ANSWERS AND SOLUTIONS TO MULTIPLE-CHOICE QUESTIONS

For each chapter set (Set 1 through Set 11), we list the correct answers, followed by brief explanations for all answers.

Answers for Set 1: Functions

1.	C	**7.**	C	**13.**	B	**19.**	A	**25.**	C
2.	E	**8.**	C	**14.**	E	**20.**	E	**26.**	A
3.	D	**9.**	B	**15.**	A	**21.**	E	**27.**	A
4.	E	**10.**	E	**16.**	B	**22.**	D	**28.**	C
5.	D	**11.**	C	**17.**	D	**23.**	B	**29.**	D
6.	B	**12.**	B	**18.**	D	**24.**	A	**30.**	C

1. C. $f(-2) = (-2)^3 - 2(-2) - 1 = -5$.

2. E. The denominator, $x^2 + 1$, is never 0.

3. D. Since $x - 2$ may not be negative, $x \geqq 2$. The denominator equals 0 at $x = 0$ and $x = 1$, but these values are not in the interval $x \geqslant 2$.

4. E. Since $g(x) = 2$, g is a constant function. Thus, for all $f(x)$, $g(f(x)) = 2$.

5. D. $f(g(x)) = f(2) = -3$.

6. B. Solve the pair of equations

$$\left\{\begin{array}{l} 4 = \;\; 1 + A + B - 3 \\ -6 = -1 + A - B - 3 \end{array}\right\}.$$

Add to get A; substitute in either equation to get B. $A = 2$ and $B = 4$.

7. C. The graph of $f(x)$ is symmetric to the origin if $f(-x) = -f(x)$. When we replace x by $-x$, we obtain $-y$ only in (C).

8. C. For g to have an inverse function it must be one-to-one. Note, on page 268, that although the graph of $y = xe^{-x^2}$ is symmetric to the origin, it is not one-to-one.

9. B. Note that $-\frac{\pi}{2} < \arctan x < \frac{\pi}{2}$ and that the sine function varies from -1 to 1 as the argument varies from $-\frac{\pi}{2}$ to $\frac{\pi}{2}$.

10. E. The maximum value of g is 2, attained when $\cos x = -1$. On $[0, 2\pi]$, $\cos x = -1$ for $x = \pi$.

11. C. f is odd if $f(-x) = -f(x)$.

Answers for Set 2: Limits and Continuity

1. B	9. C	17. B	24. C	31. E
2. D	10. D	18. E	25. D	32. C
3. C	11. B	19. A	26. D	33. B
4. A	12. C	20. C	27. E	34. B
5. D	13. B	21. B	28. E	35. D
6. B	14. C	22. B	29. A	36. E
7. A	15. A	23. C	30. A	37. E
8. E	16. B			

1. B. The limit as $x \to 2$ is $0 \div 8$.

2. D. Use the Rational Function Theorem (pages 28 and 29). The degrees of $P(x)$ and $Q(x)$ are the same.

3. C. Remove the common factor $x-3$ from numerator and denominator.

4. A. The fraction equals 1 for all nonzero x.

5. D. Note that $\frac{x^3-8}{x^2-4}=\frac{(x-2)(x^2+2x+4)}{(x-2)(x+2)}$.

6. B. Use the Rational Function Theorem.

7. A. Use the Rational Function Theorem.

8. E. Use the Rational Function Theorem.

9. C. The fraction is equivalent to $\frac{1}{2^{2x}}$; the denominator approaches ∞.

10. D. Since $\frac{2^{-x}}{2^x}=2^{-2x}$, therefore, as $x \to -\infty$, the fraction $\to +\infty$.

11. B. Because the graph of $y=\tan x$ has vertical asymptotes at $x=\pm\frac{\pi}{2}$, the graph of the inverse function $y=\arctan x$ has horizontal asymptotes at $y=\pm\frac{\pi}{2}$.

12. C. Since $\frac{x^2-9}{3x-9}=\frac{(x-3)(x+3)}{3(x-3)}=\frac{x+3}{3}$ (provided $x \neq 3$), y can be defined to be equal to 2 at $x=3$, removing the discontinuity at that point.

13. B. Note that $\frac{\sin x}{x^2+3x}=\frac{\sin x}{x(x+3)}=\frac{\sin x}{x}\cdot\frac{1}{x+3}\to 1\cdot\frac{1}{3}$.

Answers for Set 3: Differentiation

1.	E	22.	A	42.	D	62.	D	82.	B
2.	A	23.	D	43.	C	63.	E	83.	C
3.	B	24.	B	44.	B	64.	C	84.	D
4.	B	25.	E	45.	C	65.	A	85.	B
5.	E	26.	E	46.	E	66.	D	86.	D
6.	D	27.	E	47.	D	67.	E	87.	B
7.	A	28.	A	48.	C	68.	C	88.	D
8.	E	29.	D	49.	B	69.	E	89.	B
9.	C	30.	B	50.	C	70.	B	90.	B
10.	E	31.	D	51.	A	71.	D	91.	E
11.	A	32.	D	52.	D	72.	C	92.	B
12.	D	33.	E	53.	E	73.	A	93.	D
13.	D	34.	B	54.	C	74.	B	94.	E
14.	D	35.	C	55.	B	75.	C	95.	A
15.	A	36.	B	56.	B	76.	C	96.	E
16.	A	37.	B	57.	E	77.	C	97.	E
17.	D	38.	B	58.	D	78.	E	98.	D
18.	E	39.	E	59.	C	79.	A	99.	B
19.	B	40.	A	60.	A	80.	B	100.	C
20.	C	41.	B	61.	B	81.	A	101.	D
21.	C								

Many of the explanations provided include intermediate steps that would normally be reached on the way to a final algebraically simplified result. You may not need to reach the final answer.

NOTE: the formulas or rules cited in parentheses in the explanations are given on pages 46 and 47.

1. E. By the product rule, (5),

$$y' = x^5(\tan x)' + (x^5)'(\tan x).$$

2. A. By the quotient rule, (6),

$$y' = \frac{(3x+1)(-1)-(2-x)(3)}{(3x+1)^2} = -\frac{7}{(3x+1)^2}.$$

3. B. Since $y = (3-2x)^{1/2}$, by the power rule, (3),

$$y' = \frac{1}{2}(3-3x)^{-1/2} \cdot (-2) = -\frac{1}{\sqrt{3-2x}}.$$

4. B. Since $y = 2(5x+1)^{-3}, y' = -6(5x+1)^{-4}(5)$.

Answers for Set 4: Applications of Differential Calculus

1.	D	**21.**	E	**41.**	E	**61.**	A	**81.**	B
2.	A	**22.**	B	**42.**	D	**62.**	C	**82.**	B
3.	E	**23.**	D	**43.**	E	**63.**	B	**83.**	B
4.	B	**24.**	A	**44.**	D	**64.**	E	**84.**	D
5.	D	**25.**	D	**45.**	A	**65.**	C	**85.**	C
6.	C	**26.**	C	**46.**	B	**66.**	E	**86.**	C
7.	D	**27.**	E	**47.**	A	**67.**	D	**87.**	D
8.	A	**28.**	A	**48.**	D	**68.**	C	**88.**	E
9.	C	**29.**	B	**49.**	E	**69.**	E	**89.**	C
10.	B	**30.**	D	**50.**	E	**70.**	B	**90.**	C
11.	D	**31.**	C	**51.**	A	**71.**	A		
12.	E	**32.**	B	**52.**	E	**72.**	C		
13.	C	**33.**	D	**53.**	C	**73.**	D		
14.	A	**34.**	A	**54.**	E	**74.**	E		
15.	B	**35.**	B	**55.**	B	**75.**	B		
16.	E	**36.**	C	**56.**	D	**76.**	D		
17.	B	**37.**	E	**57.**	D	**77.**	D		
18.	B	**38.**	C	**58.**	D	**78.**	C		
19.	D	**39.**	A	**59.**	E	**79.**	E		
20.	A	**40.**	A	**60.**	D	**80.**	D		

1. D. Substituting $y = 2$ yields $x = 1$. We find y' implicitly.

$$3y^2y' - (2xyy' + y^2) = 0; \quad (3y^2 - 2xy)y' - y^2 = 0.$$

Replace x by 1 and y by 2; solve for y'.

2. A. $2yy' - (xy' + y) - 3 = 0$. Replace x by 0 and y by -1; solve for y'.

3. E. Find the slope of the curve at $x = \frac{\pi}{2}$: $y' = x\cos x + \sin x$. At $x = \frac{\pi}{2}$, $y' = \frac{\pi}{2} \cdot 0 + 1$. The equation is $y - \frac{\pi}{2} = 1\left(x - \frac{\pi}{2}\right)$.

4. B. Since $y' = e^{-x}(1 - x)$ and $e^{-x} > 0$ for all x, $y' = 0$ when $x = 1$.

Answers for Set 5: Integration

1.	C	**17.**	E	**33.**	A	**49.**	D	**65.**	D
2.	E	**18.**	D	**34.**	D	**50.**	A	**66.**	E
3.	A	**19.**	A	**35.**	E	**51.**	C	**67.**	B
4.	D	**20.**	D	**36.**	C	**52.**	E	**68.**	D
5.	E	**21.**	E	**37.**	E	**53.**	B	**69.**	E
6.	B	**22.**	E	**38.**	A	**54.**	B	**70.**	B
7.	A	**23.**	B	**39.**	C	**55.**	B	**71.**	D
8.	E	**24.**	C	**40.**	B	**56.**	D	**72.**	C
9.	D	**25.**	A	**41.**	E	**57.**	E	**73.**	A
10.	A	**26.**	A	**42.**	D	**58.**	A	**74.**	B
11.	D	**27.**	E	**43.**	A	**59.**	C	**75.**	E
12.	C	**28.**	B	**44.**	B	**60.**	D	**76.**	D
13.	B	**29.**	D	**45.**	C	**61.**	C	**77.**	D
14.	C	**30.**	C	**46.**	B	**62.**	E	**78.**	D
15.	B	**31.**	B	**47.**	C	**63.**	A	**79.**	C
16.	A	**32.**	E	**48.**	C	**64.**	D	**80.**	A

All the references in parentheses below are to the basic integration formulas on pages 149 and 150. In general, if u is a function of x, then $du = u'(x)\,dx$.

1. C. Use, first, formula (2), then (3), replacing u by x.

2. E. Hint: Expand. The correct answer is $\frac{x^3}{3} - x - \frac{1}{4x} + C$.

3. A. By formula (3), with $u = 4 - 2t$ and $n = \frac{1}{2}$,

$$\int \sqrt{4-2t}\,dx = -\frac{1}{2}\int \sqrt{4-2t}\cdot(-2)\,dt = -\frac{1}{2}\frac{(4-2t)^{3/2}}{3/2} + C.$$

4. D. Use (3) with $u = 2 - 3x$, noting that $du = -3\,dx$.

5. E. Rewrite:

$$\int (2y-3y^2)^{-1/2}(1-3y)\,dy = \frac{1}{2}\int (2y-3y^2)^{-1/2}(2-6y)\,dy.$$

Use (3).

6. B. Rewrite:

$$\frac{1}{3}\int (2x-1)^{-2}\,dx = \frac{1}{3}\cdot\frac{1}{2}\int (2x-1)^{-2}\cdot 2dx.$$

Using (3) yields $-\frac{1}{6(2x-1)} + C$.

Answers for Set 6: Definite Integrals

1.	C	**12.**	B	**23.**	A	**34.**	C	**45.**	A
2.	B	**13.**	E	**24.**	E	**35.**	C	**46.**	D
3.	E	**14.**	C	**25.**	C	**36.**	C	**47.**	E
4.	B	**15.**	D	**26.**	B	**37.**	B	**48.**	E
5.	D	**16.**	A	**27.**	D	**38.**	B	**49.**	C
6.	A	**17.**	C	**28.**	C	**39.**	D	**50.**	D
7.	D	**18.**	E	**29.**	E	**40.**	B	**51.**	E
8.	A	**19.**	A	**30.**	D	**41.**	C	**52.**	D
9.	C	**20.**	E	**31.**	D	**42.**	A		
10.	D	**21.**	C	**32.**	E	**43.**	C		
11.	B	**22.**	C	**33.**	C	**44.**	D		

1. C. The integral is equal to

$$\left(\frac{1}{3}x^3 - \frac{1}{2}x^2 - x\right)\Big|_{-1}^{1} = -\frac{7}{6} - \frac{1}{6}.$$

2. B. Rewrite as $\int_1^2 \left(1 - \frac{1}{3} \cdot \frac{1}{x}\right) dx$. This equals

$$\left(x - \frac{1}{3}\ln x\right)\Big|_1^2 = 2 - \frac{1}{3}\ln 2 - 1.$$

3. E. Rewrite as

$$-\int_0^3 (4-t)^{-1/2}(-1\,dt) = -2\sqrt{4-t}\,\Big|_0^3 = -2(1-2).$$

4. B. This integral equals

$$\frac{1}{3}\int_{-1}^{0} (3u+4)^{1/2} \cdot 3\,du = \frac{1}{3} \cdot \frac{2}{3}(3u+4)^{3/2}\Big|_{-1}^{0}$$
$$= \frac{2}{9}(4^{3/2} - 1^{3/2}).$$

5. D. You have:

$$\frac{1}{2}\int_2^3 \frac{2\,dy}{2y-3} = \frac{1}{2}\ln(2y-3)\Big|_2^3 = \frac{1}{2}(\ln 3 - \ln 1).$$

6. A. Rewrite as

$$-\frac{1}{2}\int_0^{\sqrt{3}} (4-x^2)^{-1/2}(-2x\,dx) = -\frac{1}{2} \cdot 2\sqrt{4-x^2}\,\Big|_0^{\sqrt{3}} = -(1-2).$$

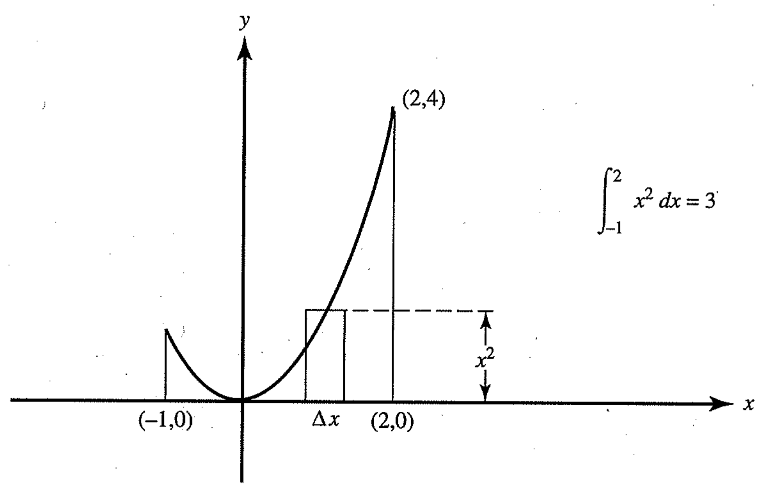
Answers for Set 7: Applications of Integration to Geometry

1. C	**14.** A	**27.** E	**40.** D	**53.** A
2. C	**15.** B	**28.** C	**41.** B	**54.** D
3. A	**16.** B	**29.** A	**42.** A	**55.** D
4. D	**17.** C	**30.** A	**43.** C	**56.** D
5. D	**18.** E	**31.** E	**44.** D	**57.** B
6. C	**19.** D	**32.** B	**45.** E	**58.** C
7. E	**20.** A	**33.** A	**46.** B	**59.** D
8. A	**21.** C	**34.** E	**47.** C	**60.** E
9. A	**22.** D	**35.** C	**48.** D	
10. D	**23.** B	**36.** A	**49.** B	
11. D	**24.** A	**37.** C	**50.** C	
12. C	**25.** D	**38.** E	**51.** B	
13. E	**26.** B	**39.** B	**52.** C	

AREA

We give below, for each of Questions 1–17, a sketch of the region, and indicate a typical element of area. The area of the region is given by the definite integral. We exploit symmetry wherever possible.

1. C.



Answers for Set 8: Further Applications of Integration

1. D	8. D	15. B	22. A	29. C
2. A	9. A	16. E	23. C	30. D
3. E	10. B	17. C	24. C	31. E
4. E	11. B	18. A	25. E	32. C
5. B	12. E	19. A	26. B	33. C
6. D	13. C	20. D	27. A	
7. D	14. D	21. B	28. D	

1. D. Velocity $v(t) = \frac{ds}{dt} = 3(t-1)(t-3)$, and changes sign both when $t = 1$ and when $t = 3$.

2. A. Since $v > 0$ for $0 \leqq t \leqq 2$, the distance is equal to $\int_0^2 (4t^3 + 3t^2 + 5)\, dt$.

3. E. The answer is 8. Since the particle reverses direction when $t = 2$, and $v > 0$ for $t > 2$ but $v < 0$ for $t < 2$, therefore, the total distance is

$$-\int_0^2 (3t^2 - 6t)\, dt + \int_2^3 (3t^2 - 6t)\, dt.$$

4. E. $\int_0^3 (3t^2 - 6t)\, dt = 0$, so there is no change in position.

5. B. Since $v = \sin t$ is positive on $0 < t \leqq 2$, the distance covered is

$\int_0^2 \sin t\, dt = 1 - \cos 2$.

6. D. Average velocity $= \frac{1}{4-0} \int_0^4 (5t - t^2 + 100)\, dt = 104\frac{2}{3}$ mph.

7. D. The velocity v of the car is linear since its acceleration is constant:

$$a = \frac{dv}{dt} = \frac{(60-0)\text{ mph}}{10\text{ sec}} = \frac{88\text{ ft/sec}}{10\text{ sec}} = 8.8\text{ ft/sec}^2$$

8. D. $\mathbf{R} = \frac{t^2}{2}\mathbf{i} + \frac{t^2 - 2t + 2}{2}\mathbf{j}$.

9. A. $\mathbf{a} = \mathbf{i} + \mathbf{j}$ for all t.

10. B. $\mathrm{v} = t\mathbf{i} + (t-1)\mathbf{j}$. $|\mathbf{v}| = \sqrt{t^2 + (t-1)^2}$ · $\frac{d|\mathbf{v}|}{dt} = \frac{2t-1}{|\mathbf{v}|}$.

Answers for Set 9: Multiple-Choice Questions on Differential Equations

1.	C	11.	A	21.	B	31.	D	41.	C
2.	B	12.	E	22.	E	32.	B	42.	E
3.	D	13.	C	23.	B	33.	D	43.	D
4.	B	14.	B	24.	D	34.	A	44.	E
5.	C	15.	E	25.	C	35.	A	45.	D
6.	E	16.	E	26.	A	36.	C	46.	B
7.	B	17.	A	27.	E	37.	C	47.	A
8.	A	18.	C	28.	A	38.	A	48.	C
9.	A	19.	E	29.	C	39.	D		
10.	E	20.	D	30.	B	40.	B		

1. C. $v(t) = 2t^2 - t + C$; $v(1) = 3$; so $C = 2$.

2. B. If $a(t) = 20t^3 - 6t$, then

$$v(t) = 5t^4 - 3t^2 + C_1,$$
$$s(t) = t^5 - t^3 + C_1 t + C_2,$$

Since

$$s(-1) = -1 + 1 - C_1 + C_2 = 2$$

and

$$s(1) = 1 - 1 + C_1 + C_2 = 4,$$

therefore

$$2C_2 = 6,\ C_2 = 3,$$
$$C_1 = 1.$$

So

$$v(t) = 5t^4 - 3t^2 + 1.$$

3. D. From Answer 2, $s(t) = t^5 - t^3 + t + 3$, so $s(0) = C_2 = 3$.

4. B. Since $a(t) = -32$, $v(t) = -32t + 40$, and the height of the stone $s(t) = -16t^2 + 40t + C$. When the stone hits the ground, 4 sec later, $s(t) = 0$, so

$$0 = -16(16) + 40(4) + C,$$
$$C = 96 \text{ ft.}$$

5. C. From Answer 4

$$s(t) = -16t^2 + 40t + 96.$$

Then

$$s'(t) = -32t + 40,$$

which is zero if $t = 5/4$, and that yields maximum height, since $s''(t) = -32$.

Answers for Set 10: Sequences and Series

1. B	11. D	21. A	31. D	41. C
2. C	12. D	22. E	32. D	42. D
3. D	13. A	23. C	33. C	43. A
4. E	14. C	24. C	34. A	44. C
5. C	15. B	25. B	35. C	45. A
6. E	16. E	26. E	36. E	46. D
7. B	17. A	27. C	37. A	47. C
8. A	18. E	28. A	38. C	
9. B	19. D	29. B	39. E	
10. D	20. E	30. B	40. D	

1. B. $a_n = -1+\frac{(-1)^n}{n}$ converges to -1.

2. C. Note that $\lim\limits_{n\to\infty} \frac{(-1)^n}{n} = 0$.

3. D. The sine function varies continuously between –1 and 1 inclusive.

4. E. Note that $a_n = \left(\frac{2}{e}\right)^n$ is a sequence of the type $s_n = r^n$ with $|r| < 1$; also that

$\lim \frac{n^2}{e^n} = 0$ by repeated application of L'Hôpital's rule.

5. C. $\lim\limits_{n\to\infty} r_n = 0$ for $|r| < 1$; $\lim\limits_{n\to\infty} r_n = 1$ for $r = 1$.

6. E. The harmonic series $\sum_1^\infty \frac{1}{k}$ is a counterexample for (A), (B), and (C).

$\sum_1^\infty \frac{(-1)^{k+1}}{k}$ shows that (D) does not follow.

7. B. $\sum_1^\infty \frac{1}{n(n+1)} = \frac{1}{1\cdot 2}+\frac{1}{2\cdot 3}+\frac{1}{3\cdot 4}+\cdots+\frac{1}{n(n+1)}+\cdots$; so

$s_n = 1-\frac{1}{2}+\frac{1}{2}-\frac{1}{3}+\frac{1}{3}-\cdots+\frac{1}{n}-\frac{1}{n+1} = 1-\frac{1}{n+1}$,

and $\lim\limits_{n\to\infty} s_n = 1$.

8. A. $S = \frac{a}{1-r} = \frac{2}{1-(-\frac{1}{2})} = \frac{4}{3}$.