Set 9: Multiple-Choice Questions on Differential Equations

Part A. Directions: Answer these questions *without* using your calculator.

In Questions 1–10, $a(t)$ denotes the acceleration function, $v(t)$ the velocity function, and $s(t)$ the position or height function at time t. (The acceleration due to gravity is -32 ft/sec^2.)

1. If $a(t) = 4t - 1$ and $v(1) = 3$, then $v(t)$ equals

(A) $2t^2 - t$ **(B)** $2t^2 - t + 1$ **(C)** $2t^2 - t + 2$
(D) $2t^2 + 1$ **(E)** $2t^2 + 2$

2. If $a(t) = 20t^3 - 6t$, $s(-1) = 2$, and $s(1) = 4$, then $v(t)$ equals

(A) $t^5 - t^3$ **(B)** $5t^4 - 3t^2 + 1$ **(C)** $5t^4 - 3t^2 + 3$
(D) $t^5 - t^3 + t + 3$ **(E)** $t^5 - t^3 + 1$

3. Given $a(t)$, $s(-1)$, and $s(1)$ as in Question 2, then $s(0)$ equals

(A) 0 **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4

4. A stone is thrown straight up from the top of a building with initial velocity 40 ft/sec and hits the ground 4 sec later. The height of the building, in feet, is

(A) 88 **(B)** 96 **(C)** 112 **(D)** 128 **(E)** 144

5. The maximum height is reached by the stone in Question 4 after

(A) 4/5 sec **(B)** 4 sec **(C)** 5/4 sec **(D)** 5/2 sec **(E)** 2 sec

6. If a car accelerates from 0 to 60 mph in 10 sec, what distance does it travel in those 10 sec? (Assume the acceleration is constant and note that 60 mph = 88 ft/sec.)

(A) 40 ft **(B)** 44 ft **(C)** 88 ft **(D)** 400 ft **(E)** 440 ft

7. A stone is thrown at a target so that its velocity after t sec is $(100 - 20t)$ ft/sec. If the stone hits the target in 1 sec, then the distance from the sling to the target is

(A) 80 ft **(B)** 90 ft **(C)** 100 ft **(D)** 110 ft **(E)** 120 ft

8. What should the initial velocity be if you want a stone to reach a height of 100 ft when you throw it straight up?

(A) 80 ft/sec **(B)** 92 ft/sec **(C)** 96 ft/sec
(D) 112 ft/sec **(E)** none of these

9. If the velocity of a car traveling in a straight line at time t is $v(t)$, then the difference in its odometer readings between times $t = a$ and $t = b$ is

(A) $\int_a^b |v(t)|\,dt$

(B) $\int_a^b v(t)\,dt$

(C) the net displacement of the car's position from $t = a$ to $t = b$
(D) the change in the car's position from $t = a$ to $t = b$
(E) none of these

10. If an object is moving up and down along the y-axis with velocity $v(t)$ and $s'(t) = v(t)$, then it is false that $\int_a^b v(t)\,dt$ gives

(A) $s(b) - s(a)$
(B) the net distance traveled by the object between $t = a$ and $t = b$
(C) the total change in $s(t)$ between $t = a$ and $t = b$
(D) the shift in the object's position from $t = a$ to $t = b$
(E) the total distance covered by the object from $t = a$ to $t = b$

11. A solution of the differential equation $y\,dy = x\,dx$ is

(A) $x^2 - y^2 = 4$ (B) $x^2 + y^2 = 4$ (C) $y^2 = 4x^2$
(D) $x^2 - 4y^2 = 0$ (E) $x^2 = 9 - y^2$

12. If $\frac{dy}{dx} = \frac{y}{2\sqrt{x}}$ and $y = 1$ when $x = 4$, then

(A) $y^2 = 4\sqrt{x} - 7$ (B) $\ln y = 4\sqrt{x} - 8$ (C) $\ln y = \sqrt{x} - 2$

(D) $y = e^{\sqrt{x}}$ (E) $y = e^{\sqrt{x}-2}$

13. If $\frac{dy}{dx} = e^y$ and $y = 0$ when $x = 1$, then

(A) $y = \ln |x|$ (B) $y = \ln |2 - x|$ (C) $e^{-y} = 2 - x$

(D) $y = -\ln |x|$ (E) $e^{-y} = x - 2$

14. If $\frac{dy}{dx} = \frac{x}{\sqrt{9 + x^2}}$ and $y = 5$ when $x = 4$, then y equals

(A) $\sqrt{9 + x^2} - 5$ (B) $\sqrt{9 + x^2}$ (C) $2\sqrt{9 + x^2} - 5$

(D) $\frac{\sqrt{9 + x^2} + 5}{2}$ (E) none of these

15. The general solution of the differential equation $x\,dy = y\,dx$ is a family of

(A) circles **(B)** hyperbolas **(C)** parallel lines
(D) parabolas **(E)** lines passing through the origin

16. The general solution of the differential equation $\frac{dy}{dx} = y$ is a family of

(A) parabolas **(B)** straight lines **(C)** hyperbolas
(D) ellipses **(E)** none of these

17. A function $f(x)$ that satisfies the equations $f(x)f'(x) = x$ and $f(0) = 1$ is

(A) $f(x) = \sqrt{x^2+1}$ **(B)** $f(x) = \sqrt{1-x^2}$ **(C)** $f(x) = x$
(D) $f(x) = e^x$ **(E)** none of these

18. The curve that passes through the point (1, 1) and whose slope at any point (x, y) is equal to $\frac{3y}{x}$ has the equation

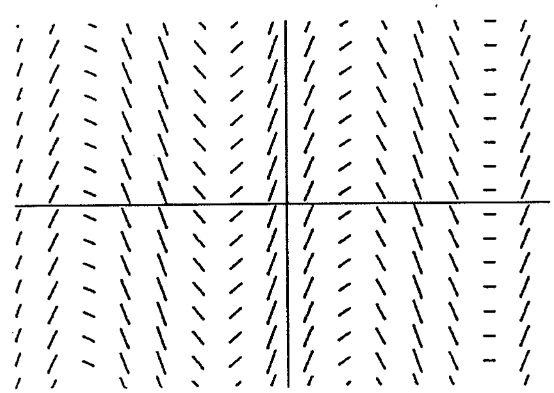
(A) $3x - 2 = y$ **(B)** $y^3 = x$ **(C)** $y = |x^3|$
(D) $3y^2 = x^2 + 2$ **(E)** $3y^2 - 2x = 1$

19. If $\frac{dy}{dx} = \frac{k}{x}$, k a constant, and if $y = 2$ when $x = 1$ and $y = 4$ when $x = e$, then, when $x = 2$, y equals

(A) 2 **(B)** 4 **(C)** ln 8 **(D)** ln 2 + 2 **(E)** ln 4 + 2

20. The slope field shown at the right is for the differential equation

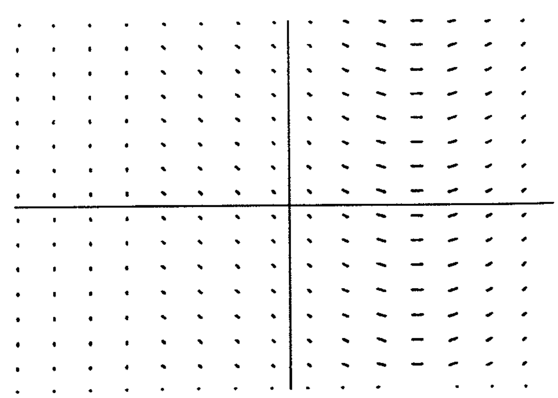
(A) $y' = x + 1$
(B) $y' = \sin x$
(C) $y' = -\sin x$
(D) $y' = \cos x$
(E) $y' = -\cos x$



$[-2\pi, 2\pi] \times [-1.5, 1.5]$

21. The slope field at the right is for the differential equation

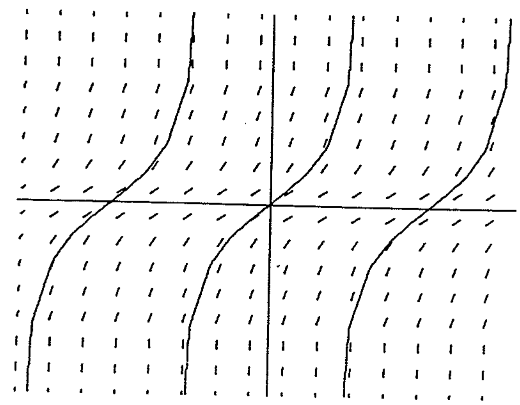
(A) $y' = 2x$
(B) $y' = 2x - 4$
(C) $y' = 4 - 2x$
(D) $y' = y$
(E) $y' = x + y$



$[-4, 4] \times [-12, 12]$

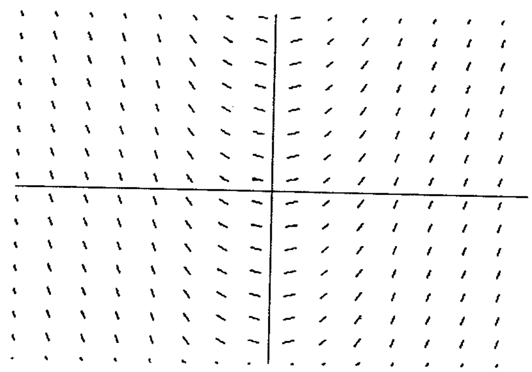
22. A solution curve has been superimposed on the slope field shown at the right. The solution is for the differential equation and initial condition

(A) $y' = \tan x$; $y(0) = 0$
(B) $y' = \cot x$, $y(\pi/4) = 1$
(C) $y' = 1 + x^2$; $y(0) = 0$
(D) $y' = \dfrac{1}{1+x^2}$; $y\left(\dfrac{\pi}{4}\right) = 1$
(E) $y' = 1 + y^2$; $y(0) = 0$

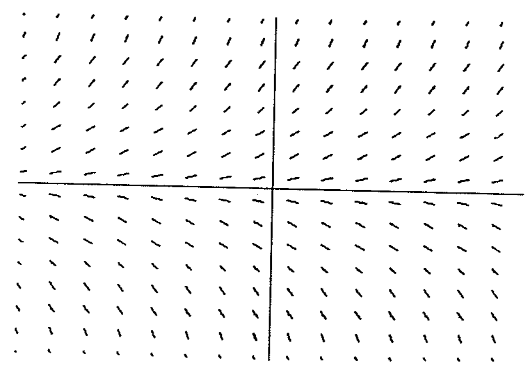


$[-4, 4] \times [-4, 4]$

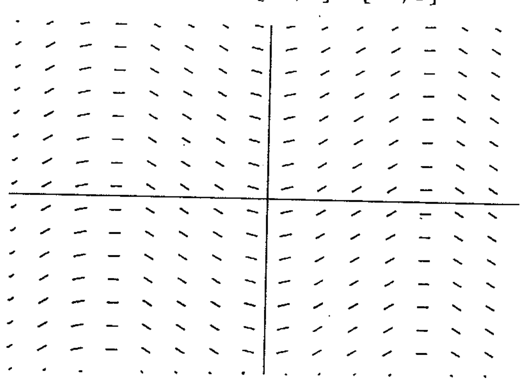
The slope fields below are for Questions 23–28.



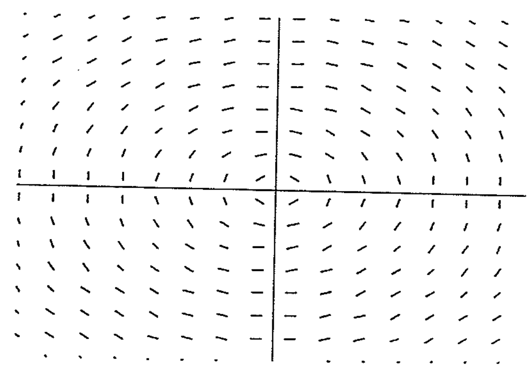
I $[-3, 3] \times [-3, 3]$



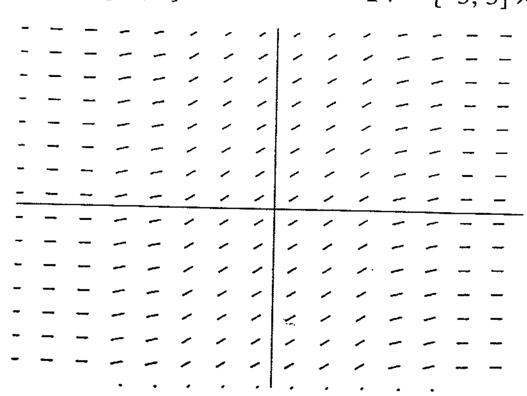
II $[-3, 3] \times [-3, 3]$



III $[-5, 5] \times [-5, 5]$



IV $[-3, 3] \times [-3, 3]$



V $[-2, 2] \times [-2, 2]$

23. Which slope field is for the differential equation $y' = y$?

(A) I (B) II (C) III (D) IV (E) V

24. Which slope field is for the differential equation $y' = -\frac{x}{y}$?

(A) I (B) II (C) III (D) IV (E) V

25. Which slope field is for the differential equation $y' = \sin x$?

(A) I (B) II (C) III (D) IV (E) V

26. Which slope field is for the differential equation $y' = 2x$?

(A) I (B) II (C) III (D) IV (E) V

27. Which slope field is for the differential equation $y' = e^{-x^2}$?

(A) I (B) II (C) III (D) IV (E) V

28. A particular solution curve of a differential equation whose slope field is shown above in II passes through the point (0,–1). The equation is

(A) $y = -e^x$ (B) $y = -e^{-x}$ (C) $y = x^2 - 1$ (D) $y = -\cos x$
(E) $y = -\sqrt{1 - x^2}$

***29.** If you use Euler's method with $\Delta x = 0.1$ for the d.e. $y' = x$, with initial value $y(1) = 5$, then, when $x = 1.2$, y is approximately

(A) 5.10 (B) 5.20 (C) 5.21 (D) 6.05 (E) 7.10

***30.** The error in using Euler's method in Question 29 is

(A) 0.005 (B) 0.010 (C) 0.050 (D) 0.500 (E) 0.720

Part B. Directions: Some of the following questions require the use of a graphing calculator.

31. If $\frac{ds}{dt} = \sin^2\left(\frac{\pi}{2}s\right)$ and if $s = 1$ when $t = 0$, then, when $s = \frac{3}{2}$, t is equal to

(A) $\frac{1}{2}$ (B) $\frac{\pi}{2}$ (C) 1 (D) $\frac{2}{\pi}$ (E) $-\frac{2}{\pi}$

32. If radium decomposes at a rate proportional to the amount present, then the amount R left after t yr, if R_0 is present initially and c is the negative constant of proportionality, is given by

(A) $R = R_0ct$ (B) $R = R_0e^{ct}$ (C) $R = R_0 + \frac{1}{2}ct^2$
(D) $R = e^{R_0ct}$ (E) $R = e^{R_0+ct}$

*An asterisk denotes a topic covered only in Calculus BC.

33. The population of a city increases continuously at a rate proportional, at any time, to the population at that time. The population doubles in 50 yr. After 75 yr the ratio of the population P to the initial population P_0 is

(A) $\frac{9}{4}$ **(B)** $\frac{5}{2}$ **(C)** $\frac{4}{1}$ **(D)** $\frac{2\sqrt{2}}{1}$ **(E)** none of these

34. If a substance decomposes at a rate proportional to the amount of the substance present, and if the amount decreases from 40 g to 10 g in 2 hr, then the constant of proportionality is

(A) $-\ln 2$ **(B)** $-\frac{1}{2}$ **(C)** $-\frac{1}{4}$ **(D)** $\ln\frac{1}{4}$ **(E)** $\ln\frac{1}{8}$

35. If $(g'(x))^2 = g(x)$ for all real x and $g(0) = 0$, $g(4) = 4$, then $g(1)$ equals

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2}$ **(C)** 1 **(D)** 2 **(E)** 4

36. The solution curve of $y' = y$ that passes through point (2, 3) is

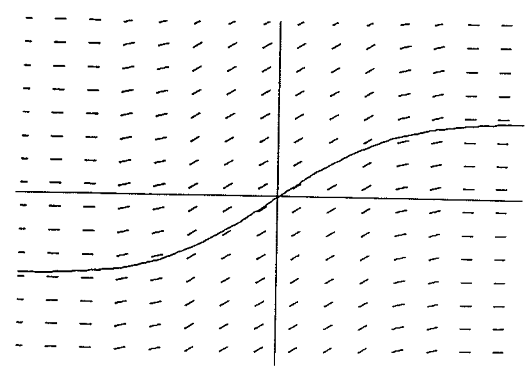
(A) $y = e^x + 3$ **(B)** $y = \sqrt{2x+5}$ **(C)** $y = 0.406e^x$
(D) $y = e^x - (e^2 + 3)$ **(E)** $y = e^x/(0.406)$

37. At any point of intersection of a solution curve of the d.e. $y' = x + y$ and the line $x + y = 0$, the function y at that point

(A) is equal to 0 **(B)** is a local maximum **(C)** is a local minimum
(D) has a point of inflection **(E)** has a discontinuity

38. The slope field for $F'(x) = e^{-x^2}$ is shown at the right with the particular solution $F(0) = 0$ superimposed. With a graphing calculator, $\lim_{x\to\infty} F(x)$ to three decimal places is

(A) 0.886 **(B)** 0.987
(C) 1.000 **(D)** 1.414
(E) ∞



$[-2, 2] \times [-2, 2]$

***39.** Which of these statements about Euler's method is(are) true?

I. It can be used to estimate solutions of differential equations numerically.

II. It cannot be applied to an equation of the form $\frac{dy}{dx} = F(x, y)$, where F is defined implicitly.

III. It should not be used on an interval on which the function becomes infinite.

(A) I only **(B)** II only **(C)** III only

(D) I and III only **(E)** I, II, and III

***40.** Which statement about Euler's method is false?

(A) If you halve the step size, you approximately halve the error.
(B) Euler's method never gives exact solutions.
(C) Euler's method assumes that the slope of a solution curve is the same at all points in a short interval.
(D) Often, when applying Euler's method, the more steps you take the smaller the error.
(E) Euler's method is used to string together a set of linearizations that approximate a curve.

41. A cup of coffee at temperature 180°F is placed on a table in a room at 68°F. The d.e. for its temperature at time t is $\frac{dy}{dt} = -0.11(y - 68)$; $y(0) = 180$. After 10 min the temperature (in °F) of the coffee is

(A) 96 **(B)** 100 **(C)** 105 **(D)** 110 **(E)** 115

42. Approximately how long does it take the temperature of the coffee in Question 41 to drop to 75°F?

(A) 10 min **(B)** 15 min **(C)** 18 min **(D)** 20 min **(E)** 25 min

43. The concentration of a medication injected into the bloodstream drops at a rate proportional to the existing concentration. If the factor of proportionality is 30% per hour, in how many hours will the concentration be one-tenth of the initial concentration?

(A) 3 **(B)** $4\frac{1}{3}$ **(C)** $6\frac{2}{3}$

(D) $7\frac{2}{3}$ **(E)** none of these

***44.** Which of the following statements characterize(s) the logistic growth of a population whose limiting value is L?

I. The rate of growth increases at first.

II. The growth rate attains a maximum when the population equals $\frac{L}{2}$.

III. The growth rate approaches 0 as the population approaches L.

(A) I only **(B)** II only **(C)** I and II only
(D) II and III only **(E)** I, II, and III

*An asterisk denotes a topic covered only in Calculus BC.

*45. Which of the following d.e.'s is not logistic?

(A) $P' = P - P^2$ (B) $\frac{dy}{dt} = 0.01y(100 - y)$

(C) $\frac{dx}{dt} = 0.8x - 0.004x^2$ (D) $\frac{dR}{dt} = 0.16(350 - R)$

(E) $f'(t) = kf(t) \cdot [A - f(t)]$ (where k and A are constants)

*46. Suppose $P(t)$ denotes the size of an animal population at time t and its growth is described by the d.e. $\frac{dP}{dt} = 0.002P(1000 - P)$. The population is growing fastest

(A) initially (B) when $P = 500$ (C) when $P = 1000$

(D) when $\frac{dP}{dt} = 0$ (E) when $\frac{d^2P}{dt^2} > 0$

47. According to Newton's law of cooling, the temperature of an object decreases at a rate proportional to the difference between its temperature and that of the surrounding air. Suppose a corpse at a temperature of 32°C arrives at a mortuary where the temperature is kept at 10°C. Then the differential equation satisfied by the temperature T of the corpse t hr later is

(A) $\frac{dT}{dt} = -k(T - 10)$ (B) $\frac{dT}{dt} = k(T - 32)$ (C) $\frac{dT}{dt} = 32e^{-kt}$

(D) $\frac{dT}{dt} = -kT(T - 10)$ (E) $\frac{dT}{dt} = kT(T - 32)$

48. If the corpse in Question 47 cools to 27°C in 1 hr, then its temperature (in °C) is given by the equation

(A) $T = 22e^{0.205t}$ (B) $T = 10e^{1.163t}$ (C) $T = 10 + 22e^{-0.258t}$

(D) $T = 32e^{-0.169t}$ (E) $T = 32 - 10e^{-0.093t}$

*An asterisk denotes a topic covered only in Calculus BC.