

## Concavity and POIs Practice

Date \_\_\_\_\_ Period \_\_\_\_\_

**For each problem, find the x-coordinates of all points of inflection and find the open intervals where the function is concave up and concave down.**

1)  $y = -x^4 + 2x^2 - 2$

2)  $y = 2\cot(x); [-\pi, \pi]$

3)  $y = -x^5 + 3x^3 + 3$

4)  $y = 2\csc(2x); [-\pi, \pi]$

5)  $y = x^3 - 3x^2 + 1$

6)  $y = x^3 - x^2 - 3$

7)  $y = x^3 + 4x^2 + 4x + 3$

8)  $y = \sin(2x); [-\pi, \pi]$

9)  $y = x^4 + 4x^3 + 4x^2$

10)  $y = x^5 - 2x^3$

11)  $y = -x^3 + x^2 + 2$

12)  $y = 2\cos(x); [-\pi, \pi]$

## Answers to Concavity and POIs Practice (ID: 1)

1) Inflection points at:  $x = -\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}$

Concave up:  $\left(-\frac{\sqrt{3}}{3}, \frac{\sqrt{3}}{3}\right)$  Concave down:  $\left(-\infty, -\frac{\sqrt{3}}{3}\right), \left(\frac{\sqrt{3}}{3}, \infty\right)$

2) Inflection points at:  $x = -\frac{\pi}{2}, \frac{\pi}{2}$

Concave up:  $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$  Concave down:  $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$

3) Inflection points at:  $x = -\frac{3\sqrt{10}}{10}, 0, \frac{3\sqrt{10}}{10}$

Concave up:  $\left(-\infty, -\frac{3\sqrt{10}}{10}\right), \left(0, \frac{3\sqrt{10}}{10}\right)$  Concave down:  $\left(-\frac{3\sqrt{10}}{10}, 0\right), \left(\frac{3\sqrt{10}}{10}, \infty\right)$

4) No inflection points exist.

Concave up:  $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$  Concave down:  $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$

5) Inflection point at:  $x = 1$

Concave up:  $(1, \infty)$  Concave down:  $(-\infty, 1)$

6) Inflection point at:  $x = \frac{1}{3}$

Concave up:  $\left(\frac{1}{3}, \infty\right)$  Concave down:  $\left(-\infty, \frac{1}{3}\right)$

7) Inflection point at:  $x = -\frac{4}{3}$

Concave up:  $\left(-\frac{4}{3}, \infty\right)$  Concave down:  $\left(-\infty, -\frac{4}{3}\right)$

8) Inflection points at:  $x = -\pi, -\frac{\pi}{2}, 0, \frac{\pi}{2}, \pi$

Concave up:  $\left(-\frac{\pi}{2}, 0\right), \left(\frac{\pi}{2}, \pi\right)$  Concave down:  $\left(-\pi, -\frac{\pi}{2}\right), \left(0, \frac{\pi}{2}\right)$

9) Inflection points at:  $x = \frac{-3 - \sqrt{3}}{3}, \frac{-3 + \sqrt{3}}{3}$

Concave up:  $\left(-\infty, \frac{-3 - \sqrt{3}}{3}\right), \left(\frac{-3 + \sqrt{3}}{3}, \infty\right)$  Concave down:  $\left(\frac{-3 - \sqrt{3}}{3}, \frac{-3 + \sqrt{3}}{3}\right)$

10) Inflection points at:  $x = -\frac{\sqrt{15}}{5}, 0, \frac{\sqrt{15}}{5}$

Concave up:  $\left(-\frac{\sqrt{15}}{5}, 0\right), \left(\frac{\sqrt{15}}{5}, \infty\right)$  Concave down:  $\left(-\infty, -\frac{\sqrt{15}}{5}\right), \left(0, \frac{\sqrt{15}}{5}\right)$

11) Inflection point at:  $x = \frac{1}{3}$

Concave up:  $\left(-\infty, \frac{1}{3}\right)$  Concave down:  $\left(\frac{1}{3}, \infty\right)$

12) Inflection points at:  $x = -\frac{\pi}{2}, \frac{\pi}{2}$

Concave up:  $\left(-\pi, -\frac{\pi}{2}\right), \left(\frac{\pi}{2}, \pi\right)$  Concave down:  $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$