Section 4-11 : Root Test

1. Determine if the following series converges or diverges.

$$\sum_{n=1}^{\infty}\left(\frac{3n+1}{4-2n}\right)^{2n}$$

Step 1
We'll need to compute *L*.

$$L=\lim_{n\to\infty}\sqrt[n]{\left|a_n\right|}=\lim_{n\to\infty}\left|\left(\frac{3n+1}{4-2n}\right)^{2n}\right|^{\frac{1}{n}}=\lim_{n\to\infty}\left|\left(\frac{3n+1}{4-2n}\right)^{2}\right|=\left(-\frac{3}{2}\right)^{2}=\frac{9}{4}$$

Step 2
Okay, we can see that $L=\frac{9}{4}>1$ and so by the Root Test the series **diverges**.

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2. Determine if the following series converges or diverges.

$$\sum_{n=0}^{\infty}\frac{n^{1-3n}}{4^{2n}}$$

Step 1
We'll need to compute *L*.

$$L=\lim_{n\to\infty}\sqrt[n]{\left|a_n\right|}=\lim_{n\to\infty}\left|\frac{n^{1-3n}}{4^{2n}}\right|^{\frac{1}{n}}=\lim_{n\to\infty}\left|\frac{n^{\frac{1}{n}-3}}{4^{2}}\right|=\left|\frac{n^{\frac{1}{n}}\ n^{-3}}{4^{2}}\right|=\frac{(1)(0)}{16}=0$$

Step 2
Okay, we can see that $L=0<1$ and so by the Root Test the series **converges**.

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3. Determine if the following series converges or diverges.

$$\sum_{n=4}^{\infty} \frac{(-5)^{1+2n}}{2^{5n-3}}$$

Step 1

We'll need to compute *L*.

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \left|\frac{(-5)^{1+2n}}{2^{5n-3}}\right|^{\frac{1}{n}} = \lim_{n\to\infty} \left|\frac{(-5)^{\frac{1}{n}+2}}{2^{5-\frac{3}{n}}}\right| = \left|\frac{(-5)^2}{2^5}\right| = \frac{25}{32}$$

Step 2

Okay, we can see that $L = \frac{25}{32} < 1$ and so by the Root Test the series **converges**.
