Answers

Error Bound Examples

1. a. Find the fourth-order Maclaurin polynomial for $f(x) = e^x$

$1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!}$

b. Use the polynomial in part a to approximate e. $x = 1$

$1 + 1 + \frac{1}{2} + \frac{1}{6} + \frac{1}{24} = \boxed{2.708}$

c. Find the Lagrange Error Bound for your approximation on the interval [0, 1]

$\frac{e^1 |1-0|^5}{5!} = \boxed{.02265}$

2. a. Use the fifth-order Maclaurin polynomial for $f(x) = \ln(1+x)$ to approximate ln(1.2).

$x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \frac{x^5}{5}$ $x = 0.2$ $.2 - \frac{.2^2}{2} + \frac{.2^3}{3} - \frac{.2^4}{4} + \frac{.2^5}{5} = \boxed{.18232}$

b. Estimate the error in your approximation using the Lagrange Error Bound

$f^{(VI)}(x) = -120(1+x)^{-6}$ $x = 0.2$ $M = \left|\frac{-120}{(1.2)^6}\right| = 40.187757$ Error $= \frac{40.187757\,(.2)^6}{6!} = \boxed{.0000035722}$

3. Estimate the error in approximating sin(-0.3) with a 3rd-degree Maclaurin polynomial using the Lagrange Error Bound. (Remember that you can use M = 1 for any trig function)

Error $= \frac{1\,|-.3|^4}{4!} = \boxed{.0003375}$

4. a. Use the second-degree Taylor polynomial for $f(x) = \sqrt{1+x}$ centered at x = 3 to approximate $\sqrt{4.2}$

$2 + \frac{1}{4}(x-3) - \frac{1}{32}\frac{(x-3)^2}{2!}$ $x = 3.2$ $2 + \frac{1}{4}(.2) - \frac{1}{32}\left(\frac{.2^2}{2}\right) = 2.049375$

b. Estimate the error in your approximation using the Lagrange Error Bound on the interval [3, 3.4]

$M = \frac{3(1+3)^{-5/2}}{8} = \frac{3}{256}$ use $x = 3.2$ in Error formula

$f'''(x) = \frac{3}{8}(1+x)^{-5/2}$ use $x = 3$ to get Max deriv $= M$ Error $= \frac{\frac{3}{256}|3.2-3|^3}{3!} = \boxed{.000015625}$

5. Suppose $f(1) = 8$, $f'(1) = 4$, $f''(1) = -2$, and $|f'''(x)| \le 10$ for all x in the domain of f.

a. Approximate $f(1.4)$

$8 + 4(x-1) - \frac{2(x-1)^2}{2}$ $8 + 4(.4) - \frac{2(.4)^2}{2} = \boxed{9.44}$

b. Estimate the error in your answer using the Lagrange Error Bound.

$M = 10$ Error $= \frac{10(.4)^3}{3!} = \boxed{.1067}$

6. Suppose $f(0) = 2$, $f'(0) = -3$, $f''(0) = 4$, and $|f'''(x)| \le 2$ for all x in the interval $[-2, 2]$ M is given as 2

a. Approximate $f(-1)$. $2 - 3(x-0) + \frac{4(x-0)^2}{2} = 2 + 3 + 2 = 7$

b. Prove that $f(-1) \ne 8.75$. $R_3(x) \le \frac{2|-1-0|^3}{3!} = \frac{2}{6} = \boxed{\frac{1}{3}}$

$f(-1)$ is within $\frac{1}{3}$ of 7. Therefore $f(-1) \ne 8.75$