AP Calculus

REVIEW FOR 3rd QUARTER MIDTERM

Name_____

Seat # _____ Date _____

Integration

DO NOT USE A CALCULATOR FOR ANY OF THESE QUESTIONS

In exercises 1–23, evaluate each integral.

1. $\int \frac{1}{6x+1}\,dx$
2. $\int \frac{x^2}{3-x^3}\,dx$
3. $\int \frac{x}{\sqrt{9-x^2}}\,dx$
4. $\int \tan(5\theta)\,d\theta$
5. $\int_1^e \frac{x^2+x+1}{x}\,dx$
6. $\int_1^2 10^t\,dt$
7. $\int \frac{e^x}{e^x+25}\,dx$
8. $\int x\cdot 2^{x^2}\,dx$
9. $\int_1^2 e^{1-x}\,dx$
10. $\int_0^{\sqrt{2}} xe^{-(x^2/2)}\,dx$
11. $\int \frac{\cos x}{2+\sin x}\,dx$
12. $\int e^x\sqrt{1+e^x}\,dx$
13. $\int \frac{e^{\sqrt{x}}}{\sqrt{x}}\,dx$
14. $\int \left(x^5+5^x\right)dx$
15. $\int 2^{\sin x}\cos x\,dx$
16. $\int_0^{\sqrt{3}/2} \frac{1}{1+4x^2}\,dx$
17. $\int \frac{1}{9+(x-1)^2}\,dx$
18. $\int_{-1/2}^{0} \frac{x}{\sqrt{1-x^2}}\,dx$
19. $\int \frac{e^{2x}}{4+e^{4x}}\,dx$
20. $\int \frac{dx}{x\sqrt{x^2-3}}$
21. $\int \frac{5-x}{\sqrt{1-x^2}}\,dx$
22. $\int \frac{x-2}{x^2+9}\,dx$

23. Find the area of the region bounded by the graphs of $y=\frac{1}{\sqrt{4-x^2}}$, $y=0$, $x=0$, $x=1$.

24. Solve the differential equation $\frac{ds}{d\theta}=\tan 2\theta$ if $s=2$ when $\theta=0$.

25. Find the average value of the function $y=\frac{5}{2x+3}$ in the interval $[0, 3]$.

26. Determine whether the function $y=\ln(x-3)$ is one-to-one and therefore has an inverse. Justify your answer. Do not use your calculator to graph the function!

27. Find the inverse of $f(x)=\frac{\sqrt[3]{2x-1}}{5}$.

SEE OTHER SIDE!

Just for fun! In spite of yesterday's proof, you probably still do not believe that 2 = 1. Maybe if we use Calculus to prove the same fact you will start to believe it…

You probably never thought of squares this way…

$$2^2 = \underbrace{2+2}_{2}$$

$$3^2 = \underbrace{3+3+3}_{3}$$

$$4^2 = \underbrace{4+4+4+4}_{4}$$

$$5^2 = \underbrace{5+5+5+5+5}_{5}$$

and so on

In general, we can say

$$x^2 = \underbrace{x+x+\ldots+x}_{x \text{ times}}$$

Let's use Calculus to take the derivatives of both sides of the equation

$$2x = \underbrace{1+1+\ldots+1}_{x \text{ times}}$$

If we add up 1 x number of times, we will get x. So

$$2x = x$$

Divide by x both sides, and, yes, here you have it again!

$$2 = 1$$

Calculus is power!

AP Calculus

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INTEGRATION

ANSWER KEY

Integration

1. $\frac{1}{6}\ln|6x+1|+C$

2. $-\frac{1}{3}\ln|3-x^3|+C$

3. $-\sqrt{9-x^2}+C$

4. $-\frac{1}{5}\ln|\cos 5\theta|+C$

5. $\frac{e^2}{2}+e-\frac{1}{2}$

6. $\frac{90}{\ln 10}$

7. $\ln\left(e^x+25\right)+C$

8. $\frac{2^{x^2}}{2\cdot\ln 2}+C$

9. $1-e^{-1}$

10. $1-e^{-1}$

11. $\ln(2+\sin x)+C$

12. $\frac{2}{3}\left(1+e^x\right)^{\frac{3}{2}}+C$

13. $2e^{\sqrt{x}}+C$

14. $\frac{x^6}{6}+\frac{5^x}{\ln 5}+C$

15. $\frac{2^{\sin x}}{\ln 2}+C$

16. $\frac{\pi}{6}$

17. $\frac{1}{3}\tan^{-1}\left(\frac{x-1}{3}\right)+C$

18. $\frac{\sqrt{3}}{2}-1$

19. $\frac{1}{4}\tan^{-1}\left(\frac{e^{2x}}{2}\right)+C$

20. $\frac{1}{\sqrt{3}}\sec^{-1}\left(\frac{|x|}{\sqrt{3}}\right)+C$

21. $5\sin^{-1}x+\sqrt{1-x^2}+C$

22. $\frac{1}{2}\ln\left(x^2+9\right)-\frac{2}{3}\tan^{-1}\left(\frac{x}{3}\right)+C$

23. $\int_0^1 \frac{1}{\sqrt{4-x^2}}\cdot dx=\frac{\pi}{6}$

24. $s=-\frac{1}{2}\ln|\cos 2\theta|+2$

25. $\frac{1}{3-0}\cdot\int_0^3 \frac{5}{2x+3}\cdot dx=\frac{5}{6}\ln 3$

26. $y'=\frac{1}{x-3}>0$ for $x>3$. Since the domain of $y=\ln(x-3)$ is $x>3$, the function is monotonic and therefore one-to-one.

27. $f^{-1}(x)=\frac{125x^3+1}{2}$.