Set 3: Multiple-Choice Questions on Differentiation

Part A. Directions: Answer these questions *without* using your calculator.

In each of Questions 1–20 a function is given. Choose the alternative that is the derivative, $\frac{dy}{dx}$, of the function.

1. $y = x^5 \tan x$

(A) $5x^4 \tan x$ (B) $x^5 \sec^2 x$ (C) $5x^4 \sec^2 x$

(D) $5x^4 + \sec^2 x$ (E) $5x^4 \tan x + x^5 \sec^2 x$

2. $y = \frac{2-x}{3x+1}$

(A) $-\frac{7}{(3x+1)^2}$ (B) $\frac{6x-5}{(3x+1)^2}$ (C) $-\frac{9}{(3x+1)^2}$

(D) $\frac{7}{(3x+1)^2}$ (E) $\frac{7-6x}{(3x+1)^2}$

3. $y = \sqrt{3-2x}$

(A) $\frac{1}{2\sqrt{3-2x}}$ (B) $-\frac{1}{\sqrt{3-2x}}$ (C) $-\frac{(3-2x)^{3/2}}{3}$

(D) $-\frac{1}{3-2x}$ (E) $\frac{2}{3}(3-2x)^{3/2}$

4. $y = \frac{2}{(5x+1)^3}$

(A) $-\frac{30}{(5x+1)^2}$ (B) $-30(5x+1)^{-4}$ (C) $\frac{-6}{(5x+1)^4}$

(D) $-\frac{10}{3}(5x+1)^{-4/3}$ (E) $\frac{30}{(5x+1)^4}$

5. $y = 3x^{2/3} - 4x^{1/2} - 2$

(A) $2x^{1/3} - 2x^{-1/2}$ (B) $3x^{-1/3} - 2x^{-1/2}$ (C) $\frac{9}{5}x^{5/3} - 8x^{3/2}$

(D) $\frac{2}{x^{1/3}} - \frac{2}{x^{1/2}} - 2$ (E) $2x^{-1/3} - 2x^{-1/2}$

6. $y = 2\sqrt{x} - \frac{1}{2\sqrt{x}}$

(A) $x + \frac{1}{x\sqrt{x}}$ **(B)** $x^{-1/2} + x^{-3/2}$ **(C)** $\frac{4x-1}{4x\sqrt{x}}$

(D) $\frac{1}{\sqrt{x}} + \frac{1}{4x\sqrt{x}}$ **(E)** $\frac{4}{\sqrt{x}} + \frac{1}{x\sqrt{x}}$

7. $y = \sqrt{x^2 + 2x - 1}$

(A) $\frac{x+1}{y}$ **(B)** $4y(x+1)$ **(C)** $\frac{1}{2\sqrt{x^2+2x-1}}$

(D) $-\frac{x+1}{(x^2+2x-1)^{3/2}}$ **(E)** none of these

8. $y = \frac{x}{\sqrt{1-x^2}}$

(A) $\frac{1-2x^2}{(1-x^2)^{3/2}}$ **(B)** $\frac{1}{1-x^2}$ **(C)** $\frac{1}{\sqrt{1-x^2}}$

(D) $\frac{1-2x^2}{(1-x^2)^{1/2}}$ **(E)** none of these

9. $y = \ln \frac{e^x}{e^x - 1}$

(A) $x - \frac{e^x}{e^x - 1}$ **(B)** $\frac{1}{e^x - 1}$ **(C)** $-\frac{1}{e^x - 1}$

(D) 0 **(E)** $\frac{e^x - 2}{e^x - 1}$

10. $y = \tan^{-1} \frac{x}{2}$

(A) $\frac{4}{4+x^2}$ **(B)** $\frac{1}{2\sqrt{4-x^2}}$ **(C)** $\frac{2}{\sqrt{4-x^2}}$

(D) $\frac{1}{2+x^2}$ **(E)** $\frac{2}{x^2+4}$

11. $y = \ln(\sec x + \tan x)$

(A) $\sec x$ **(B)** $\frac{1}{\sec x}$ **(C)** $\tan x + \frac{\sec^2 x}{\tan x}$

(D) $\frac{1}{\sec x + \tan x}$ **(E)** $-\frac{1}{\sec x + \tan x}$

12. $y = \frac{e^x - e^{-x}}{e^x + e^{-x}}$

(A) 0 (B) 1 (C) $\frac{2}{(e^x + e^{-x})^2}$

(D) $\frac{4}{(e^x + e^{-x})^2}$ (E) $\frac{1}{e^{2x} + e^{-2x}}$

13. $y = \ln (x\sqrt{x^2+1})$

(A) $1 + \frac{x}{x^2+1}$ (B) $\frac{1}{x\sqrt{x^2+1}}$ (C) $\frac{2x^2+1}{x\sqrt{x^2+1}}$

(D) $\frac{2x^2+1}{x(x^2+1)}$ (E) none of these

14. $y = x^2 \sin \frac{1}{x}$ $(x \neq 0)$

(A) $2x \sin \frac{1}{x} - x^2 \cos \frac{1}{x}$ (B) $-\frac{2}{x} \cos \frac{1}{x}$ (C) $2x \cos \frac{1}{x}$

(D) $2x \sin \frac{1}{x} - \cos \frac{1}{x}$ (E) $-\cos \frac{1}{x}$

15. $y = \frac{1}{2 \sin 2x}$

(A) $-\csc 2x \cot 2x$ (B) $\frac{1}{4 \cos 2x}$ (C) $-4 \csc 2x \cot 2x$

(D) $\frac{\cos 2x}{2\sqrt{\sin 2x}}$ (E) $-\csc^2 2x$

16. $y = e^{-x} \cos 2x$

(A) $-e^{-x}(\cos 2x + 2 \sin 2x)$
(B) $e^{-x}(\sin 2x - \cos 2x)$
(C) $2e^{-x} \sin 2x$
(D) $-e^{-x}(\cos 2x + \sin 2x)$
(E) $-e^{-x} \sin 2x$

17. $y = \sec^2 \sqrt{x}$

(A) $\frac{\sec \sqrt{x} \tan \sqrt{x}}{\sqrt{x}}$ (B) $\frac{\tan \sqrt{x}}{\sqrt{x}}$ (C) $2 \sec \sqrt{x} \tan^2 \sqrt{x}$

(D) $\frac{\sec^2 \sqrt{x} \tan \sqrt{x}}{\sqrt{x}}$ (E) $2 \sec^2 \sqrt{x} \tan \sqrt{x}$

18. $y = x \ln^3 x$

(A) $\frac{3 \ln^2 x}{x}$ **(B)** $3 \ln^2 x$ **(C)** $3x \ln^2 x + \ln^3 x$

(D) $3(\ln x + 1)$ **(E)** none of these

19. $y = \frac{1+x^2}{1-x^2}$

(A) $-\frac{4x}{(1-x^2)^2}$ **(B)** $\frac{4x}{(1-x^2)^2}$ **(C)** $\frac{-4x^3}{(1-x^2)^2}$

(D) $\frac{2x}{1-x^2}$ **(E)** $\frac{4}{1-x^2}$

20. $y = \sin^{-1} x - \sqrt{1-x^2}$

(A) $\frac{1}{2\sqrt{1-x^2}}$ **(B)** $\frac{2}{\sqrt{1-x^2}}$ **(C)** $\frac{1+x}{\sqrt{1-x^2}}$

(D) $\frac{x^2}{\sqrt{1-x^2}}$ **(E)** $\frac{1}{\sqrt{1+x}}$

In each of Questions 21–24, y is a differentiable function of x. Choose the alternative that is the derivative $\frac{dy}{dx}$.

21. $x^3 - xy + y^3 = 1$

(A) $\frac{3x^2}{x-3y^2}$ **(B)** $\frac{3x^2-1}{1-3y^2}$ **(C)** $\frac{y-3x^2}{3y^2-x}$

(D) $\frac{3x^2+3y^2-y}{x}$ **(E)** $\frac{3x^2+3y^2}{x}$

22. $x + \cos(x + y) = 0$

(A) $\csc(x+y) - 1$ **(B)** $\csc(x+y)$ **(C)** $\frac{x}{\sin(x+y)}$

(D) $\frac{1}{\sqrt{1-x^2}}$ **(E)** $\frac{1-\sin x}{\sin y}$

23. $\sin x - \cos y - 2 = 0$

(A) $-\cot x$ **(B)** $-\cot y$ **(C)** $\frac{\cos x}{\sin y}$

(D) $-\csc y \cos x$ **(E)** $\frac{2-\cos x}{\sin y}$

24. $3x^2 - 2xy + 5y^2 = 1$

(A) $\frac{3x+y}{x-5y}$ (B) $\frac{y-3x}{5y-x}$ (C) $3x+5y$

(D) $\frac{3x+4y}{x}$ (E) none of these

***25.** If $x = t^2 - 1$ and $y = t^4 - 2t^3$, then, when $t = 1$, $\frac{d^2y}{dx^2}$ is

(A) 1 (B) −1 (C) 0 (D) 3 (E) $\frac{1}{2}$

26. If $f(x) = x^4 - 4x^3 + 4x^2 - 1$, then the set of values of x for which the derivative equals zero is

(A) $\{1, 2\}$ (B) $\{0, -1, -2\}$ (C) $\{-1, +2\}$
(D) $\{0\}$ (E) $\{0, 1, 2\}$

27. If $f(x) = 16\sqrt{x}$, then $f''(4)$ is equal to

(A) −32 (B) −16 (C) −4 (D) −2 (E) $-\frac{1}{2}$

28. If $f(x) = \ln x^3$, then $f''(3)$ is

(A) $-\frac{1}{3}$ (B) −1 (C) −3 (D) 1 (E) none of these

29. If a point moves on the curve $x^2 + y^2 = 25$, then, at (0, 5), $\frac{d^2y}{dx^2}$ is

(A) 0 (B) $\frac{1}{5}$ (C) −5 (D) $-\frac{1}{5}$ (E) nonexistent

30. If $y = a \sin ct + b \cos ct$, where a, b, and c are constants, then $\frac{d^2y}{dt^2}$ is

(A) $ac^2(\sin t + \cos t)$ (B) $-c^2y$ (C) $-ay$
(D) $-y$ (E) $a^2c^2 \sin ct - b^2c^2 \cos ct$

31. If $f(x) = 5^x$ and $5^{1.002} \simeq 5.016$, which is closest to $f'(1)$?

(A) 0.016 (B) 1.0 (C) 5.0 (D) 8.0 (E) 32.0

32. If $y = e^x(x - 1)$, then $y''(0)$ equals

(A) −2 (B) −1 (C) 0 (D) 1 (E) none of these

*An asterisk denotes a topic covered only in Calculus BC.

*33. If $x = e^{\theta} \cos \theta$ and $y = e^{\theta} \sin \theta$, then, when $\theta = \frac{\pi}{2}$, $\frac{dy}{dx}$ is

(A) 1 (B) 0 (C) $e^{\pi/2}$ (D) nonexistent (E) -1

*34. If $x = \cos t$ and $y = \cos 2t$, then $\frac{d^2y}{dx^2}$ ($\sin t \neq 0$) is

(A) $4 \cos t$ (B) 4 (C) $\frac{4y}{x}$ (D) -4 (E) $-4 \cot t$

35. $\lim_{h \to 0} \frac{(1+h)^6 - 1}{h}$ is

(A) 0 (B) 1 (C) 6 (D) ∞ (E) nonexistent

36. $\lim_{h \to 0} \frac{\sqrt[3]{8+h} - 2}{h}$ is

(A) 0 (B) $\frac{1}{12}$ (C) 1 (D) 192 (E) ∞

37. $\lim_{h \to 0} \frac{\ln(e+h) - 1}{h}$ is

(A) 0 (B) $\frac{1}{e}$ (C) 1 (D) e (E) nonexistent

38. $\lim_{x \to 0} \frac{\cos x - 1}{x}$ is

(A) -1 (B) 0 (C) 1 (D) ∞ (E) none of these

39. The function $f(x) = x^{2/3}$ on $[-8, 8]$ does not satisfy the conditions of the Mean Value Theorem because

(A) $f(0)$ is not defined (B) $f(x)$ is not continuous on $[-8, 8]$
(C) $f'(-1)$ does not exist (D) $f(x)$ is not defined for $x < 0$
(E) $f'(0)$ does not exist

40. If $f(x) = 2x^3 - 6x$, at what point on the interval $0 \leq x \leq \sqrt{3}$, if any, is the tangent to the curve parallel to the secant line?

(A) 1 (B) -1 (C) $\sqrt{2}$ (D) 0 (E) nowhere

41. If h is the inverse function of f and if $f(x) = \frac{1}{x}$, then $h'(3) =$

(A) -9 (B) $-\frac{1}{9}$ (C) $\frac{1}{9}$ (D) 3 (E) 9

*42. $\lim_{x \to \infty} \frac{e^x}{x^{50}}$ equals

(A) 0 (B) 1 (C) $\frac{1}{50!}$ (D) ∞ (E) none of these

*An asterisk denotes a topic covered only in Calculus BC.

43. If $\sin(xy) = x$, then $\frac{dy}{dx} =$

(A) $\sec(xy)$ (B) $\frac{\sec(xy)}{x}$ (C) $\frac{\sec(xy) - y}{x}$

(D) $-\frac{1 + \sec(xy)}{x}$ (E) $\sec(xy) - 1$

44. $\lim\limits_{x \to 0} \frac{\sin 2x}{x}$ is

(A) 1 (B) 2 (C) $\frac{1}{2}$ (D) 0 (E) ∞

45. $\lim\limits_{x \to 0} \frac{\sin 3x}{\sin 4x}$ is

(A) 1 (B) $\frac{4}{3}$ (C) $\frac{3}{4}$ (D) 0 (E) nonexistent

46. $\lim\limits_{x \to 0} \frac{1 - \cos x}{x}$ is

(A) nonexistent (B) 1 (C) 2 (D) ∞ (E) none of these

47. $\lim\limits_{x \to 0} \frac{\tan \pi x}{x}$ is

(A) $\frac{1}{\pi}$ (B) 0 (C) 1 (D) π (E) ∞

48. $\lim\limits_{x \to \infty} x^2 \sin \frac{1}{x}$

(A) is 1 (B) is 0 (C) is ∞

(D) oscillates between -1 and 1 (E) is none of these

***49.** The graph in the xy-plane represented by $x = 3 + 2 \sin t$ and $y = 2 \cos t - 1$, for $-\pi \leqslant t \leqslant \pi$, is

(A) a semicircle (B) a circle (C) an ellipse

(D) half of an ellipse (E) a hyperbola

50. $\lim\limits_{x \to 0} \frac{\sec x - \cos x}{x^2}$ equals

(A) 0 (B) $\frac{1}{2}$ (C) 1 (D) 2 (E) none of these

*An asterisk denotes a topic covered only in Calculus BC.

In each of Questions 51–54 a pair of equations that represent a curve parametrically is given. Choose the alternative that is the derivative $\frac{dy}{dx}$.

***51.** $x = t - \sin t$ and $y = 1 - \cos t$

(A) $\frac{\sin t}{1 - \cos t}$ (B) $\frac{1 - \cos t}{\sin t}$ (C) $\frac{\sin t}{\cos t - 1}$

(D) $\frac{1 - x}{y}$ (E) $\frac{1 - \cos t}{t - \sin t}$

***52.** $x = \cos^3 \theta$ and $y = \sin^3 \theta$

(A) $\tan^3 \theta$ (B) $-\cot \theta$ (C) $\cot \theta$ (D) $-\tan \theta$ (E) $-\tan^2 \theta$

***53.** $x = 1 - e^{-t}$ and $y = t + e^{-t}$

(A) $\frac{e^{-t}}{1 - e^{-t}}$ (B) $e^{-t} - 1$ (C) $e^t + 1$ (D) $e^t - e^{-2t}$ (E) $e^t - 1$

***54.** $x = \frac{1}{1-t}$ and $y = 1 - \ln(1 - t)$ $(t < 1)$

(A) $\frac{1}{1-t}$ (B) $t - 1$ (C) $\frac{1}{x}$ (D) $\frac{(1-t)^2}{t}$ (E) $1 + \ln x$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

In Questions 55–62, differentiable functions f and g have the values shown in the table.

x	f	f'	g	g'
0	2	1	5	−4
1	3	2	3	−3
2	5	3	1	−2
3	10	4	0	−1

55. If $A = f + 2g$, then $A'(3) =$

(A) −2 (B) 2 (C) 7 (D) 8 (E) 10

56. If $B = f \cdot g$, then $B'(2) =$

(A) −20 (B) −7 (C) −6 (D) −1 (E) 13

57. If $D = \frac{1}{g}$, then $D'(1) =$

(A) $-\frac{1}{2}$ (B) $-\frac{1}{3}$ (C) $-\frac{1}{9}$ (D) $\frac{1}{9}$ (E) $\frac{1}{3}$

*An asterisk denotes a topic covered only in Calculus BC.

58. If $H(x) = \sqrt{f(x)}$, then $H'(3) =$

(A) $\frac{1}{4}$ **(B)** $\frac{1}{2\sqrt{10}}$ **(C)** 2 **(D)** $\frac{2}{\sqrt{10}}$ **(E)** $4\sqrt{10}$

59. If $K(x) = \left(\frac{f}{g}\right)(x)$, then $K'(0) =$

(A) $\frac{-13}{25}$ **(B)** $-\frac{1}{4}$ **(C)** $\frac{13}{25}$ **(D)** $\frac{13}{16}$ **(E)** $\frac{22}{25}$

60. If $M(x) = f(g(x))$, then $M'(1) =$

(A) −12 **(B)** −6 **(C)** 4 **(D)** 6 **(E)** 12

61. If $P(x) = f(x^3)$, then $P'(1) =$

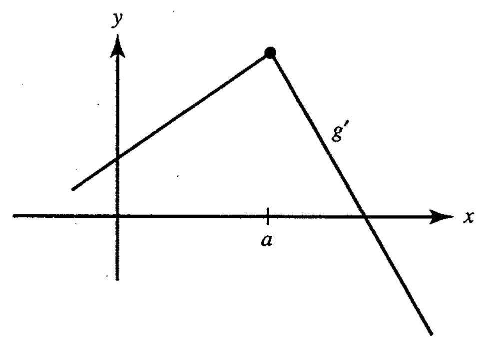
(A) 2 **(B)** 6 **(C)** 8 **(D)** 12 **(E)** 54

62. If $S(x) = f^{-1}(x)$, then $S'(3) =$

(A) −2 **(B)** $-\frac{1}{25}$ **(C)** $\frac{1}{4}$ **(D)** $\frac{1}{2}$ **(E)** 2

63. The graph of g' is shown here. Which of the following statements is (are) true of g at $x = a$?

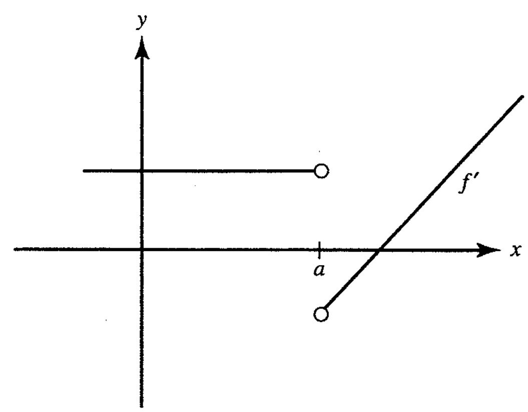
I. g is continuous.
II. g is differentiable.
III. g is increasing.



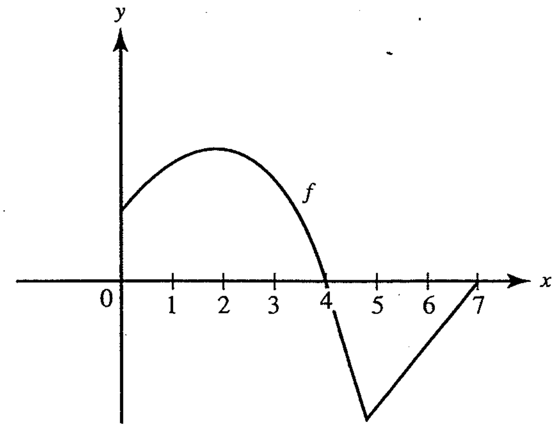
(A) I only **(B)** III only **(C)** I and III only
(D) II and III only **(E)** I, II, and III

64. A function f has the derivative shown. Which of the following statements must be false?

(A) f is continuous at $x = a$.
(B) $f(a) = 0$.
(C) f has a vertical asymptote at $x = a$.
(D) f has a jump discontinuity at $x = a$.
(E) f has a removable discontinuity at $x = a$.



65. The function f whose graph is shown has $f' = 0$ at $x =$



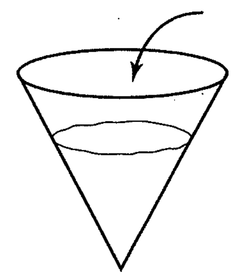
(A) 2 only
(B) 2 and 5
(C) 4 and 7
(D) 2, 4, and 7
(E) 2, 4, 5, and 7

66. A differentiable function f has the values shown. Estimate $f'(1.5)$.

x	1.0	1.2	1.4	1.6
$f(x)$	8	10	14	22

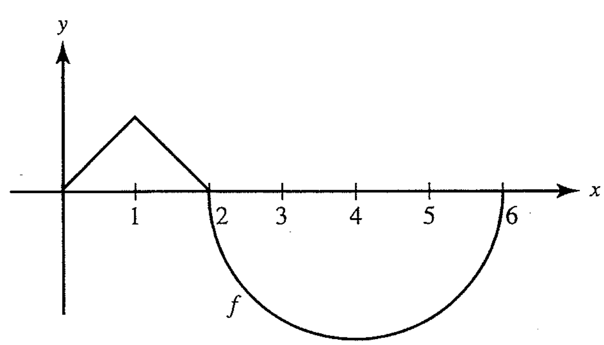
(A) 8 **(B)** 12 **(C)** 18 **(D)** 40 **(E)** 80

67. Water is poured into a conical reservoir at a constant rate. If $h(t)$ is the rate of change of the depth of the water, then h is



(A) constant
(B) linear and increasing
(C) linear and decreasing
(D) nonlinear and increasing
(E) nonlinear and decreasing

Use the graph to answer Questions 68–70. It consists of two line segments and a semicircle.



68. $f'(x) = 0$ for $x =$

(A) 1 only
(B) 2 only
(C) 4 only
(D) 1 and 4
(E) 2 and 6

69. $f'(x)$ does not exist for $x =$

(A) 1 only **(B)** 2 only **(C)** 1 and 2
(D) 2 and 6 **(E)** 1, 2, and 6

70. $f'(5) =$

(A) $\frac{1}{2}$ **(B)** $\frac{1}{\sqrt{3}}$ **(C)** 1 **(D)** 2 **(E)** $\sqrt{3}$

71. At how many points on the interval [–5,5] is a tangent to $y = x + \cos x$ parallel to the secant line?

(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** more than 3

72. From the values of f shown, estimate $f'(2)$.

x	1.92	1.94	1.96	1.98	2.00
$f(x)$	6.00	5.00	4.40	4.10	4.00

(A) –0.10 **(B)** –0.20 **(C)** –5 **(D)** –10 **(E)** –25

73. Using the values shown in the table for Question 72, estimate $(f^{-1})'(4)$.

(A) –0.2 **(B)** –0.1 **(C)** –5 **(D)** –10 **(E)** –25

74. The "left half" of the parabola defined by $y = x^2 - 8x + 10$ for $x \le 4$ is a one-to-one function; therefore its inverse is also a function. Call that inverse g. Find $g'(3)$.

(A) $-\frac{1}{2}$ **(B)** $-\frac{1}{6}$ **(C)** $\frac{1}{6}$ **(D)** $\frac{1}{2}$ **(E)** $\frac{11}{2}$

75. For $f(x) = 5^x$, what is the estimate of $f'(2)$ obtained by using the symmetric difference quotient with $h = 0.03$?

(A) 25.029 **(B)** 40.236 **(C)** 40.252 **(D)** 41.223 **(E)** 80.503

76. If f is differentiable and difference quotients overestimate the slope of f at $x = a$ for all $h > 0$, which must be true?

(A) $f'(a) > 0$ **(B)** $f'(a) < 0$ **(C)** $f''(a) > 0$
(D) $f''(a) < 0$ **(E)** none of these

77. If $f(u) = \sin u$ and $u = g(x) = x^2 - 9$, then $(f \circ g)'(3)$ equals

(A) 0 **(B)** 1 **(C)** 6 **(D)** 9 **(E)** none of these

78. If $f(x) = \frac{x}{(x-1)^2}$, then the set of x's for which $f'(x)$ exists is

(A) all reals
(B) all reals except $x = 1$ and $x = -1$
(C) all reals except $x = -1$
(D) all reals except $x = \frac{1}{3}$ and $x = -1$
(E) all reals except $x = 1$

79. If $y = \sqrt{x^2+1}$, then the derivative of y^2 with respect to x^2 is

(A) 1 **(B)** $\frac{x^2+1}{2x}$ **(C)** $\frac{x}{2(x^2+1)}$ **(D)** $\frac{2}{x}$ **(E)** $\frac{x^2}{x^2+1}$

80. If $f(x) = \frac{1}{x^2+1}$ and $g(x) = \sqrt{x}$, then the derivative of $f(g(x))$ is

(A) $\frac{-\sqrt{x}}{(x^2+1)^2}$ **(B)** $-(x+1)^{-2}$ **(C)** $\frac{-2x}{(x^2+1)^2}$

(D) $\frac{1}{(x+1)^2}$ **(E)** $\frac{1}{2\sqrt{x}(x+1)}$

81. If $y = x^2 + x$, then the derivative of y with respect to $\frac{1}{1-x}$ is

(A) $(2x+1)(x-1)^2$ **(B)** $\frac{2x+1}{(1-x)^2}$ **(C)** $2x+1$

(D) $\frac{3-x}{(1-x)^3}$ **(E)** none of these

82. If $f(a) = f(b) = 0$ and $f(x)$ is continuous on $[a, b]$, then

(A) $f(x)$ must be identically zero
(B) $f'(x)$ may be different from zero for all x on $[a, b]$
(C) there exists at least one number c, $a < c < b$, such that $f'(c) = 0$
(D) $f'(x)$ must exist for every x on (a, b)
(E) none of the preceding is true

83. Suppose $y = f(x) = 2x^3 - 3x$. If $h(x)$ is the inverse function of f, then $h'(-1) =$

(A) -1 **(B)** $\frac{1}{5}$ **(C)** $\frac{1}{3}$ **(D)** 1 **(E)** 3

84. Suppose $f(1) = 2$, $f'(1) = 3$, and $f'(2) = 4$. Then $(f^{-1})'(2)$

(A) equals $-\frac{1}{3}$ **(B)** equals $-\frac{1}{4}$ **(C)** equals $\frac{1}{4}$

(D) equals $\frac{1}{3}$ **(E)** cannot be determined

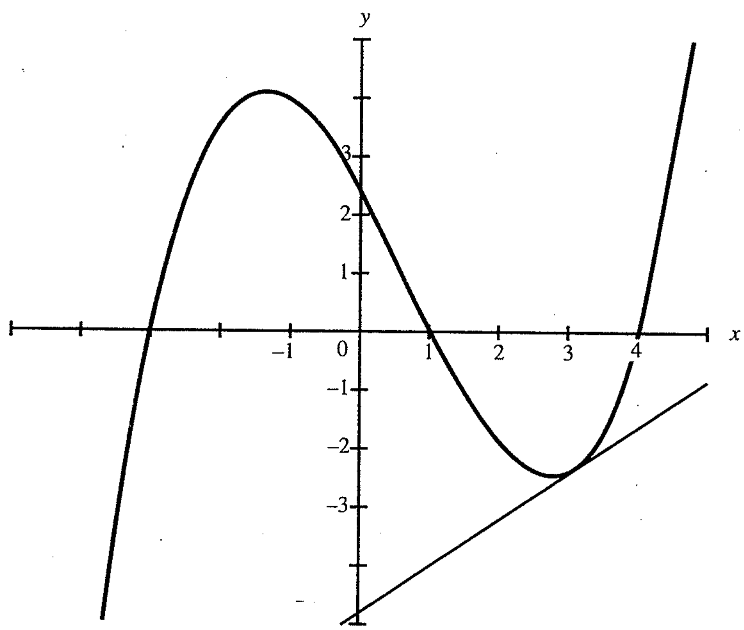
85. If $f(x) = x^3 - 3x^2 + 8x + 5$ and $g(x) = f^{-1}(x)$, then $g'(5) =$

(A) 8 **(B)** $\frac{1}{8}$ **(C)** 1 **(D)** $\frac{1}{53}$ **(E)** 53

86. Suppose $\lim\limits_{x \to 0} \frac{g(x) - g(0)}{x} = 1$. It follows necessarily that

(A) g is not defined at $x = 0$
(B) g is not continuous at $x = 0$
(C) the limit of $g(x)$ as x approaches 0 equals 1
(D) $g'(0) = 1$
(E) $g'(1) = 0$

Use this graph of $y = f(x)$ for Questions 87 and 88.



87. $f'(3)$ is most closely approximated by

(A) 0.3 **(B)** 0.8 **(C)** 1.5 **(D)** 1.8 **(E)** 2

88. The rate of change of $f(x)$ is least at $x \simeq$

(A) –3 **(B)** –1.3 **(C)** 0 **(D)** 0.7 **(E)** 2.7

Use the following definition of the *symmetric difference quotient for* $f'(x_0)$ for Questions 89–91: For small values of h,

$$f'(x_0) = \frac{f(x_0+h) - f(x_0-h)}{2h}.$$

89. $f'(x_0)$ equals the exact value of the derivative at $x = x_0$

(A) only when f is linear
(B) whenever f is quadratic
(C) if and only if $f'(x_0)$ exists
(D) whenever $|h| < 0.001$
(E) in none of these cases

90. To how many places is the symmetric difference quotient accurate when it is used to approximate $f'(0)$ for $f(x) = 4^x$ and $h = 0.08$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

91. To how many places is $f'(x_0)$ accurate when it is used to approximate $f'(0)$ for $f(x) = 4^x$ and $h = 0.001$?

(A) 1 (B) 2 (C) 3 (D) 4 (E) more than 4

92. The value of $f'(0)$ obtained using the symmetric difference quotient with $f(x) = |x|$ and $h = 0.001$ is

(A) -1 (B) 0 (C) ± 1 (D) 1 (E) indeterminate

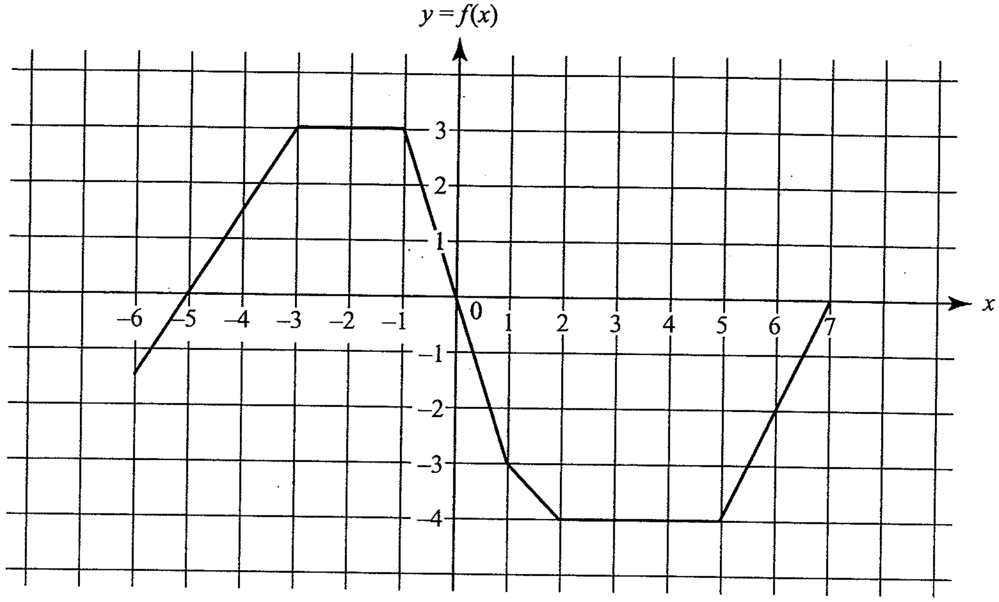
93. If $\frac{d}{dx} f(x) = g(x)$ and $h(x) = \sin x$, then $\frac{d}{dx} f(h(x))$ equals

(A) $g(\sin x)$ (B) $\cos x \cdot g(x)$ (C) $g'(x)$
(D) $\cos x \cdot g(\sin x)$ (E) $\sin x \cdot g(\sin x)$

94. Let $f(x) = 3^x - x^3$. The tangent to the curve is parallel to the secant through (0,1) and (3,0) for x equal

(A) only to 0.984 (B) only to 1.244 (C) only to 2.727
(D) to 0.984 and 2.804 (E) to 1.244 and 2.727

Questions 95–99 are based on the following graph of $f(x)$, sketched on $-6 \leqslant x \leqslant 7$. Assume the horizontal and vertical grid lines are equally spaced at unit intervals.



95. On the interval $1 < x < 2, f(x)$ equals

(A) $-x-2$ (B) $-x-3$ (C) $-x-4$ (D) $-x+2$ (E) $x-2$

97. Over which of the following intervals does $f'(x)$ equal zero?

I. $(-6,-3)$ II. $(-3,-1)$ III. $(2,5)$

(A) I only (B) II only (C) I and II only
(D) I and III only (E) II and III only

97. How many points of discontinuity does $f'(x)$ have on the interval $-6 < x < 7$?

(A) none (B) 2 (C) 3 (D) 4 (E) 5

98. For $-6 < x -3, f'(x)$ equals

(A) $-\frac{3}{2}$ (B) -1 (C) 1 (D) $\frac{3}{2}$ (E) 2

99. Which of the following statements about the graph of $f'(x)$ is false?

(A) It consists of six horizontal segments.
(B) It has four jump discontinuities.
(C) $f'(x)$ is discontinuous at each x in the set $\{-3,-1,1,2,5\}$.
(D) $f'(x)$ ranges from -3 to 2.
(E) On the interval $-1 < x < 1, f'(x) = -3$.

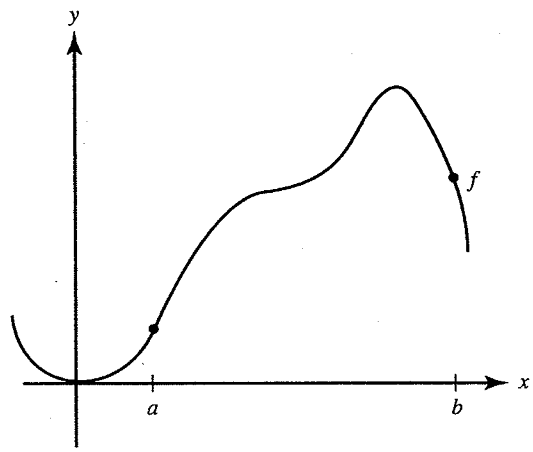
100. The table gives the values of a function f that is differentiable on the interval [0,1]:

x	0.10	0.20	0.30	0.40	0.50	0.60
$f(x)$	0.171	0.288	0.357	0.384	0.375	0.336

According to this table, the best approximation of $f'(0.10)$ is

(A) 0.12 **(B)** 1.08 **(C)** 1.17 **(D)** 1.77 **(E)** 2.88

101. At how many points on the interval $[a,b]$ does the function graphed satisfy the Mean Value Theorem?



(A) none **(B)** 1 **(C)** 2 **(D)** 3 **(E)** 4