Geometric Sequences Explicit and Recursive Formulas

Recall that a geometric sequence has a pattern of multiplying by the same value over and over again. We can model that type of sequence in two ways. One is called an explicit formula and the other a recursive formula. First lets look at explicit formulas.

<u>Explicit formula of a Geometric Sequences</u>

The explicit form of a sequence is used to find the general term, or "nth" term, by plugging in the number of term we want to know. The ***explicit form of a geometric sequence*** is $t_n = t_1(r)^{n-1}$ where t_n is the general term, t_1 is the first term of the sequence, r is the common ratio, and n is the number of term to plug in.

<u>For example:</u> In the equation $t_n = 6(2)^{n-1}$ 6 is the first term of the sequence and 2 is the common ratio. We could write out the first several terms of this sequence and get 6, 12, 24, 48, ... Suppose we want to know what the 10th term of this sequence is without having to go through finding all of the middle terms. We can first plug in 10 for n in the formula. Then use order of operations to evaluate for the tenth term.

$$t_n = 6(2)^{n-1}$$
$$t_{10} = 6(2)^{10-1}$$
$$t_{10} = 6(2)^9$$
$$t_{10} = 6(512)$$
$$t_{10} = 3072$$

So the tenth term of this sequence is 3072.

Likewise we can write an explicit formula for a geometric sequence by plugging in the first term and common ratio into $t_n = t_1(r)^{n-1}$.

<u>For example:</u> In the sequence 2, 6, 18, 54, ...
To write the explicit formula for this sequence we need to know the first term and the common ratio. We can see that the first term is 2. To find the common ratio divide any successive pair of terms such that the second in the pair is always the numerator and the first in the pair is the denominator.

2 & 6	6 & 18	18 & 54
$r = \frac{6}{2} = 3$	$r = \frac{18}{6} = 3$	$r = \frac{54}{18} = 3$

Now we know that the common ratio is 3.
Using $t_1 = 2$ and $r = 3$ we get $t_n = 2(3)^{n-1}$

Recursive formula of a Geometric Sequence

The recursive form of a sequence is slightly different. For a recursive equation we much be given one term (usually the first term) and how to use that term to get to the next term. Often we see this written as NEXT/NOW form. Let's look at our previous example

2, 6, 18, 54, ...

Suppose I tell you that the first term is 2. How can I get from 2 to 6 or from 6 to 18 or from 18 to 54? Clearly we can multiply by 3 each time. So one way to show that would be to say $NEXT = NOW(3)$ since each term is three times the previous term.

To write that recursively we plug it into the following form for a ***recursive geometric sequence***. $t_1 = \_\_\_\_\_;\ t_n = t_{n-1}(r)$ for $n > 1$ Notice that NEXT is replaced with t_n and NOW is replaced with t_{n-1} . Plug in the correct values for first term and common ratio just like we did for the explicit formula.

$t_1 = \_\_\_\_\_;\ t_n = t_{n-1}(r)$ for $n > 1$

$t_1 = 2;\ t_n = t_{n-1}(3)$ for $n > 1$

Translating between Explicit and Recursive formulas

We can clearly se that both the explicit and recursive forms use the same two key pieces of information about the sequence, the first term and the common ratio. That being the case it is fairly easy to translate between the two. Remember the explicit form is $t_n = t_1(r)^{n-1}$ and the recursive form is $t_1 = \_\_\_\_\_;\ t_n = t_{n-1}(r)$ for $n > 1$. We simply need to know where to look to find the first term and the common ratio to change forms.

Example: Write the following explicit geometric sequence as a recursive geometric sequence.

$t_n = 18(1.5)^{n-1}$

We can now identify t_1 as 18 and r as 1.5. So we can plug them into the recursive form and get $t_1 = 18;\ t_n = t_{n-1}(1.5)$ for $n > 1$.

Example: Write the following recursive geometric sequence as a explicit geometric sequence.

$t_1 = 30;\ t_n = t_{n-1}\left(\frac{1}{3}\right)$ for $n > 1$

We can now identify t_1 as 30 and r as 1/3 . So we can plug them into the explicit form and get $t_n = 30\left(\frac{1}{3}\right)^{n-1}$