AP Calculus

REVIEW FOR 3rd QUARTER MIDTERM

Name_____

Seat # _____ Date _____

Differentiation

DO NOT USE A CALCULATOR FOR ANY OF THESE QUESTIONS

In exercises 1–24, find the derivative of each function. Simplify your answer as much as possible.

1. $h(x)=\ln\left(x^2+3\right)$
2. $g(x)=\ln\sqrt{x}$
3. $f(x)=\sqrt{(\ln x)}$
4. $y=\ln\left(\dfrac{x}{x+1}\right)$
5. $h(t)=\dfrac{\ln t}{t}$
6. $y=\ln\sqrt{\dfrac{x+1}{x-1}}$
7. $f(t)=e^{1-t}$
8. $g(x)=e^{\sqrt{x}}$
9. $y=x^2e^{-x}$
10. $h(x)=\dfrac{e^x}{1+x}$
11. $f(t)=\ln\left(1+e^{2t}\right)$
12. $y=\dfrac{e^x-e^{-x}}{2}$
13. $g(x)=2^{-x}$
14. $y=\log_3\left(x^2-4\right)$
15. $h(t)=t\left(7^{-3t}\right)$
16. $y=\log_2\left(\dfrac{x-1}{x}\right)$
17. $f(t)=\dfrac{3^{2t}}{t}$
18. $f(x)=\log\sqrt[3]{x^2}$
19. $y=\sin^{-1}\left(x^2\right)$
20. $f(x)=\tan^{-1}\left(e^x\right)$
21. $h(x)=x\cdot\arctan x$
22. $g(t)=\left(\sec^{-1}t\right)^2$
23. $y=(\operatorname{arccot}x)\cdot\ln x$
24. $y=\cos^{-1}\sqrt{x}$

In exercises 25 and 26, use implicit differentiation to find $\dfrac{dy}{dx}$.

25. $x^3-\ln y=5+xy$
26. $xe^y-10x+3y^2=9$

27. Find the domain of $f(x)=\dfrac{x}{\ln\left(1-x^2\right)}$. Justify your answer.

28. a) Find the domain of the function $y=x-\ln x$.
    b) Find any relative extrema and inflection points for $y=x-\ln x$. Justify your answer.

29. Find an equation of the tangent line to the curve $y=e^{-x}\sin x$ at $x=\pi$.

30. Find an equation of the tangent line to the curve $y=3\arccos\left(\frac{x}{2}\right)$ at $x=1$.

Just for fun! You can use math to prove some things that go beyond the wildest imagination…Check the example below!

Suppose you have two variables a and b, both of them non-zero and such that

$$a=b$$

Multiply both sides of the equation by a and you will have

$$a^2=a\cdot b$$

Subtract b^2 from both sides to obtain

$$a^2-b^2=a\cdot b-b^2$$

If we now factor both sides of the equation, we will get

$$(a+b)(a-b)=b\cdot(a-b)$$

Divide both sides of the equation by $(a-b)$ to obtain

$$a+b=b$$

Since we started with the fact that $a=b$, we can substitute a by b

$$b+b=b$$

And from here, it easily follows that

$$2b=b$$

So

$$2=1$$

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DIFFERENTIATION

ANSWER KEY

Differentiation

1. $h'(x)=\dfrac{2x}{x^2+3}$

2. $g'(x)=\dfrac{1}{2x}$

3. $f'(x)=\dfrac{1}{2x\sqrt{(\ln x)}}$

4. $y'=\dfrac{1}{x\cdot(x+1)}$

5. $h'(t)=\dfrac{1-\ln t}{t^2}$

6. $y'=\dfrac{-1}{x^2-1}$

7. $f'(t)=-e^{1-t}$

8. $g'(x)=\dfrac{e^{\sqrt{x}}}{2\sqrt{x}}$

9. $y'=e^{-x}(2x-x^2)$

10. $h'(x)=\dfrac{xe^x}{(1+x)^2}$

11. $f'(t)=\dfrac{2e^{2t}}{1+e^{2t}}$

12. $y'=\dfrac{e^x+e^{-x}}{2}$

13. $g'(x)=-2^{-x}\cdot\ln 2$

14. $y'=\dfrac{2x}{(x^2-4)\cdot\ln 3}$

15. $h'(t)=7^{-3t}(1-3t\cdot\ln 7)$

16. $y'=\dfrac{1}{x\cdot(x-1)\cdot\ln 2}$

17. $f'(t)=\dfrac{3^{2t}(2t\ln 3-1)}{t^2}$

18. $f'(x)=\dfrac{2}{3x\cdot\ln 10}$

19. $y'=\dfrac{2x}{\sqrt{1-x^4}}$

20. $f'(x)=\dfrac{e^x}{e^{2x}+1}$

21. $h(x)=\tan^{-1}x+\dfrac{x}{x^2+1}$

22. $g(t)=\dfrac{2(\sec^{-1}t)}{|t|\cdot\sqrt{t^2-1}}$

23. $y=\dfrac{-\ln x}{x^2+1}+\dfrac{\cot^{-1}x}{x}$

24. $y=\dfrac{-1}{2\sqrt{x}\sqrt{1-x}}$

25. $\dfrac{dy}{dx}=\dfrac{3x^2y-y^2}{xy+1}$

26. $\dfrac{dy}{dx}=\dfrac{10-e^y}{xe^y+6y}$

27. $(-1, 0)$ and $(0, 1)$

28. a) Domain: $x > 0$
    b) Local minimum with value $y = 1$ at $x = 1$. No local maxima or POI.

29. $y=-\dfrac{1}{e^{\pi}}\cdot(x-\pi)$

30. $y-\pi=-\sqrt{3}\cdot(x-1)$