AP CALCULUS Name Answer Key

INTRO TO PARTICLE MOTION PRACTICE (No calculator!)

1) A particle travels along the x-axis so that at any time $t \geq 0$, its position is given by $x(t) = t^3 - 9t^2 + 24t + 2$. For what value(s) of t is the velocity equal to zero?

A) $t = 4$, only
B) $t = 2$, only
C) $t = 0$ and $t = 3$
D) $t = 3$, only
(E) $t = 2$ and $t = 4$

$v(t) = 3t^2 - 18t + 24$
$0 = 3(t^2 - 6t + 8)$
$0 = 3(t-4)(t-2)$
$t = 4 \quad t = 2$

2) A particle moves on the x-axis so that its position is given by $x(t) = t^4 - 6t^2 + 8$ for $t \geq 0$. For what times t is the velocity of the particle increasing?

Take deriv of # line!

A) $t > 0$
B) $0 < t < \sqrt{3}$
C) $t > \sqrt{3}$
D) $0 < t < 1$
(E) $1 < t < \sqrt{3}$ ← Best ans.

$v(t) = 4t^3 - 12t$
$v'(t) = 12t^2 - 12$
$0 = 12(t^2 - 1)$
$t = \pm 1$ — can't have neg. time

v dec / v inc: v': $-$ on $(0,1)$, $+$ after 1

3) The position of a particle moving on a horizontal axis for time t, where $t \geq 0$, is $S(t) = 3\sin\frac{1}{2}t + 1$. What is the average velocity of the particle for $0 \leq t \leq \frac{3\pi}{2}$?

A) $\frac{\pi}{\sqrt{2}}$
(B) $\frac{\sqrt{2}}{\pi}$
C) $\frac{3\sqrt{2}}{\pi}$
D) $\frac{-\sqrt{2}}{\pi}$
E) $-\frac{\pi}{\sqrt{2}}$

$\frac{\Delta s}{\Delta t} = \frac{s(\frac{3\pi}{2}) - s(0)}{\frac{3\pi}{2} - 0} = \frac{\frac{3\sqrt{2}}{2} + 1 - 1}{\frac{3\pi}{2}} = \frac{\sqrt{2}}{\pi}$

$3\sin(\frac{3\pi}{4}) + 1 = 3(\frac{\sqrt{2}}{2}) + 1 \qquad 3\sin(0) + 1 = 3(0) + 1$

4) What is the maximum acceleration of a particle on the interval $0 \leq t \leq 3$ if its position is given by $s(t) = t^4 - 4t^3$?

Take deriv. of # line + to −

(A) 36
B) -16
C) 0
D) -12
E) 24

$v(t) = 4t^3 - 12t^2$
$a(t) = 12t^2 - 24t$
$a'(t) = 24t - 24$
$0 = 24t - 24$
$t = 1$

a': max! at 0, $-$, min! at 1, $+$, max! at 3

$a(0) = 0$
$a(3) = 12t(t-2) = 36(1) = 36$

5) The table below shows the position of a particle, S, at various times, t, as it moves along a straight line.

t (sec)	1.0	1.4	1.8	2.2	2.6
s (ft)	6.0	7.0	10.0	15.0	21.0

What is an estimated value of the velocity of the particle at time $t = 2$?

A) 15 ft/sec
(B) 12.5 ft/sec
C) 20 ft/sec
D) 10 ft/sec
E) 5 ft/sec

vel is $\frac{ds}{dt}$, estimate w/ $\frac{\Delta s}{\Delta t}$

$\frac{\Delta s}{\Delta t} = \frac{15 - 10}{2.2 - 1.8} = \frac{5}{0.4} = \frac{5}{\frac{4}{10}} = \frac{5}{\frac{2}{5}} = \frac{25}{2} = 12.5$

6) If the position of a particle moving on the x-axis at any time t is given by $x(t) = 2t^3 - 3t^2$, what is the average acceleration of the particle for $0 \leq t \leq 3$?

A) 15
B) 18
C) 8
D) 9
(E) 12

$v(t) = 6t^2 - 6t$

$\frac{\Delta v}{\Delta t} = \frac{v(3) - v(0)}{3 - 0} = \frac{36 - 0}{3} = 12$

$v(3) = 6t(t-1) = 18(2) = 36 \qquad v(0) = 0$

7) A particle moves along the x-axis so that at any time $t \geq 0$, its position is given by $x(t) = 2t + \sin(\pi t)$. What is the acceleration of the particle at time $t = \frac{3}{2}$?

Two deriv. from position

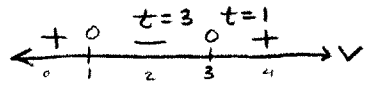
A) $-\pi^2$
B) 2
C) π
(D) π^2
E) 0

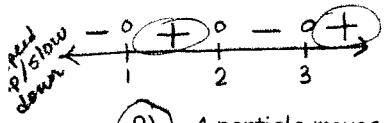
$v(t) = 2 + \pi\cos(\pi t)$
$a(t) = -\pi^2 \sin(\pi t)$
$a(\frac{3}{2}) = -\pi^2 \sin(\frac{3\pi}{2}) = -\pi^2(-1) = \pi^2$

8) A particle moves along the x-axis so that is position at any time t is given by $x(t) = t^3 - 6t^2 + 9t + 12$. During what times is the speed of the particle increasing? speeding up → Look at signs of vel + acc.

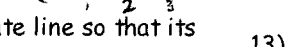
A) $t < 1$ or $2 < t < 3$
B) $1 < t < 2$ or $t > 3$
C) $t < 2$ or $t > 3$
D) $1 < t < 3$
E) $t < 1$ or $t > 3$

$v(t) = 3t^2 - 12t + 9$
$0 = 3(t^2 - 4t + 3)$
$= 3(t-3)(t-1)$
$t = 3 \quad t = 1$





$a(t) = 6t - 12$
$0 = 6t - 12$
$t = 2$



9) A particle moves along a coordinate line so that its position is given by $s(t) = 2\sin\frac{1}{2}t + \frac{1}{2}\cos 2t$ for $0 \le t \le 2\pi$. What is the acceleration of the particle at $t = \pi$? Two deriv. from position

A) $-\frac{3}{2}$
B) $-\frac{1}{2}$
C) 1
D) $-\frac{5}{2}$
E) $\frac{5}{2}$

$v(t) = \cos(\frac{1}{2}t) - \sin(2t)$
$a(t) = -\frac{1}{2}\sin(\frac{1}{2}t) - 2\cos(2t)$
$a(\pi) = -\frac{1}{2}\sin(\frac{\pi}{2}) - 2\cos(2\pi)$
$= -\frac{1}{2}(1) - 2(1)$
$= -\frac{1}{2} - 2$
$= -\frac{5}{2}$

10) A particle moves along the x-axis in such a way that its position at any time t is given by $x(t) = t^4 - 8t^3 + 18t^2 + 2$ for $t > 0$. At what time is acceleration of the particle equal to 36? Two deriv!

A) 3
B) 4
C) 12
D) 6
E) 2

$v(t) = 4t^3 - 24t^2 + 36t$
$a(t) = 12t^2 - 48t + 36$
$36 = 12t^2 - 48t + 36$
$0 = 12t^2 - 48t$
$0 = 12t(t-4)$
$t = 0 \quad t = 4$

11) A particle moves on the x-axis such that its position at any time $t > 0$ is given by $x(t) = t^3 - 9t^2 + 24t$. What is the velocity of the particle when its acceleration is zero?

A) 105
B) 24
C) -3
D) 3
E) 0

$v(t) = 3t^2 - 18t + 24$
$a(t) = 6t - 18$
$0 = 6t - 18$
$t = 3$

$v(t) = 3(t^2 - 6t + 8)$
$v(t) = 3(t-4)(t-2)$
$v(3) = 3(-1)(1)$
$v(3) = -3$

12) A particle moves along a horizontal axis so that its position is defined by $s(t) = 4\cos\frac{\pi}{2}t$ for $0 \le t \le 5$. What is the velocity of the particle at the time its acceleration is first equal to zero?

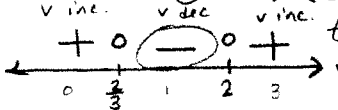
A) 2π
B) -2π
C) -4π
D) 4π
E) $-\pi^2$

$v(t) = -2\pi\sin(\frac{\pi}{2}t)$
$a(t) = -\pi^2\cos(\frac{\pi}{2}t)$
$0 = -\pi^2\cos(\frac{\pi}{2}t)$
$0 = \cos(\frac{\pi}{2}t)$
$\frac{\pi}{2}t = \frac{\pi}{2} \quad t = 1$
$v(1) = -2\pi\sin(\frac{\pi}{2})$
$= -2\pi(1)$
$= -2\pi$

13) A particle moves along a horizontal coordinate line so that its position at time t, $0 \le t \le 4$ is given by $s(t) = t^4 - \frac{16}{3}t^3 + 8t^2 + 1$. For what times t is the velocity of the particle decreasing? Take Deriv of vel. # line!

A) $\frac{2}{3} < t < 2$
B) $t > \frac{2}{3}$
C) $0 < t < 2$
D) $0 < t < 4$
E) $2 < t < 4$

$v(t) = 4t^3 - 16t^2 + 16t$
$v'(t) = 12t^2 - 32t + 16$
$0 = 4(3t^2 - 8t + 4)$
$0 = 4(3t-2)(t-2)$
$t = \frac{2}{3} \quad t = 2$



14) The table below shows velocity of a particle at various times t of a particle that moves along a horizontal line.

t (sec)	0.5	1.0	1.5	2.0	2.5
v (m/sec)	8.3	9.2	9.8	10.6	11.9

What is an approximate value of the acceleration of the particle at time $t = 2$?

A) 1.2 ft/sec^2
B) -0.8 ft/sec^2
C) 1.6 ft/sec^2
D) -1.6 ft/sec^2
E) 1.8 ft/sec^2

$\frac{\Delta v}{\Delta t} = \frac{11 - 9.8}{2.5 - 1.5}$
$= \frac{1.2}{1}$
$= 1.2$