Calculus　　　　Name____________________________　　ID: 1

Linear Approximation (cont.)

Date________________　Period____

Use differentials to solve each problem.

1) The hypotenuse of a right triangle is known to be exactly 5 in. One of the acute angles is measured to be 45°, with a possible error of ±1°. Estimate the possible propagated error in the side opposite to the measured angle.

2) The hypotenuse of a right triangle is known to be exactly 7 cm. One of the acute angles is measured to be 30°, with a possible error of ±3°. Estimate the possible propagated error in the side opposite to the measured angle.

3) The radius of a circle is measured to be 6 cm, with a possible error of $\pm\frac{1}{10}$ cm. Estimate the possible propagated error in the calculated area.

4) The sides of a square are measured to be 7 ft, with a possible error of $\pm\frac{3}{10}$ ft. Estimate the possible propagated error in the calculated area.

5) The sides of a cube are measured to be 6 in, with a possible error of $\pm\frac{1}{10}$ in. Estimate the possible propagated error in the calculated volume.

6) The hypotenuse of a right triangle is known to be exactly 10 in. One of the acute angles is measured to be 60°, with a possible error of ±1°. Estimate the possible propagated error in the side opposite to the measured angle.

7) The sides of a square are measured to be 4 ft, with a possible error of $\pm\frac{1}{10}$ ft. Estimate the possible propagated error in the calculated area.

8) The sides of a cube are measured to be 5 in, with a possible error of $\pm\frac{1}{5}$ in. Estimate the possible propagated error in the calculated volume.

9) The radius of a circle is measured to be 7 cm, with a possible error of $\pm\frac{1}{5}$ cm. Estimate the possible propagated error in the calculated area.

10) The hypotenuse of a right triangle is known to be exactly 7 ft. One of the acute angles is measured to be 30°, with a possible error of ±1°. Estimate the possible propagated error in the side opposite to the measured angle.

Answers to Linear Approximation (cont.) (ID: 1)

1) $x = 5\sin\theta$, $dx = 5\cos\theta\ d\theta$

$\theta = \frac{\pi}{4}$ radians, $d\theta = \pm\frac{\pi}{180}$ radians

$\Delta x \approx dx = \pm\frac{\pi\sqrt{2}}{72} \approx \pm 0.0617$ in

2) $x = 7\sin\theta$, $dx = 7\cos\theta\ d\theta$

$\theta = \frac{\pi}{6}$ radians, $d\theta = \pm\frac{\pi}{60}$ radians

$\Delta x \approx dx = \pm\frac{7\pi\sqrt{3}}{120} \approx \pm 0.3174$ cm

3) $A = \pi r^2$, $dA = 2\pi r\ dr$

$r = 6$, $dr = \pm 0.1$

$\Delta A \approx dA = \pm\frac{6\pi}{5} \approx \pm 3.7699$ cm²

4) $A = s^2$, $dA = 2s\ ds$

$s = 7$, $ds = \pm 0.3$

$\Delta A \approx dA = \pm\frac{21}{5} = \pm 4.2$ ft²

5) $V = s^3$, $dV = 3s^2\ ds$

$s = 6$, $ds = \pm 0.1$

$\Delta V \approx dV = \pm\frac{54}{5} = \pm 10.8$ in³

6) $x = 10\sin\theta$, $dx = 10\cos\theta\ d\theta$

$\theta = \frac{\pi}{3}$ radians, $d\theta = \pm\frac{\pi}{180}$ radians

$\Delta x \approx dx = \pm\frac{\pi}{36} \approx \pm 0.0873$ in

7) $A = s^2$, $dA = 2s\ ds$

$s = 4$, $ds = \pm 0.1$

$\Delta A \approx dA = \pm\frac{4}{5} = \pm 0.8$ ft²

8) $V = s^3$, $dV = 3s^2\ ds$

$s = 5$, $ds = \pm 0.2$

$\Delta V \approx dV = \pm 15$ in³

9) $A = \pi r^2$, $dA = 2\pi r\ dr$

$r = 7$, $dr = \pm 0.2$

$\Delta A \approx dA = \pm\frac{14\pi}{5} \approx \pm 8.7965$ cm²

10) $x = 7\sin\theta$, $dx = 7\cos\theta\ d\theta$

$\theta = \frac{\pi}{6}$ radians, $d\theta = \pm\frac{\pi}{180}$ radians

$\Delta x \approx dx = \pm\frac{7\pi\sqrt{3}}{360} \approx \pm 0.1058$ ft