Section 3-1 : Parametric Equations and Curves

For problems 1 – 6 eliminate the parameter for the given set of parametric equations, sketch the graph of the parametric curve and give any limits that might exist on *x* and *y*.

1. $x = 4 - 2t \qquad y = 3 + 6t - 4t^2$

2. $x = 4 - 2t \qquad y = 3 + 6t - 4t^2 \qquad 0 \le t \le 3$

3. $x = \sqrt{t+1} \qquad y = \dfrac{1}{t+1} \qquad t > -1$

4. $x = 3\sin\left(t\right) \qquad y = -4\cos\left(t\right) \qquad 0 \le t \le 2\pi$

5. $x = 3\sin\left(2t\right) \qquad y = -4\cos\left(2t\right) \qquad 0 \le t \le 2\pi$

6. $x = 3\sin\left(\tfrac{1}{3}t\right) \qquad y = -4\cos\left(\tfrac{1}{3}t\right) \qquad 0 \le t \le 2\pi$

For problems 7 – 11 the path of a particle is given by the set of parametric equations. Completely describe the path of the particle. To completely describe the path of the particle you will need to provide the following information.

(*i*) A sketch of the parametric curve (including direction of motion) based on the equation you get by eliminating the parameter.
(*ii*) Limits on *x* and *y*.
(*iii*) A range of *t*'s for a single trace of the parametric curve.
(*iv*) The number of traces of the curve the particle makes if an overall range of *t*'s is provided in the problem.

7. $x = 3 - 2\cos\left(3t\right) \qquad y = 1 + 4\sin\left(3t\right)$

8. $x = 4\sin\left(\tfrac{1}{4}t\right) \qquad y = 1 - 2\cos^2\left(\tfrac{1}{4}t\right) \qquad -52\pi \le t \le 34\pi$

9. $x = \sqrt{4 + \cos\left(\tfrac{5}{2}t\right)} \qquad y = 1 + \tfrac{1}{3}\cos\left(\tfrac{5}{2}t\right) \qquad -48\pi \le t \le 2\pi$

10. $x = 2\mathbf{e}^{t} \qquad y = \cos\left(1 + \mathbf{e}^{3t}\right) \qquad 0 \le t \le \tfrac{3}{4}$

11. $x = \tfrac{1}{2}\mathbf{e}^{-3t} \qquad y = \mathbf{e}^{-6t} + 2\mathbf{e}^{-3t} - 8$

For problems 12 – 14 write down a set of parametric equations for the given equation that meets the given extra conditions (if any).

12. $y = 3x^2 - \ln(4x + 2)$

13. $x^2 + y^2 = 36$ and the parametric curve resulting from the parametric equations should be at $(6,0)$ when $t = 0$ and the curve should have a counter clockwise rotation.

14. $\frac{x^2}{4} + \frac{y^2}{49} = 1$ and the parametric curve resulting from the parametric equations should be at $(0,-7)$ when $t = 0$ and the curve should have a clockwise rotation.

Section 3-2 : Tangents with Parametric Equations

For problems 1 and 2 compute $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the given set of parametric equations.

1. $x = 4t^3 - t^2 + 7t \qquad y = t^4 - 6$

2. $x = \mathbf{e}^{-7t} + 2 \qquad y = 6\mathbf{e}^{2t} + \mathbf{e}^{-3t} - 4t$

For problems 3 and 4 find the equation of the tangent line(s) to the given set of parametric equations at the given point.

3. $x = 2\cos(3t) - 4\sin(3t) \qquad y = 3\tan(6t)$ at $t = \frac{\pi}{2}$

4. $x = t^2 - 2t - 11 \qquad y = t(t-4)^3 - 3t^2(t-4)^2 + 7$ at $(-3,7)$

5. Find the values of *t* that will have horizontal or vertical tangent lines for the following set of parametric equations.

$$x = t^5 - 7t^4 - 3t^3 \qquad y = 2\cos(3t) + 4t$$

Section 3-3 : Area with Parametric Equations

For problems 1 and 2 determine the area of the region below the parametric curve given by the set of parametric equations. For each problem you may assume that each curve traces out exactly once from left to right for the given range of *t*. For these problems you should only use the given parametric equations to determine the answer.

1. $x = 4t^3 - t^2 \qquad y = t^4 + 2t^2 \qquad 1 \le t \le 3$

2. $x = 3 - \cos^3\left(t\right) \qquad y = 4 + \sin\left(t\right) \qquad 0 \le t \le \pi$

Section 3-4 : Arc Length with Parametric Equations

For all the problems in this section you should only use the given parametric equations to determine the answer.

For problems 1 and 2 determine the length of the parametric curve given by the set of parametric equations. For these problems you may assume that the curve traces out exactly once for the given range of *t*'s.

1. $x = 8t^{\frac{3}{2}} \qquad y = 3 + \left(8 - t\right)^{\frac{3}{2}} \qquad 0 \le t \le 4$

2. $x = 3t + 1 \qquad y = 4 - t^2 \qquad -2 \le t \le 0$

3. A particle travels along a path defined by the following set of parametric equations. Determine the total distance the particle travels and compare this to the length of the parametric curve itself.

$$x = 4\sin\left(\tfrac{1}{4}t\right) \qquad y = 1 - 2\cos^2\left(\tfrac{1}{4}t\right) \qquad -52\pi \le t \le 34\pi$$

For problems 4 and 5 set up, but do not evaluate, an integral that gives the length of the parametric curve given by the set of parametric equations. For these problems you may assume that the curve traces out exactly once for the given range of *t*'s.

4. $x = 2 + t^2 \qquad y = \mathbf{e}^t \sin\left(2t\right) \qquad 0 \le t \le 3$

5. $x = \cos^3\left(2t\right) \qquad y = \sin\left(1 - t^2\right) \qquad -\tfrac{3}{2} \le t \le 0$

