ACRAHS #16 Solis CALC

Integration MIXED Practice

Name ________________________ PER _______ DATE __________

In your practice notebooks, indicate *which integration method* you would use to complete the practice below and state your reasoning. Then carry out the problem, referring to your notes or online resources.

1.

$$\int (2x^4 + 3x^3 - 5x^2 - 2x + 1)\, dx$$

2.

$$\int (5x+4)^5\, dx$$

3. Find c in [a,b] such that f(c)= favg

$f(x) = x^2 + 3x + 2$ on the interval [1,4]

4.

$$\int 3t^2(t^3+4)^5\, dt$$

5.

$$\int \left(\frac{5}{x^3} - \frac{2}{x} + \frac{3}{\sqrt{x}} + x + 1\right) dx$$

6.

$$\int_1^9 \left(\sqrt{x} - \frac{4}{\sqrt{x}}\right) dx$$

7.

$$\int \sqrt{x^3 + x^2}\,(3x^2 + 2x)\, dx$$

8.

. **a)** Approximate the value of the integral:

$$\int_1^3 x^3 - 3\,dx$$

with a Riemann sum, using three sub-intervals and right endpoints.

b) using five sub-intervals and left endpoints.

9.

Find the exact average value of $f(x) = x^2 + x$ over $[1, 4]$ (exact)

10.

Approximate the area under the curve

$$f(x) = \sqrt{x+1}, \quad -1 \leq x \leq 0$$

with a Riemann sum, using 4 sub-intervals and left endpoints.

AP PRACTICE

11.

. A rectangular canal, 5m wide and 100m long has an uneven bottom. Depth measurements are taken at every 20m along the length of the canal. Use these depth measurements to construct a Riemann sum using right endpoints to estimate the volume of water in the canal.

Distance	0m	20m	40m	60m	80m	100m
Depth	2.0m	1.6m	1.8m	2.1m	2.1m	1.9m

Answer Key

1. Power Rule

$$\frac{2}{5}x^5 + \frac{3}{4}x^4 - \frac{5}{3}x^3 - x^2 + x + C$$

2. U-SUB

(a) Let $u = 5x + 4$

(b) Then $du = 5\,dx$ or $\frac{1}{5}\,du = dx$.

(c) Now substitute

$$\begin{aligned}\int (5x+4)^5\,dx &= \int u^5 \cdot \frac{1}{5}\,du \\ &= \int \frac{1}{5}u^5\,du \\ &= \frac{1}{30}u^6 + C \\ &= \frac{1}{30}(5x+4)^6 + C\end{aligned}$$

3. MVTi

First let's notice that the function is a polynomial and so is continuous on the given interval. This means that we can use the Mean Value Theorem. So, let's do that.

$$\int_1^4 x^2 + 3x + 2\,dx = \left(c^2 + 3c + 2\right)(4-1)$$

$$\left(\frac{1}{3}x^3 + \frac{3}{2}x^2 + 2x\right)\Bigg|_1^4 = 3\left(c^2 + 3c + 2\right)$$

$$\frac{99}{2} = 3c^2 + 9c + 6$$

$$0 = 3c^2 + 9c - \frac{87}{2}$$

This is a quadratic equation that we can solve. Using the quadratic formula we get the following two solutions,

$$c = \frac{-3 + \sqrt{67}}{2} = 2.593$$

$$c = \frac{-3 - \sqrt{67}}{2} = -5.593$$

Clearly the second number is not in the interval and so that isn't the one that we're after. The first however is in the interval and so that's the number we want.

Note that it is possible for both numbers to be in the interval so don't expect only one to be in the interval.

4. USUB

(a) Let $u = t^3 + 4$

(b) Then $du = 3t^2\,dt$

(c) Now substitute

$$\begin{aligned}
\int 3t^2(t^3+4)^5\,dt &= \int (t^3+4)^5 \cdot 3t^2\,dt \\
&= \int u^5 \cdot du \\
&= \frac{1}{6}u^6 + C \\
&= \frac{1}{6}(t^3+4)^6 + C
\end{aligned}$$

5. Power Rule

$$-\tfrac{5}{2}x^{-2} - 2\ln|x| + 6x^{1/2} + \tfrac{1}{2}x^2 + x + C$$

6. Power Rule

- $\frac{4}{3}$

7. USUB

(a) Let $u = x^3 + x^2$

(b) Then $du = (3x^2 + 2x)\,dx$

Now substitute

$$\begin{aligned}
\int \sqrt{x^3+x^2} \cdot (3x^2+2x)\,dx &= \int \sqrt{u}\,du \\
&= \int u^{1/2}\,du \\
&= \frac{2}{3}u^{3/2} + C \\
&= \frac{2}{3}(x^3+x^2)^{3/2} + C
\end{aligned}$$

8. Riemann's Sum

$$a = -2,\ b = 1,\quad \Delta x = \frac{b-a}{n} = \frac{1-(-2)}{6} = \frac{1}{2},\quad f(x) = x^2 + 2$$

$$\text{Area} \simeq \left(\sum_i f(x_i)\Delta x\right) = \Delta x\left(f(-1.5) + f(-1) + f(-0.5) + f(0) + f(0.5) + f(1)\right)$$

$$= \frac{1}{2}\left((-1.5)^2 + 2 + (-1)^2 + 2 + (-0.5)^2 + (0)^2 + 2 + (0.5)^2 + 2 + (1)^2 + 2\right)$$

$$= 8.375$$

9. MVTi

$$f_{ave} = \frac{19}{2}$$

10. Riemann's Sum

$$a = -1,\ b = 0,\quad \Delta x = \frac{b-a}{n} = \frac{0-(-1)}{4} = \frac{1}{4},\quad f(x) = \sqrt{x+1}$$

$$\text{Area} \simeq \left(\sum_i f(x_i)\Delta x\right) = \Delta x\left(f(-1) + f(-0.75) + f(-0.5) + f(-0.25)\right)$$

$$= 0.25\left(\sqrt{0} + \sqrt{0.25} + \sqrt{0.5} + \sqrt{0.75}\right) \simeq 0.5183$$

AP PRACTICE

$$\text{Volume} = (\text{width})(\text{cross section area})$$

$$\simeq (5\text{m})\left(\sum_i f(x_i)\Delta x\right) = (5\text{m})\Delta x\left(f(20) + f(40) + f(60) + f(80) + f(100)\right)$$

Δx here is the distance between our measurements, 20m, so we get:

$$\text{Volume} \simeq 100\left(f(20) + f(40) + f(60) + f(80) + f(100)\right)$$

$$= 100(1.6 + 1.8 + 2.1 + 2.1 + 1.9)$$

$$= 950\text{m}^3$$