AP CALCULUS
Miss Brown

Name ______________________________
Date _______________________________

Differential Equation Applications

PART 1: WRITE THE DIFFERENTIAL EQUATION

1. A radioactive substance decays at a rate directly proportional to the amount of substance present.

2. Uranium disintegrates at a rate proportional to the square of the amount present.

3. The temperature of an object changes at a rate proportional to the positive difference between the temperature of the object and the temperature of the environment. This is known as Newton's Law of Cooling!

PART 2: PROPORTIONAL TO THE AMOUNT PRESENT $\frac{d\square}{dt} = k\square$ →

4. The rate at which the population of a bacteria culture grows is proportional to the number of bacteria present. If the number of bacteria grew from 1000 to 5000 in 10 hours, find the number of bacteria after 15 hours.

5. Radium decomposes at a rate proportional to the amount present. If 200 mg reduces to 180 mg in 100 years, how many milligrams will remain at the end of 500 years? What is the half-life of radium?

6. The bacteria in a certain culture increase at a rate proportional to the number of bacteria present. If the number of bacteria triples in 8 hours, how many bacteria are there in 12 hours?

7. Oil is being pumped continuously from a certain oil well at a rate proportional to the amount of oil left in the well. Initially there were 1,000,000 gallons of oil in the well, and 6 years later there were 500,000 gallons. It is no longer profitable to pump oil when there are fewer than 50,000 gallons remaining.

   A. Write an equation for y, the amount of oil remaining in the well at any time t.
   B. At what rate is the amount of oil in the well decreasing when there are 600,000 gallons of oil remaining?
   C. In order not to lose money, at what time t should oil no longer be pumped from the well?

8. Research has provided data which substantiate the model that the risk (r%) of having an automobile accident is related to the blood alcohol level (b%) by the differential equation $\frac{dr}{db} = kr$ where k is a constant. The risk is 1% if the blood alcohol level is 0% and the risk is 20% if the blood alcohol level is 14%. At what blood alcohol level will the risk of having an accident be 80%?

PART 3: PROPORTIONAL TO A DIFFERENCE OF TWO QUANTITIES

9. Let P(t) represent the number of wolves in a population at time t years, when $t \geq 0$. The population P(t) is increasing at a rate directly proportional to $800 - P(t)$, where the constant of proportionality is k.

   A. If $P(0) = 500$, find P(t) in terms of t and k.
   B. If $P(2) = 700$, find k.
   C. Find $\lim_{t \to \infty} P(t)$.

10. A roast is put in a 300° F oven and heats according to the differential equation $\frac{dT}{dt} = k(300 - T)$ where k is a positive constant and T(t) is the temperature of the roast after t minutes.

    A. If the roast is at 50° F when put in the oven, find T(t) in terms of k and t.
    B. If $T(30) = 200^\circ$ F, find k.

11. Let F(t) be the temperature, in degrees Fahrenheit, of a cup of tea at time t minutes, $t \geq 0$. Room temperature is 70° and the initial temperature of the tea is 180°. The tea's temperature at time t is described by the differential equation $\frac{dF}{dt} = -0.1(F - 70)$, with the initial condition $F(0) = 180$.

    A. Find an expression for F in terms of t, where t is measured in minutes.
    B. How hot is the tea after 10 minutes?
    C. If the tea is safe to drink without burning your mouth when its temperature is less than 120°, after how many full minutes is the tea safe to drink?

12. The population of a species of jellyfish in Sting Harbor increases at a rate directly proportional to $450 - Q(t)$, where Q is the population (in thousands) and t is the time (in years). At $t = 0$ (1990) the population was 100,000, and in 1992, the population was 300,000.

    A. What was the population in 1993?
    B. In what year did the population reach 400,000?

PART 4: PROPORTIONAL TO OTHER THINGS

13. A scientist has discovered a radioactive substance that disintegrates in such a way that at time t, the rate of disintegration is proportional to the square of the amount present.

    A. A 100 gram sample of the substance dwindles to 80 grams in 1 day. How much will be left after 6 days?
    B. When will only 10 grams be left?

14. A certain population increases at a rate proportional to the square root of the population. If the population goes from 2500 to 3600 in five years, what is the population at the end of 10 years?