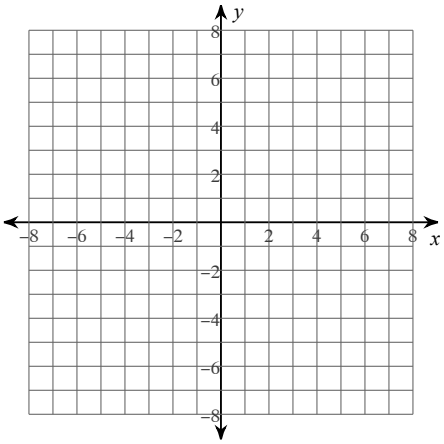
Calculus Name_____ ID: 1

Curve sketching Practice!

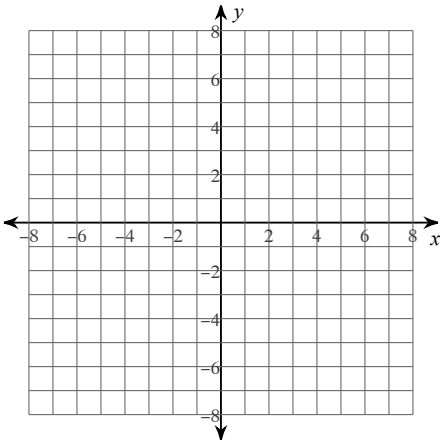
Date_____ Period____

For each problem, find the: x and y intercepts, asymptotes, x-coordinates of the critical points, open intervals where the function is increasing and decreasing, x-coordinates of the inflection points, open intervals where the function is concave up and concave down, and relative minima and maxima. Using this information, sketch the graph of the function.

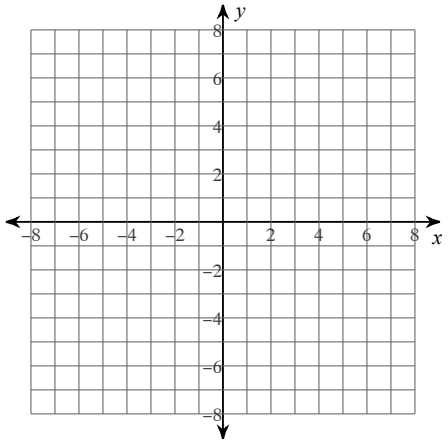
1) $y = -\frac{x^3}{12} + \frac{x^2}{12}$



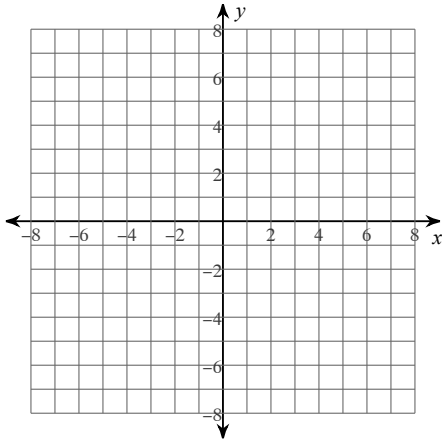
2) $y = -2x^3 + 4x^2$



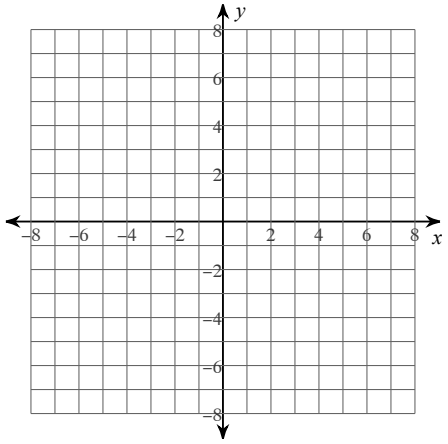
3) $y = -\frac{x^3}{12} + \frac{x^2}{6}$



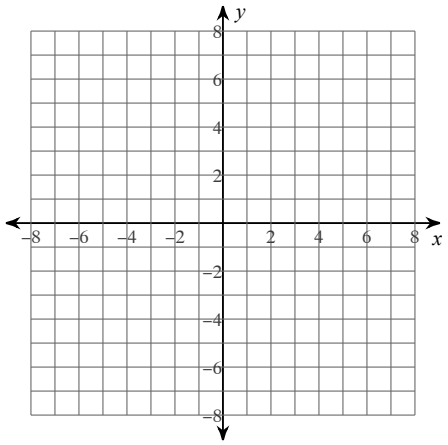
4) $y = -x^3 - 2x^2$



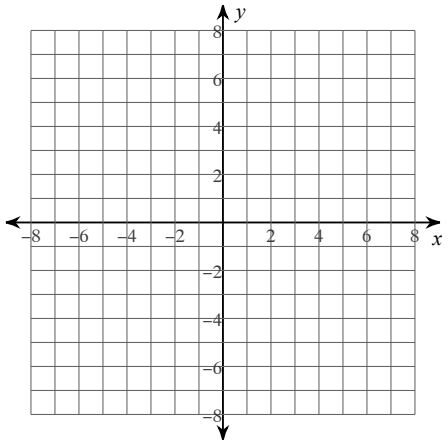
5) $y = x^4 - x^2$



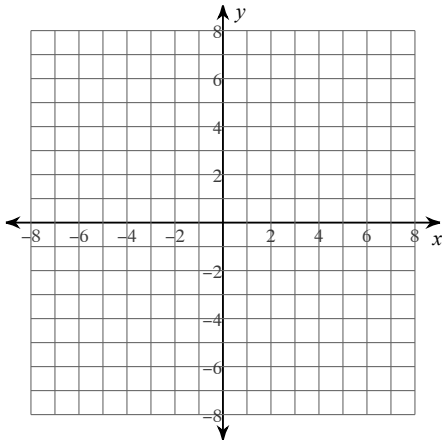
6) $y = x^2 + 4x - 1$



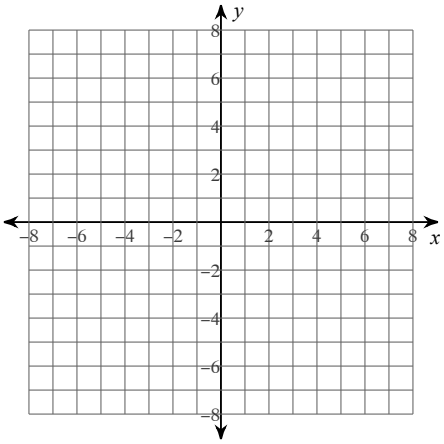
7) $y = -\frac{x^3}{6} - \frac{x^2}{6} + \frac{5x}{6}$



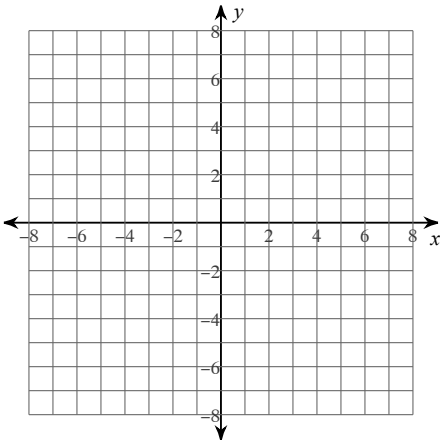
8) $y = \frac{x^3}{6} + \frac{x^2}{3} - \frac{2x}{3}$



9) $y = \dfrac{1}{x - 3}$

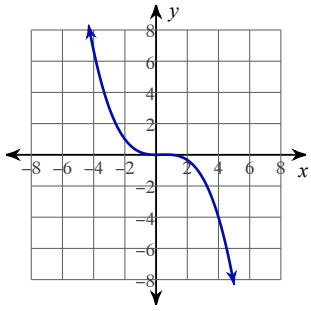


10) $y = \dfrac{x^4}{2} - x^2 + \dfrac{1}{2}$



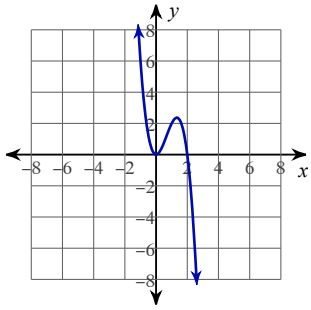
Answers to Curve sketching Practice! (ID: 1)

1)



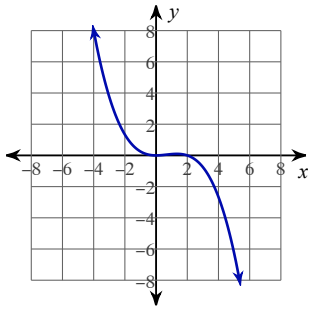
x-intercepts at $x = 0, 1$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = 0, \frac{2}{3}$
Increasing: $\left(0, \frac{2}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{2}{3}, \infty\right)$
Inflection point at: $x = \frac{1}{3}$
Concave up: $\left(-\infty, \frac{1}{3}\right)$ Concave down: $\left(\frac{1}{3}, \infty\right)$
Relative minimum: $(0, 0)$ Relative maximum: $\left(\frac{2}{3}, \frac{1}{81}\right)$

2)



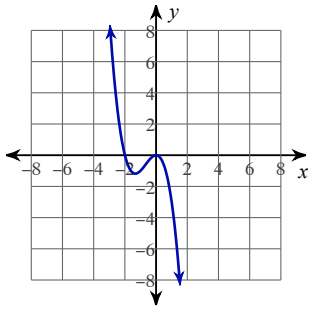
x-intercepts at $x = 0, 2$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = 0, \frac{4}{3}$
Increasing: $\left(0, \frac{4}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{4}{3}, \infty\right)$
Inflection point at: $x = \frac{2}{3}$
Concave up: $\left(-\infty, \frac{2}{3}\right)$ Concave down: $\left(\frac{2}{3}, \infty\right)$
Relative minimum: $(0, 0)$ Relative maximum: $\left(\frac{4}{3}, \frac{64}{27}\right)$

3)



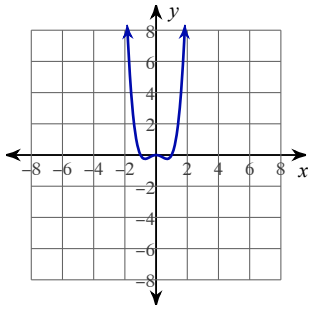
x-intercepts at $x = 0, 2$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = 0, \frac{4}{3}$
Increasing: $\left(0, \frac{4}{3}\right)$ Decreasing: $(-\infty, 0), \left(\frac{4}{3}, \infty\right)$
Inflection point at: $x = \frac{2}{3}$
Concave up: $\left(-\infty, \frac{2}{3}\right)$ Concave down: $\left(\frac{2}{3}, \infty\right)$
Relative minimum: $(0, 0)$ Relative maximum: $\left(\frac{4}{3}, \frac{8}{81}\right)$

4)



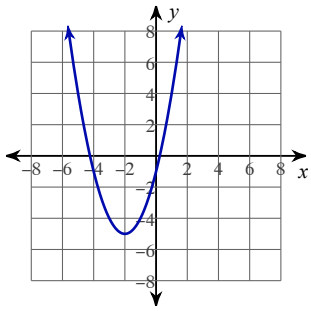
x-intercepts at $x = -2, 0$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\frac{4}{3}, 0$
Increasing: $\left(-\frac{4}{3}, 0\right)$ Decreasing: $\left(-\infty, -\frac{4}{3}\right), (0, \infty)$
Inflection point at: $x = -\frac{2}{3}$
Concave up: $\left(-\infty, -\frac{2}{3}\right)$ Concave down: $\left(-\frac{2}{3}, \infty\right)$
Relative minimum: $\left(-\frac{4}{3}, -\frac{32}{27}\right)$ Relative maximum: $(0, 0)$

5)



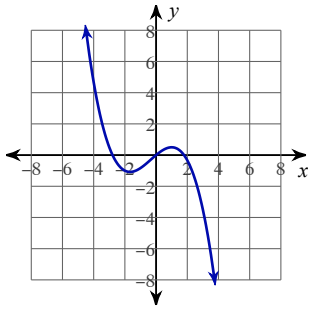
x-intercepts at $x = -1, 0, 1$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\frac{\sqrt{2}}{2}, 0, \frac{\sqrt{2}}{2}$
Increasing: $\left(-\frac{\sqrt{2}}{2}, 0\right), \left(\frac{\sqrt{2}}{2}, \infty\right)$ Decreasing: $\left(-\infty, -\frac{\sqrt{2}}{2}\right), \left(0, \frac{\sqrt{2}}{2}\right)$
Inflection points at: $x = -\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}$
Concave up: $\left(-\infty, -\frac{\sqrt{6}}{6}\right), \left(\frac{\sqrt{6}}{6}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{6}}{6}, \frac{\sqrt{6}}{6}\right)$
Relative minima: $\left(-\frac{\sqrt{2}}{2}, -\frac{1}{4}\right), \left(\frac{\sqrt{2}}{2}, -\frac{1}{4}\right)$ Relative maximum: $(0, 0)$

6)



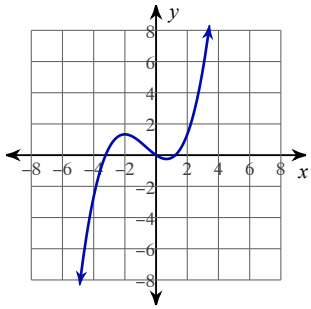
x-intercepts at $x = -2 - \sqrt{5}, -2 + \sqrt{5}$ y-intercept at $y = -1$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical point at: $x = -2$
Increasing: $(-2, \infty)$ Decreasing: $(-\infty, -2)$
No inflection points exist.
Concave up: $(-\infty, \infty)$ Concave down: No intervals exist.
Relative minimum: $(-2, -5)$ No relative maxima.

7)



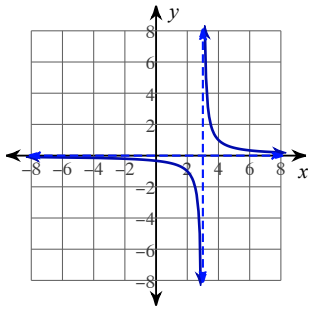
x-intercepts at $x = \frac{-1 - \sqrt{21}}{2}, 0, \frac{-1 + \sqrt{21}}{2}$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -\frac{5}{3}, 1$
Increasing: $\left(-\frac{5}{3}, 1\right)$ Decreasing: $\left(-\infty, -\frac{5}{3}\right), (1, \infty)$
Inflection point at: $x = -\frac{1}{3}$
Concave up: $\left(-\infty, -\frac{1}{3}\right)$ Concave down: $\left(-\frac{1}{3}, \infty\right)$
Relative minimum: $\left(-\frac{5}{3}, -\frac{175}{162}\right)$ Relative maximum: $\left(1, \frac{1}{2}\right)$

8)



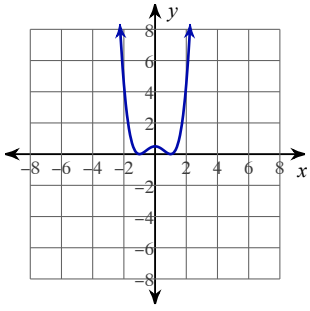
x-intercepts at $x = -1 - \sqrt{5}, 0, -1 + \sqrt{5}$ y-intercept at $y = 0$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -2, \frac{2}{3}$
Increasing: $(-\infty, -2), \left(\frac{2}{3}, \infty\right)$ Decreasing: $\left(-2, \frac{2}{3}\right)$
Inflection point at: $x = -\frac{2}{3}$
Concave up: $\left(-\frac{2}{3}, \infty\right)$ Concave down: $\left(-\infty, -\frac{2}{3}\right)$
Relative minimum: $\left(\frac{2}{3}, -\frac{20}{81}\right)$ Relative maximum: $\left(-2, \frac{4}{3}\right)$

9)



No x-intercepts. y-intercept at $y = -\frac{1}{3}$
Vertical asymptote at: $x = 3$
Horizontal asymptote at: y = 0
No critical points exist.
Increasing: No intervals exist. Decreasing: $(-\infty, 3), (3, \infty)$
No inflection points exist.
Concave up: $(3, \infty)$ Concave down: $(-\infty, 3)$
No relative minima. No relative maxima.

10)



x-intercepts at $x = -1, 1$ y-intercept at $y = \frac{1}{2}$
No vertical asymptotes exist.
No horizontal asymptotes exist.
Critical points at: $x = -1, 0, 1$
Increasing: $(-1, 0), (1, \infty)$ Decreasing: $(-\infty, -1), (0, 1)$
Inflection points at: $x = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$
Concave up: $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$ Concave down: $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$
Relative minima: $(-1, 0), (1, 0)$ Relative maximum: $\left(0, \frac{1}{2}\right)$