Set 6: Multiple-Choice Questions on Definite Integrals

Part A. Directions: Answer these questions *without* using your calculator.

1. $\int_{-1}^{1} (x^2 - x - 1)\, dx =$

(A) $\frac{2}{3}$ (B) 0 (C) $-\frac{4}{3}$ (D) -2 (E) -1

2. $\int_{1}^{2} \frac{3x-1}{3x}\, dx =$

(A) $\frac{3}{4}$ (B) $1 - \frac{1}{3}\ln 2$ (C) $1 - \ln 2$ (D) $-\frac{1}{3}\ln 2$ (E) 1

3. $\int_{0}^{3} \frac{dt}{\sqrt{4-t}} =$

(A) 1 (B) -2 (C) 4 (D) -1 (E) 2

4. $\int_{-1}^{0} \sqrt{3u+4}\, du =$

(A) 2 (B) $\frac{14}{9}$ (C) $\frac{14}{3}$ (D) 6 (E) $\frac{7}{2}$

5. $\int_{2}^{3} \frac{dy}{2y-3} =$

(A) $\ln 3$ (B) $\frac{1}{2}\ln\frac{3}{2}$ (C) $\frac{16}{9}$ (D) $\ln\sqrt{3}$ (E) $\sqrt{3} - 1$

6. $\int_{0}^{\sqrt{3}} \frac{x}{\sqrt{4-x^2}}\, dx =$

(A) 1 (B) $\frac{\pi}{6}$ (C) $\frac{\pi}{3}$ (D) -1 (E) 2

7. $\int_{0}^{1} (2t-1)^3\, dt =$

(A) $\frac{1}{4}$ (B) 6 (C) $\frac{1}{2}$ (D) 0 (E) 4

8. $\int_4^9 \frac{2+x}{2\sqrt{x}}\,dx =$

(A) $\frac{25}{3}$ (B) $\frac{41}{3}$ (C) $\frac{100}{3}$ (D) $\frac{5}{3}$ (E) $\frac{1}{3}$

9. $\int_{-3}^{3} \frac{dx}{9+x^2} =$

(A) $\frac{\pi}{2}$ (B) 0 (C) $\frac{\pi}{6}$ (D) $-\frac{\pi}{2}$ (E) $\frac{\pi}{3}$

10. $\int_0^1 e^{-x}\,dx =$

(A) $\frac{1}{e} - 1$ (B) $1 - e$ (C) $-\frac{1}{e}$ (D) $1 - \frac{1}{e}$ (E) $\frac{1}{e}$

11. $\int_0^1 xe^{x^2}\,dx =$

(A) $e - 1$ (B) $\frac{1}{2}(e-1)$ (C) $2(e-1)$ (D) $\frac{e}{2}$ (E) $\frac{e}{2} - 1$

12. $\int_0^{\pi/4} \sin 2\theta\, d\theta =$

(A) 2 (B) $\frac{1}{2}$ (C) -1 (D) $-\frac{1}{2}$ (E) -2

13. $\int_1^2 \frac{dz}{3-z} =$

(A) $-\ln 2$ (B) $\frac{3}{4}$ (C) $2(\sqrt{2} - 1)$ (D) $\frac{1}{2}\ln 2$ (E) $\ln 2$

*14. If we let $x = 2\sin\theta$, then $\int_1^2 \frac{\sqrt{4-x^2}}{x}\,dx$ is equivalent to

(A) $2\int_0^2 \frac{\cos^2\theta}{\sin\theta}\,d\theta$ (B) $\int_{\pi/6}^{\pi/2} \frac{\cos\theta}{\sin\theta}\,d\theta$ (C) $2\int_{\pi/6}^{\pi/2} \frac{\cos^2\theta}{\sin\theta}\,d\theta$

(D) $\int_1^2 \frac{\cos\theta}{\sin\theta}\,d\theta$ (E) none of these

15. $\int_0^{\pi} \cos^2\theta \sin\theta\, d\theta =$

(A) $-\frac{2}{3}$ (B) $\frac{1}{3}$ (C) 1 (D) $\frac{2}{3}$ (E) 0

*An asterisk denotes a topic covered only in Calculus BC.

16. $\int_1^e \frac{\ln x}{x}\, dx =$

(A) $\frac{1}{2}$ **(B)** $\frac{1}{2}(e^2 - 1)$ **(C)** 0 **(D)** 1 **(E)** $e - 1$

***17.** $\int_0^1 xe^x\, dx =$

(A) -1 **(B)** $e + 1$ **(C)** 1 **(D)** $e - 1$ **(E)** $\frac{1}{2}(e - 1)$

18. $\int_0^{\pi/6} \frac{\cos \theta}{1 + 2 \sin \theta}\, d\theta =$

(A) $\ln 2$ **(B)** $\frac{3}{8}$ **(C)** $-\frac{1}{2}\ln 2$ **(D)** $\frac{3}{2}$ **(E)** $\ln \sqrt{2}$

19. $\int_{\sqrt{2}}^2 \frac{u}{u^2 - 1}\, du =$

(A) $\ln \sqrt{3}$ **(B)** $\frac{8}{9}$ **(C)** $\ln \frac{3}{2}$ **(D)** $\ln 3$ **(E)** $1 - \sqrt{3}$

20. $\int_{\sqrt{2}}^2 \frac{u\, du}{(u^2 - 1)^2} =$

(A) $-\frac{1}{3}$ **(B)** $-\frac{2}{3}$ **(C)** $\frac{2}{3}$ **(D)** -1 **(E)** $\frac{1}{3}$

21. $\int_0^{\pi/4} \cos^2 \theta\, d\theta =$

(A) $\frac{1}{2}$ **(B)** $\frac{\pi}{8}$ **(C)** $\frac{\pi}{8} + \frac{1}{4}$ **(D)** $\frac{\pi}{8} + \frac{1}{2}$ **(E)** $\frac{\pi}{8} - \frac{1}{4}$

22. $\int_{\pi/12}^{\pi/4} \frac{\cos 2x\, dx}{\sin^2 2x} =$

(A) $-\frac{1}{4}$ **(B)** 1 **(C)** $\frac{1}{2}$ **(D)** $-\frac{1}{2}$ **(E)** -1

23. $\int_0^1 \frac{e^{-x} + 1}{e^{-x}}\, dx =$

(A) e **(B)** $2 + e$ **(C)** $\frac{1}{e}$ **(D)** $1 + e$ **(E)** $e - 1$

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24. $\int_0^1 \frac{e^x}{e^x+1}\,dx =$

(A) $\ln 2$ (B) e (C) $1+e$ (D) $-\ln 2$ (E) $\ln \frac{e+1}{2}$

25. If we let $x = \tan\theta$, then $\int_1^{\sqrt{3}} \sqrt{1+x^2}\,dx$ is equivalent to

(A) $\int_{\pi/4}^{\pi/3} \sec\theta\,d\theta$ (B) $\int_1^{\sqrt{3}} \sec^3\theta\,d\theta$ (C) $\int_{\pi/4}^{\pi/3} \sec^3\theta\,d\theta$

(D) $\int_{\pi/4}^{\pi/3} \sec^2\theta\tan\theta\,d\theta$ (E) $\int_1^{\sqrt{3}} \sec\theta\,d\theta$

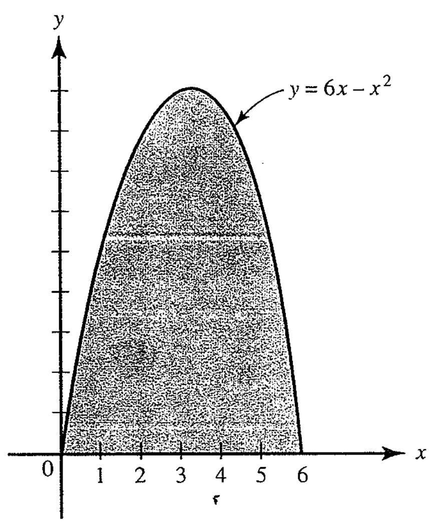
26. If the substitution $u = \sqrt{x+1}$ is used, then $\int_0^3 \frac{dx}{x\sqrt{x+1}}$ is equivalent to

(A) $\int_1^2 \frac{du}{u^2-1}$ (B) $\int_1^2 \frac{2\,du}{u^2-1}$ (C) $2\int_0^3 \frac{du}{(u-1)(u+1)}$

(D) $2\int_1^2 \frac{du}{u(u^2-1)}$ (E) $2\int_0^3 \frac{du}{u(u-1)}$

27. Using $M(3)$, we find that the approximate area of the shaded region below is

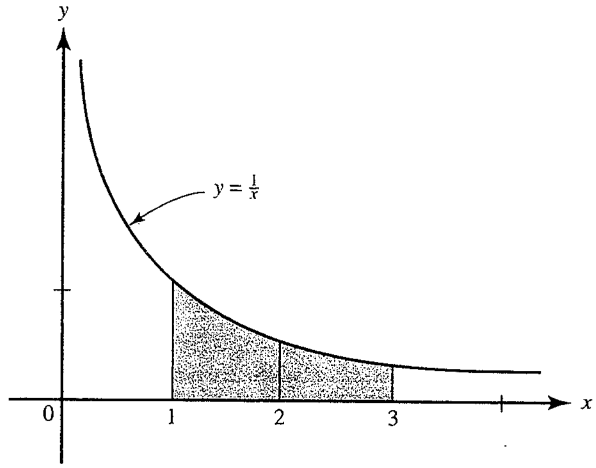
(A) 9 (B) 19 (C) 36 (D) 38 (E) 54



28. Using $T(6)$, we find that the approximate area of the shaded region above is

(A) 17.5 (B) 30 (C) 35 (D) 36 (E) 60

29. The area of the shaded region in the figure is equal exactly to ln 3. If we approximate ln 3 using $L(2)$ and $R(2)$, which inequality follows?



(A) $\frac{1}{2} < \int_1^2 \frac{1}{x}\,dx < 1$ **(B)** $\frac{1}{3} < \int_1^3 \frac{1}{x}\,dx < 2$ **(C)** $\frac{1}{2} < \int_0^2 \frac{1}{x}\,dx < 2$

(D) $\frac{1}{3} < \int_2^3 \frac{1}{x}\,dx < \frac{1}{2}$ **(E)** $\frac{5}{6} < \int_1^3 \frac{1}{x}\,dx < \frac{3}{2}$

30. Let $A = \int_0^1 \cos x\,dx$. We estimate A using the L, R, and T approximations with $n = 100$ subintervals. Which is true?

(A) $L < A < T < R$
(B) $L < T < A < R$
(C) $R < A < T < L$
(D) $R < T < A < L$
(E) The order cannot be determined.

31. $\int_{-1}^{3} |x|\,dx =$

(A) $\frac{7}{2}$ **(B)** 4 **(C)** $\frac{9}{2}$ **(D)** 5 **(E)** $\frac{11}{2}$

32. $\int_{-3}^{2} |x+1|\,dx =$

(A) $\frac{5}{2}$ **(B)** $\frac{7}{2}$ **(C)** 5 **(D)** $\frac{11}{2}$ **(E)** $\frac{13}{2}$

33. The average value of $y = \sqrt{64 - x^2}$ on its domain is

(A) 2 **(B)** 4 **(C)** 2π **(D)** 4π **(E)** none of these

34. The average value of $\cos x$ over the interval $\frac{\pi}{3} \leqq x \leqq \frac{\pi}{2}$ is

(A) $\frac{3}{\pi}$ **(B)** $\frac{1}{2}$ **(C)** $\frac{3(2-\sqrt{3})}{\pi}$ **(D)** $\frac{3}{2\pi}$ **(E)** $\frac{2}{3\pi}$

35. The average value of $\csc^2 x$ over the interval from $x = \frac{\pi}{6}$ to $x = \frac{\pi}{4}$ is

(A) $\frac{3\sqrt{3}}{\pi}$ **(B)** $\frac{\sqrt{3}}{\pi}$ **(C)** $\frac{12}{\pi}(\sqrt{3}-1)$

(D) $3\sqrt{3}$ **(E)** $3(\sqrt{3}-1)$

Part B. Directions: Some of the following questions require the use of a graphing calculator.

36. To three decimal places, $\int_0^1 \frac{dx}{\sqrt{4-x^2}} =$

(A) 0.262 **(B)** 0.268 **(C)** 0.524 **(D)** 0.536 **(E)** 1.047

37. The integral $\int_{-4}^{4} \sqrt{16-x^2}\, dx$ gives the area of

(A) a circle of radius 4
(B) a semicircle of radius 4
(C) a quadrant of a circle of radius 4
(D) an ellipse whose semimajor axis is 4
(E) none of these

38. $\int_0^{\pi/4} \sqrt{1-\cos 2\alpha}\, d\alpha =$

(A) 0.25 **(B)** 0.414 **(C)** 1.000 **(D)** 1.414 **(E)** 2.000

39. If $f(x)$ is continuous on the interval $a \leq x \leq b$ and $a < c < b$, then $\int_c^b f(x)\, dx$ is equal to

(A) $\int_a^c f(x)\, dx + \int_c^b f(x)\, dx$ **(B)** $\int_a^c f(x)\, dx - \int_a^b f(x)\, dx$

(C) $\int_c^a f(x)\, dx + \int_b^a f(x)\, dx$ **(D)** $\int_a^b f(x)\, dx - \int_a^c f(x)\, dx$

(E) $\int_a^c f(x)\, dx - \int_b^c f(x)\, dx$

40. If $f(x)$ is continuous on $a \le x \le b$, then

(A) $\int_a^b f(x)\,dx = f(b) - f(a)$ (B) $\int_a^b f(x)\,dx = -\int_b^a f(x)\,dx$

(C) $\int_a^b f(x)\,dx \geqq 0$ (D) $\frac{d}{dx}\int_a^x f(t)\,dt = f'(x)$

(E) $\frac{d}{dx}\int_a^x f(t)\,dt = f(x) - f(a)$

41. If $f(x)$ is continuous on the interval $a \leqq x \leqq b$, if this interval is partitioned into n equal subintervals of length Δx, and if x_k is a number in the kth subinterval, then $\lim_{n\to\infty}\sum_1^n f(x_k)\ \Delta x$ is equal to

(A) $f(b) - f(a)$

(B) $F(x) + C$, where $\frac{dF(x)}{dx} = f(x)$ and C is an arbitrary constant

(C) $\int_a^b f(x)\,dx$

(D) $F(b - a)$, where $\frac{dF(x)}{dx} = f(x)$

(E) none of these

42. If $F'(x) = G'(x)$ for all x, then

(A) $\int_a^b F'(x)\,dx = \int_a^b G'(x)\,dx$ (B) $\int F(x)\,dx = \int G(x)\,dx$

(C) $\int_a^b F(x)\,dx = \int_a^b G(x)\,dx$ (D) $\int F(x)\,dx = \int G(x)\,dx + C$

(E) None of the preceding is necessarily true

43. If $f(x)$ is continuous on the closed interval $[a,b]$, then there exists at least one number c, $a < c < b$, such that $\int_a^b f(x)\,dx$ is equal to

(A) $\frac{f(c)}{b-a}$ (B) $f'(c)(b-a)$ (C) $f(c)(b-a)$

(D) $\frac{f'(c)}{b-a}$ (E) $f(c)[f(b) - f(a)]$

44. If $f(x)$ is continuous on the closed interval $[a,b]$ and k is a constant, then $\int_a^b kf(x)\,dx$ is equal to

(A) $k(b-a)$ (B) $k[f(b)-f(a)]$ (C) $kF(b-a)$, where $\frac{dF(x)}{dx}=f(x)$

(D) $k\int_a^b f(x)\,dx$ (E) $\left.\frac{[kf(x)]^2}{2}\right]_a^b$

45. $\frac{d}{dt}\int_0^t \sqrt{x^3+1}\,dx =$

(A) $\sqrt{t^3+1}$ (B) $\frac{\sqrt{t^3+1}}{3t^2}$ (C) $\frac{2}{3}(t^3+1)(\sqrt{t^3+1}-1)$

(D) $3x^2\sqrt{x^3+1}$ (E) none of these

46. If $F(u)=\int_1^u (2-x^2)^3\,dx$, then $F'(u)$ is equal to

(A) $-6u(2-u^2)^2$ (B) $\frac{(2-u^2)^4}{4}-\frac{1}{4}$ (C) $(2-u^2)^3-1$

(D) $(2-u^2)^3$ (E) $-2u(2-u^2)^3$

47. $\frac{d}{dx}\int_{\pi/2}^{x^2} \sqrt{\sin t}\,dt =$

(A) $\sqrt{\sin t^2}$ (B) $2x\sqrt{\sin x^2}-1$ (C) $\frac{2}{3}(\sin^{3/2} x^2-1)$

(D) $\sqrt{\sin x^2}-1$ (E) $2x\sqrt{\sin x^2}$

48. If $x=4\cos\theta$ and $y=3\sin\theta$, then $\int_2^4 xy\,dx$ is equivalent to

(A) $48\int_{\pi/3}^{0} \sin\theta\cos^2\theta\,d\theta$ (B) $48\int_2^4 \sin^2\theta\cos\theta\,d\theta$

(C) $36\int_2^4 \sin\theta\cos^2\theta\,d\theta$ (D) $-48\int_0^{\pi/3} \sin\theta\cos^2\theta\,d\theta$

(E) $48\int_0^{\pi/3} \sin^2\theta\cos\theta\,d\theta$

*49. A curve is defined by the parametric equations $y = 2a\cos^2\theta$ and $x = 2a\tan\theta$, where $0 \leqq \theta \leqq \pi$. Then the definite integral $\pi\int_0^{2a} y^2\,dx$ is equivalent to

(A) $4\pi a^2\int_0^{\pi/4}\cos^4\theta\,d\theta$ (B) $8\pi a^3\int_{\pi/2}^{\pi}\cos^2\theta\,d\theta$ (C) $8\pi a^3\int_0^{\pi/4}\cos^2\theta\,d\theta$

(D) $8\pi a^3\int_0^{2a}\cos^2\theta\,d\theta$ (E) $8\pi a^3\int_0^{\pi/4}\sin\theta\cos^2\theta\,d\theta$

*50. A curve is given parametrically by $x = 1 - \cos t$ and $y = t - \sin t$, where $0 \leqq t \leqq \pi$. Then $\int_0^{3/2} y\,dx$ is equivalent to

(A) $\int_0^{3/2}\sin t(t - \sin t)\,dt$ (B) $\int_{2\pi/3}^{\pi}\sin t(t - \sin t)\,dt$

(C) $\int_0^{2\pi/3}(t - \sin t)\,dt$ (D) $\int_0^{2\pi/3}\sin t(t - \sin t)\,dt$

(E) $\int_0^{3/2}(t - \sin t)\,dt$

51. When $\int_0^1\sqrt{1+x^2}\,dx$ is estimated using $n = 5$ subintervals, which is (are) true?

I. $L(5) = \left(1+\sqrt{1+0.2^2}+\sqrt{1+0.4^2}+\sqrt{1+0.6^2}+\sqrt{1+0.8^2}\right)$

II. $M(5) = \left(\sqrt{1+0.1^2}+\sqrt{1+0.3^2}+\sqrt{1+0.5^2}+\sqrt{1+0.7^2}+\sqrt{1+0.9^2}\right)\cdot(0.2)$

III. $T(5) = \frac{0.2}{2}\left(1+2\sqrt{1+0.2^2}+2\sqrt{1+0.4^2}+2\sqrt{1+0.6^2}+2\sqrt{1+0.8^2}+\sqrt{2}\right)$

(A) II only
(B) III only
(C) I and II only
(D) I and III only
(E) II and III only

52. Find the value of x at which the function $y = x^2$ reaches its average value on the interval [0,10].

(A) 4.642 (B) 5 (C) 5.313 (D) 5.774 (E) 7.071

*An asterisk denotes a topic covered only in Calculus BC.