Set 2: Multiple-Choice Questions on Limits and Continuity

Part A. Directions: Answer these questions *without* using your calculator.

1. $\lim_{x\to 2} \frac{x^2-4}{x^2+4}$ is

(A) 1 **(B)** 0 **(C)** $-\frac{1}{2}$ **(D)** −1 **(E)** ∞

2. $\lim_{x\to \infty} \frac{4-x^2}{x^2-1}$ is

(A) 1 **(B)** 0 **(C)** −4 **(D)** −1 **(E)** ∞

3. $\lim_{x\to 3} \frac{x-3}{x^2-2x-3}$ is

(A) 0 **(B)** 1 **(C)** $\frac{1}{4}$ **(D)** ∞ **(E)** none of these

4. $\lim_{x\to 0} \frac{x}{x}$ is

(A) 1 **(B)** 0 **(C)** ∞ **(D)** −1 **(E)** nonexistent

5. $\lim_{x\to 2} \frac{x^3-8}{x^2-4}$ is

(A) 4 **(B)** 0 **(C)** 1 **(D)** 3 **(E)** ∞

6. $\lim_{x\to \infty} \frac{4-x^2}{4x^2-x-2}$ is

(A) −2 **(B)** $-\frac{1}{4}$ **(C)** 1 **(D)** 2 **(E)** nonexistent

7. $\lim_{x\to -\infty} \frac{5x^3+27}{20x^2+10x+9}$ is

(A) $-\infty$ **(B)** −1 **(C)** 0 **(D)** 3 **(E)** ∞

8. $\lim_{x\to \infty} \frac{3x^2+27}{x^3-27}$ is

(A) 3 **(B)** ∞ **(C)** 1 **(D)** −1 **(E)** 0

9. $\lim_{x\to \infty} \frac{2^{-x}}{2^x}$ is

(A) −1 **(B)** 1 **(C)** 0 **(D)** ∞ **(E)** none of these

10. $\lim_{x\to-\infty} \frac{2^{-x}}{2^x}$ is

(A) -1 **(B)** 1 **(C)** 0 **(D)** ∞ **(E)** none of these

11. The graph of $y = \arctan x$ has

(A) vertical asymptotes at $x = 0$ and $x = \pi$
(B) horizontal asymptotes at $y = \pm\frac{\pi}{2}$
(C) horizontal asymptotes at $y = 0$ and $y = \pi$
(D) vertical asymptotes at $x = \pm\frac{\pi}{2}$
(E) none of these

12. The graph of $y = \frac{x^2-9}{3x-9}$ has

(A) a vertical asymptote at $x = 3$ **(B)** a horizontal asymptote at $y = \frac{1}{3}$
(C) a removable discontinuity at $x = 3$ **(D)** an infinite discontinuity at $x = 3$
(E) none of these

13. $\lim_{x\to 0} \frac{\sin x}{x^2+3x}$ is

(A) 1 **(B)** $\frac{1}{3}$ **(C)** 3 **(D)** ∞ **(E)** $\frac{1}{4}$

14. $\lim_{x\to 0} \sin\frac{1}{x}$ is

(A) ∞ **(B)** 1 **(C)** nonexistent **(D)** -1 **(E)** none of these

15. Which statement is true about the curve $y = \frac{2x^2+4}{2+7x-4x^2}$?

(A) The line $x = -\frac{1}{4}$ is a vertical asymptote.
(B) The line $x = 1$ is a vertical asymptote.
(C) The line $y = -\frac{1}{4}$ is a horizontal asymptote.
(D) The graph has no vertical or horizontal asymptote.
(E) The line $y = 2$ is a horizontal asymptote.

16. $\lim_{x\to\infty} \frac{2x^2+1}{(2-x)(2+x)}$ is

(A) -4 **(B)** -2 **(C)** 1 **(D)** 2 **(E)** nonexistent

17. $\lim_{x\to 0} \frac{|x|}{x}$ is

(A) 0 **(B)** nonexistent **(C)** 1 **(D)** -1 **(E)** none of these

18. $\lim_{x\to\infty} x\sin\frac{1}{x}$ is

(A) 0 **(B)** ∞ **(C)** nonexistent **(D)** -1 **(E)** 1

19. $\lim_{x\to\pi} \frac{\sin(\pi-x)}{\pi-x}$ is

(A) 1 **(B)** 0 **(C)** ∞ **(D)** nonexistent **(E)** none of these

20. Let $f(x)=\begin{cases} \frac{x^2-1}{x-1} & \text{if } x\neq 1, \\ 4 & \text{if } x=1. \end{cases}$

Which of the following statements is (are) true?

I. $\lim_{x\to 1} f(x)$ exists. II. $f(1)$ exists. III. f is continuous at $x=1$.

(A) I only **(B)** II only **(C)** I and II
(D) none of them **(E)** all of them

21. If $\begin{cases} f(x)=\frac{x^2-x}{2x} \text{ for } x\neq 0, \\ f(0)=k, \end{cases}$

and if f is continuous at $x=0$, then $k=$

(A) -1 **(B)** $-\frac{1}{2}$ **(C)** 0 **(D)** $\frac{1}{2}$ **(E)** 1

22. Suppose $\begin{cases} f(x)=\frac{3x(x-1)}{x^2-3x+2} \text{ for } x\neq 1,2, \\ f(1)=-3, \\ f(2)=4. \end{cases}$

Then $f(x)$ is continuous

(A) except at $x=1$ **(B)** except at $x=2$ **(C)** except at $x=1$ or 2
(D) except at $x=0$, 1, or 2 **(E)** at each real number

23. The graph of $f(x)=\frac{4}{x^2-1}$ has

(A) one vertical asymptote, at $x=1$
(B) the y-axis as vertical asymptote
(C) the x-axis as horizontal asymptote and $x=\pm 1$ as vertical asymptotes
(D) two vertical asymptotes, at $x=\pm 1$, but no horizontal asymptote
(E) no asymptote

24. The graph of $y = \frac{2x^2+2x+3}{4x^2-4x}$ has

(A) a horizontal asymptote at $y = +\frac{1}{2}$ but no vertical asymptote
(B) no horizontal asymptote but two vertical asymptotes, at $x = 0$ and $x = 1$
(C) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = 0$ and $x = 1$
(D) a horizontal asymptote at $x = 2$ but no vertical asymptote
(E) a horizontal asymptote at $y = \frac{1}{2}$ and two vertical asymptotes, at $x = \pm 1$

25. Let $f(x) = \begin{cases} \frac{x^2+x}{x} & \text{if } x \neq 0 \\ 1 & \text{if } x = 0 \end{cases}$.

Which of the following statements is (are) true?

I. $f(0)$ exists. II. $\lim_{x \to 0} f(x)$ exists. III. f is continuous at $x = 0$.

(A) I only **(B)** II only **(C)** I and II only
(D) all of them **(E)** none of them

Part B. Directions: Some of the following questions require the use of a graphing calculator.

26. If $[x]$ is the greatest integer not greater than x, then $\lim_{x \to 1/2}[x]$ is

(A) $\frac{1}{2}$ **(B)** 1 **(C)** nonexistent **(D)** 0 **(E)** none of these

27. (With the same notation) $\lim_{x \to -2}[x]$ is

(A) -3 **(B)** -2 **(C)** -1 **(D)** 0 **(E)** none of these

28. $\lim_{x \to \infty} \sin x$

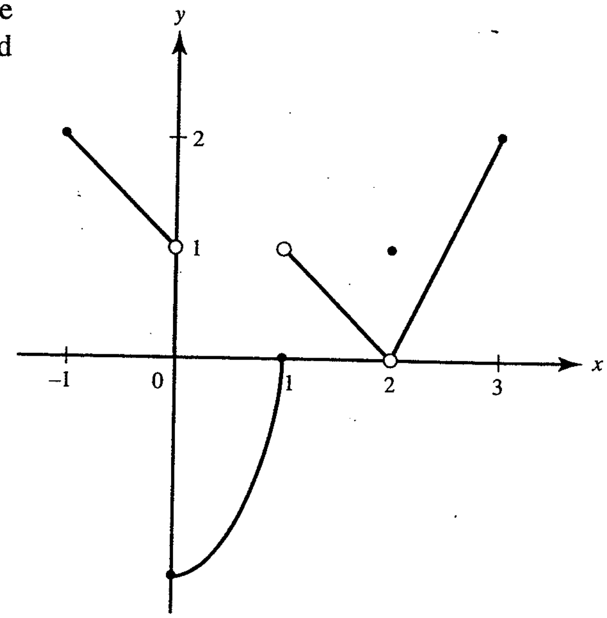
(A) is -1 **(B)** is infinity **(C)** oscillates between -1 and 1
(D) is zero **(E)** does not exist

29. The function $f(x) = \begin{cases} x^2/x & (x \neq 0) \\ 0 & (x = 0) \end{cases}$

(A) is continuous everywhere
(B) is continuous except at $x = 0$
(C) has a removable discontinuity at $x = 0$
(D) has an infinite discontinuity at $x = 0$
(E) has $x = 0$ as a vertical asymptote

Questions 30–34 are based on the function f shown in the graph and defined below:

$$f(x)=\begin{cases}1-x & (-1\leqslant x<0)\\ 2x^2-2 & (0\leqslant x\leqslant 1)\\ -x+2 & (1<x<2)\\ 1 & (x=2)\\ 2x-4 & (2<x\leqslant 3)\end{cases}$$



30. $\lim_{x\to 2} f(x)$

(A) equals 0 **(B)** equals 1 **(C)** equals 2
(D) does not exist **(E)** none of these

31. The function f is defined on $[-1,3]$

(A) if $x\neq 0$ **(B)** if $x\neq 1$ **(C)** if $x\neq 2$
(D) if $x\neq 3$ **(E)** at each x in $[-1,3]$

32. The function f has a removable discontinuity at

(A) $x=0$ **(B)** $x=1$ **(C)** $x=2$ **(D)** $x=3$ **(E)** none of these

33. On which of the following intervals is f continuous?

(A) $-1\leqslant x\leqslant 0$ **(B)** $0<x<1$ **(C)** $1\leqslant x\leqslant 2$
(D) $2\leqslant x\leqslant 3$ **(E)** none of these

34. The function f has a jump discontinuity at

(A) $x=-1$ **(B)** $x=1$ **(C)** $x=2$
(D) $x=3$ **(E)** none of these

35. Suppose $\lim_{x\to -3^-} f(x) = -1$, $\lim_{x\to -3^+} f(x) = -1$, and $f(-3)$ is not defined. Which of the following statements is (are) true?

I. $\lim_{x\to -3} f(x) = -1$.
II. f is continuous everywhere except at $x = -3$.
III. f has a removable discontinuity at $x = -3$.

(A) None of them **(B)** I only **(C)** III only
(D) I and III only **(E)** All of them

36. If $y = \dfrac{1}{2 + 10^{\frac{1}{x}}}$, then $\lim_{x\to 0} y$ is

(A) 0 **(B)** $\dfrac{1}{12}$ **(C)** $\dfrac{1}{2}$ **(D)** $\dfrac{1}{3}$ **(E)** nonexistent

37. $\lim_{x\to 0} \sqrt{3 + \arctan \dfrac{1}{x}}$ is

(A) $-\infty$ **(B)** $\sqrt{3 - \dfrac{\pi}{2}}$ **(C)** $\sqrt{3 + \dfrac{\pi}{2}}$

(D) ∞ **(E)** none of these