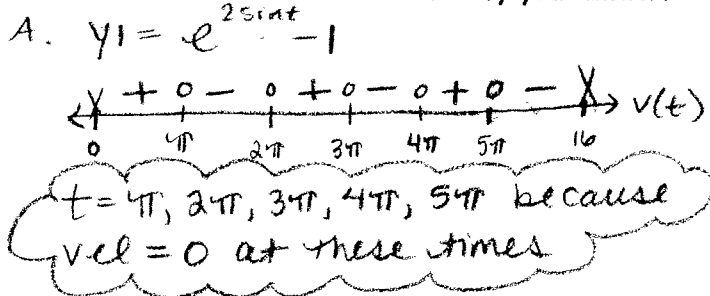


$v(2) = y_1(2) = -2.728$
 Since $v(2)$ is negative and $a(2)$ is positive, the particle's speed is decreasing.



4. (2002-B) A particle moves along the x-axis so that its velocity at any time t , for $0 \leq t \leq 16$, is given by $v(t) = e^{2\sin t} - 1$. At time $t = 0$, the particle is at the origin.

- A. At what time is the particle at rest? Justify your answer.
- B. During what intervals of time is the particle moving to the left? Give a reason for your answer.
- C. What is the acceleration of the particle at time $t = 2$? Is the particle's speed increasing or decreasing at time $t = 2$? Justify your answer.



B. The particle moves left when $t \in (\pi, 2\pi) \cup (3\pi, 4\pi) \cup (5\pi, 16)$ because the velocity is negative.

C. $y_2 = \text{ndDeriv}(y_1, x, x)$
 In H.W. $\rightarrow y_2(2)$
 $a(2) = y_2(2) = -5.130$
 $v(2) = y_1(2) = 5.163$
 Since $v(2)$ is positive and $a(2)$ is negative, the particle's speed is decreasing.

5. (2002) An object moves along the x-axis with initial position $x(0) = 2$. The velocity of the object at time $t \geq 0$ is given by $v(t) = \sin\left(\frac{\pi}{3}t\right)$.

A. $y_1 = \sin\left(\frac{\pi}{3}t\right)$
 $y_2 = \text{ndDeriv}(y_1, x, x)$
 $a(4) = y_2(4) = -0.524$

- A. What is the acceleration of the object at time $t = 4$?
- B. Consider the following two statements.

Statement I: For $3 < t < 4.5$, the velocity of the object is decreasing.
 Statement II: For $3 < t < 4.5$, the speed of the object is increasing.

$v(4) = -0.8660$
 $a(4) = -0.524$
 A sample value in $3 < t < 4.5$

Are either or both of these statements correct? For each statement provide a reason why it is correct or not correct.

B. Both statements are correct! Statement I is correct because vel. is dec. when acc. is negative. Statement II is correct because both $v(4)$ and $a(4)$ are negative meaning that the particle's speed is increasing.