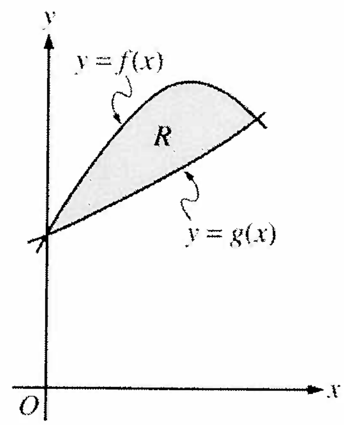
Area + Volumes AP Ques. Answer Key!



## Question 1

①

Let $f$ and $g$ be the functions given by $f(x) = 1 + \sin(2x)$ and $g(x) = e^{x/2}$. Let $R$ be the shaded region in the first quadrant enclosed by the graphs of $f$ and $g$ as shown in the figure above.




(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the $x$-axis.

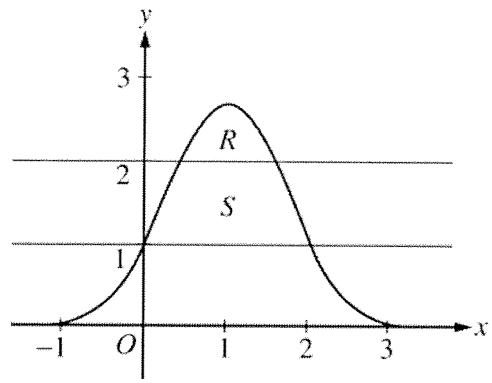
(c) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $x$-axis are semicircles with diameters extending from $y = f(x)$ to $y = g(x)$. Find the volume of this solid.

---

| Solution | Points |
|---|---|
| The graphs of $f$ and $g$ intersect in the first quadrant at $(S, T) = (1.13569, 1.76446)$. | 1 : correct limits in an integral in (a), (b), or (c) |
| (a) Area $= \int_0^S (f(x) - g(x))\,dx$ $= \int_0^S \left(1 + \sin(2x) - e^{x/2}\right) dx$ $= 0.429$ | 2 : { 1 : integrand; 1 : answer } |
| (b) Volume $= \pi \int_0^S \left((f(x))^2 - (g(x))^2\right) dx$ $= \pi \int_0^S \left((1 + \sin(2x))^2 - \left(e^{x/2}\right)^2\right) dx$ $= 4.266$ or $4.267$ | 3 : { 2 : integrand; ⟨−1⟩ each error; Note: 0/2 if integral not of form $c\int_a^b \left(R^2(x) - r^2(x)\right) dx$; 1 : answer } |
| (c) Volume $= \int_0^S \frac{\pi}{2}\left(\frac{f(x) - g(x)}{2}\right)^2 dx$ $= \int_0^S \frac{\pi}{2}\left(\frac{1 + \sin(2x) - e^{x/2}}{2}\right)^2 dx$ $= 0.077$ or $0.078$ | 3 : { 2 : integrand; 1 : answer } |

### Question 1

Let $R$ be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal line $y = 2$, and let $S$ be the region bounded by the graph of $y = e^{2x-x^2}$ and the horizontal lines $y = 1$ and $y = 2$, as shown above.




(a) Find the area of $R$.

(b) Find the area of $S$.

(c) Write, but do not evaluate, an integral expression that gives the volume of the solid generated when $R$ is rotated about the horizontal line $y = 1$.

---

$e^{2x-x^2} = 2$ when $x = 0.446057, 1.553943$

Let $P = 0.446057$ and $Q = 1.553943$

(a) Area of $R = \int_P^Q \left(e^{2x-x^2} - 2\right) dx = 0.514$

$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

(b) $e^{2x-x^2} = 1$ when $x = 0, 2$

Area of $S = \int_0^2 \left(e^{2x-x^2} - 1\right) dx$ − Area of $R$

= 2.06016 − Area of $R$ = 1.546

OR

$$\int_0^P \left(e^{2x-x^2} - 1\right) dx + (Q - P)\cdot 1 + \int_Q^2 \left(e^{2x-x^2} - 1\right) dx$$

$= 0.219064 + 1.107886 + 0.219064 = 1.546$

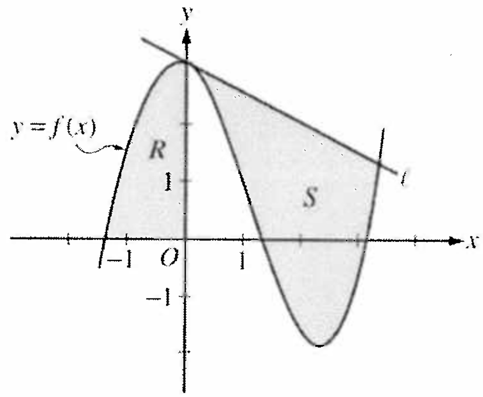
$$3 : \begin{cases} 1 : \text{integrand} \\ 1 : \text{limits} \\ 1 : \text{answer} \end{cases}$$

(c) Volume $= \pi \int_P^Q \left(\left(e^{2x-x^2} - 1\right)^2 - (2-1)^2\right) dx$

$$3 : \begin{cases} 2 : \text{integrand} \\ 1 : \text{constant and limits} \end{cases}$$

## Question 1

Let $f$ be the function given by $f(x) = \frac{x^3}{4} - \frac{x^2}{3} - \frac{x}{2} + 3\cos x$. Let $R$ be the shaded region in the second quadrant bounded by the graph of $f$, and let $S$ be the shaded region bounded by the graph of $f$ and line $\ell$, the line tangent to the graph of $f$ at $x = 0$, as shown above.




(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is rotated about the horizontal line $y = -2$.

(c) Write, but do not evaluate, an integral expression that can be used to find the area of $S$.

---

For $x < 0$, $f(x) = 0$ when $x = -1.37312$.
Let $P = -1.37312$.

(a) Area of $R = \int_P^0 f(x)\,dx = 2.903$

$2 : \begin{cases} 1 : \text{integral} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_P^0 \left( (f(x) + 2)^2 - 4 \right) dx = 59.361$

$4 : \begin{cases} 1 : \text{limits and constant} \\ 2 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) The equation of the tangent line $\ell$ is $y = 3 - \frac{1}{2}x$.

The graph of $f$ and line $\ell$ intersect at $A = 3.38987$.

Area of $S = \int_0^A \left( \left(3 - \frac{1}{2}x\right) - f(x) \right) dx$

$3 : \begin{cases} 1 : \text{tangent line} \\ 1 : \text{integrand} \\ 1 : \text{limits} \end{cases}$

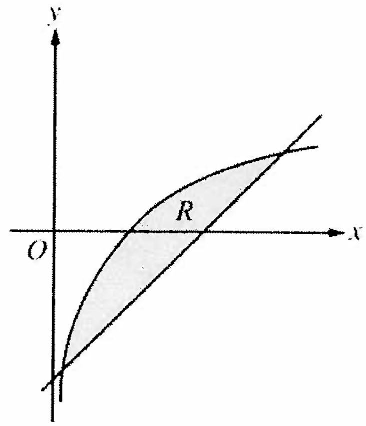
### Question 1

Let $R$ be the shaded region bounded by the graph of $y = \ln x$ and the line $y = x - 2$, as shown above.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is rotated about the horizontal line $y = -3$.

(c) Write, but do not evaluate, an integral expression that can be used to find the volume of the solid generated when $R$ is rotated about the $y$-axis.




---

$\ln(x) = x - 2$ when $x = 0.15859$ and $3.14619$.
Let $S = 0.15859$ and $T = 3.14619$

| | |
|---|---|
| (a) Area of $R = \int_S^T (\ln(x) - (x-2))\,dx = 1.949$ | 3 : { 1 : integrand; 1 : limits; 1 : answer |
| (b) Volume $= \pi \int_S^T \left( (\ln(x) + 3)^2 - (x - 2 + 3)^2 \right) dx = 34.198$ or $34.199$ | 3 : { 2 : integrand; 1 : limits, constant, and answer |
| (c) Volume $= \pi \int_{S-2}^{T-2} \left( (y+2)^2 - \left(e^y\right)^2 \right) dy$ | 3 : { 2 : integrand; 1 : limits and constant |

# AP® CALCULUS AB
# 2007 SCORING GUIDELINES

## Question 1

Let $R$ be the region in the first and second quadrants bounded above by the graph of $y = \frac{20}{1 + x^2}$ and below by the horizontal line $y = 2$.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is rotated about the $x$-axis.

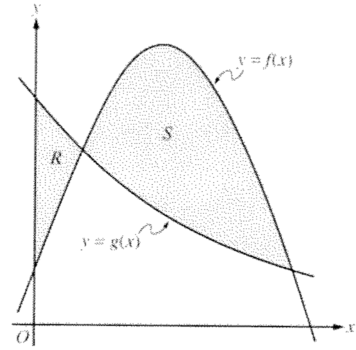
(c) The region $R$ is the base of a solid. For this solid, the cross sections perpendicular to the $x$-axis are semicircles. Find the volume of this solid.

| | |
|---|---|
| $\frac{20}{1 + x^2} = 2$ when $x = \pm 3$ | 1 : correct limits in an integral in (a), (b), or (c) |
| (a) Area $= \int_{-3}^{3} \left( \frac{20}{1 + x^2} - 2 \right) dx = 37.961$ or $37.962$ | 2 : { 1 : integrand; 1 : answer |
| (b) Volume $= \pi \int_{-3}^{3} \left( \left( \frac{20}{1 + x^2} \right)^2 - 2^2 \right) dx = 1871.190$ | 3 : { 2 : integrand; 1 : answer |
| (c) Volume $= \frac{\pi}{2} \int_{-3}^{3} \left( \frac{1}{2} \left( \frac{20}{1 + x^2} - 2 \right) \right)^2 dx$ $= \frac{\pi}{8} \int_{-3}^{3} \left( \frac{20}{1 + x^2} - 2 \right)^2 dx = 174.268$ | 3 : { 2 : integrand; 1 : answer |

### Question 1

(6)

Let $f$ and $g$ be the functions given by $f(x) = \frac{1}{4} + \sin(\pi x)$ and $g(x) = 4^{-x}$. Let $R$ be the shaded region in the first quadrant enclosed by the $y$-axis and the graphs of $f$ and $g$, and let $S$ be the shaded region in the first quadrant enclosed by the graphs of $f$ and $g$, as shown in the figure above.




(a) Find the area of $R$.

(b) Find the area of $S$.

(c) Find the volume of the solid generated when $S$ is revolved about the horizontal line $y = -1$.

---

$f(x) = g(x)$ when $\frac{1}{4} + \sin(\pi x) = 4^{-x}$.

$f$ and $g$ intersect when $x = 0.178218$ and when $x = 1$.
Let $a = 0.178218$.

| Solution | Points |
|---|---|
| (a) $\int_0^a (g(x) - f(x))\,dx = 0.064$ or $0.065$ | 3 : { 1 : limits<br>1 : integrand<br>1 : answer |
| (b) $\int_a^1 (f(x) - g(x))\,dx = 0.410$ | 3 : { 1 : limits<br>1 : integrand<br>1 : answer |
| (c) $\pi \int_a^1 \left( (f(x)+1)^2 - (g(x)+1)^2 \right) dx = 4.558$ or $4.559$ | 3 : { 2 : integrand<br>1 : limits, constant, and answer |

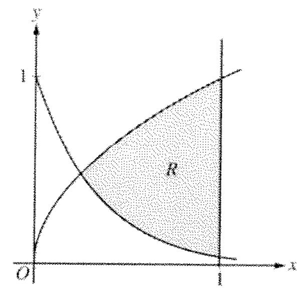
## Question 1

Let $R$ be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the horizontal line $y = 1$.

(c) The region $R$ is the base of a solid. For this solid, each cross section perpendicular to the $x$-axis is a rectangle whose height is 5 times the length of its base in region $R$. Find the volume of this solid.




| | |
|---|---|
| Point of intersection<br>$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734,\ 0.488604)$ | 1: Correct limits in an integral in (a), (b), or (c) |
| (a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$<br>$= 0.442$ or $0.443$ | 2 : { 1 : integrand<br>1 : answer |
| (b) Volume $= \pi \int_T^1 \left( \left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2 \right) dx$<br>$= 0.453\pi$ or $1.423$ or $1.424$ | 3 : { 2 : integrand<br>< −1 > reversal<br>< −1 > error with constant<br>< −1 > omits 1 in one radius<br>< − 2 > other errors<br>1 : answer |
| (c) Length $= \sqrt{x} - e^{-3x}$<br>Height $= 5\left(\sqrt{x} - e^{-3x}\right)$<br>Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$ | 3 : { 2 : integrand<br>< − 1 > incorrect but has $\sqrt{x} - e^{-3x}$ as a factor<br>1 : answer |

# AP® CALCULUS AB
# 2004 SCORING GUIDELINES (Form B)

## Question 1

Let $R$ be the region enclosed by the graph of $y = \sqrt{x-1}$, the vertical line $x = 10$, and the $x$-axis.

(a) Find the area of $R$.

(b) Find the volume of the solid generated when $R$ is revolved about the horizontal line $y = 3$.

(c) Find the volume of the solid generated when $R$ is revolved about the vertical line $x = 10$.

---

(a) Area $= \int_1^{10} \sqrt{x-1}\, dx = 18$

$3 : \begin{cases} 1 : \text{limits} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(b) Volume $= \pi \int_1^{10} \left(9 - \left(3 - \sqrt{x-1}\right)^2\right) dx$

$= 212.057$ or $212.058$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$

(c) Volume $= \pi \int_0^{3} \left(10 - \left(y^2 + 1\right)\right)^2 dy$

$= 407.150$

$3 : \begin{cases} 1 : \text{limits and constant} \\ 1 : \text{integrand} \\ 1 : \text{answer} \end{cases}$