

Finding Limits Analytically

1. Limits of continuous functions can be evaluated with direct substitution.

a. $\lim_{x \rightarrow -5} [3x^2 + 4x - 5]$ b. $\lim_{p \rightarrow -2} \frac{2p + 4}{3p}$ c. $\lim_{x \rightarrow 0} [3e^x - \sin x + \ln(x + 1)]$

2. Use factoring and/or expanding, then use cancellation

a. $\lim_{x \rightarrow -2} \frac{2x + 4}{x^2 - 3x - 10}$
b. $\lim_{x \rightarrow -3} \frac{x^3 + 27}{x + 3}$ (Hint: Factor sum of cubes)
c. $\lim_{t \rightarrow 1} \frac{2t^2 + 3t - 5}{1 - t}$
d. $\lim_{h \rightarrow 0} \frac{(-1 + h)^2 - 1}{h}$
e. $\lim_{h \rightarrow 0} \frac{(3 + h)^3 - 27}{h}$

3. Use our “rationalizing the denominator/numerator” technique

a. $\lim_{x \rightarrow -10} \frac{\sqrt{x + 19} - 3}{x + 10}$
b. $\lim_{h \rightarrow 0} \frac{\sqrt{4 + h} - 2}{h}$
c. $\lim_{x \rightarrow 1} \frac{1 - \sqrt{x}}{x - 1}$

4. Solve by simplifying the compound fraction

a. $\lim_{h \rightarrow 0} \frac{\frac{1}{1+h} - 1}{h}$
b. $\lim_{x \rightarrow -4} \frac{\frac{8}{x+6} - 4}{x + 4}$
c. $\lim_{x \rightarrow 2} \frac{\frac{2-x}{x} - 1}{\frac{2}{x} - 1}$

5. Limits involving trigonometric functions

a. $\lim_{x \rightarrow 0} \frac{\sin x}{2x}$
b. $\lim_{t \rightarrow 0} \frac{\sin 3t}{5t}$
c. $\lim_{x \rightarrow 0} \frac{\tan 4x}{x}$