

## Section 5-3 : Substitution Rule for Indefinite Integrals

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1. Evaluate  $\int (8x-12)(4x^2-12x)^4 dx$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an “obvious” inside function then there is at least a chance that the “inside” function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 4x^2 - 12x$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $x$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dx$  as well as the remaining  $x$ 's in the integrand.

$$du = (8x - 12)dx$$

Recall that, in most cases, we will also need to do a little manipulation of the differential prior to doing the substitution. In this case we don't need to do that.

Step 3

Doing the substitution and evaluating the integral gives,

$$\int (8x-12)(4x^2-12x)^4 dx = \int u^4 du = \frac{1}{5}u^5 + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int (8x-12)(4x^2-12x)^4 dx = \boxed{\frac{1}{5}(4x^2-12x)^5 + c}$$

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2. Evaluate  $\int 3t^{-4}(2+4t^{-3})^{-7} dt$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an “obvious” inside function then there is at least a chance that the “inside” function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 2 + 4t^{-3}$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $t$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dt$  as well as the remaining  $t$ 's in the integrand.

$$du = -12t^{-4} dt$$

To help with the substitution let's do a little rewriting of this to get,

$$3t^{-4} dt = -\frac{1}{4} du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int 3t^{-4} (2 + 4t^{-3})^{-7} dt = -\frac{1}{4} \int u^{-7} du = \frac{1}{24} u^{-6} + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int 3t^{-4} (2 + 4t^{-3})^{-7} dt = \boxed{\frac{1}{24} (2 + 4t^{-3})^{-6} + c}$$

3. Evaluate  $\int (3 - 4w)(4w^2 - 6w + 7)^{10} dw$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 4w^2 - 6w + 7$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $w$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dw$  as well as the remaining  $w$ 's in the integrand.

$$du = (8w - 6)dw$$

To help with the substitution let's do a little rewriting of this to get,

$$du = -2(3 - 4w)dw \quad \Rightarrow \quad (3 - 4w)dw = -\frac{1}{2}du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int (3 - 4w)(4w^2 - 6w + 7)^{10} dw = -\frac{1}{2} \int u^{10} du = -\frac{1}{22}u^{11} + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int (3 - 4w)(4w^2 - 6w + 7)^{10} dw = \boxed{-\frac{1}{22}(4w^2 - 6w + 7)^{11} + c}$$

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4. Evaluate  $\int 5(z - 4) \sqrt[3]{z^2 - 8z} dz$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = z^2 - 8z$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $z$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dz$  as well as the remaining  $z$ 's in the integrand.

$$du = (2z - 8) dz$$

To help with the substitution let's do a little rewriting of this to get,

$$du = (2z - 8) dz = 2(z - 4) dz \quad \Rightarrow \quad (z - 4) dz = \frac{1}{2} du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int 5(z - 4) \sqrt[3]{z^2 - 8z} dz = \frac{5}{2} \int u^{\frac{1}{3}} du = \frac{15}{8} u^{\frac{4}{3}} + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int 5(z - 4) \sqrt[3]{z^2 - 8z} dz = \boxed{\frac{15}{8} (z^2 - 8z)^{\frac{4}{3}} + c}$$

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5. Evaluate  $\int 90x^2 \sin(2 + 6x^3) dx$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 2 + 6x^3$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $x$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dx$  as well as the remaining  $x$ 's in the integrand.

$$du = 18x^2 dx$$

Recall that, in most cases, we will also need to do a little manipulation of the differential prior to doing the substitution. In this case we don't need to do that. When doing the substitution just notice that  $90 = (18)(5)$ .

Step 3

Doing the substitution and evaluating the integral gives,

$$\int 90x^2 \sin(2 + 6x^3) dx = \int 5 \sin(u) du = -5 \cos(u) + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int 90x^2 \sin(2 + 6x^3) dx = \boxed{-5 \cos(2 + 6x^3) + c}$$

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6. Evaluate  $\int \sec(1 - z) \tan(1 - z) dz$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 1 - z$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $z$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dz$  as well as the remaining  $z$ 's in the integrand.

$$du = -dz$$

To help with the substitution let's do a little rewriting of this to get,

$$dz = -du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int \sec(1 - z) \tan(1 - z) dz = -\int \sec(u) \tan(u) du = -\sec(u) + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int \sec(1-z) \tan(1-z) dz = \boxed{-\sec(1-z) + c}$$

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7. Evaluate  $\int (15t^{-2} - 5t) \cos(6t^{-1} + t^2) dt$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 6t^{-1} + t^2$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $t$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dt$  as well as the remaining  $t$ 's in the integrand.

$$du = (-6t^{-2} + 2t) dt$$

To help with the substitution let's do a little rewriting of this to get,

$$du = (-6t^{-2} + 2t) dt = -2\left(\frac{3}{5}\right)(3t^{-2} - t) dt \quad \Rightarrow \quad (15t^{-2} - 5t) dt = -\frac{5}{2} du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int (15t^{-2} - 5t) \cos(6t^{-1} + t^2) dt = -\frac{5}{2} \int \cos(u) du = -\frac{5}{2} \sin(u) + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int (15t^{-2} - 5t) \cos(6t^{-1} + t^2) dt = \boxed{-\frac{5}{2} \sin(6t^{-1} + t^2) + c}$$

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8. Evaluate  $\int (7y - 2y^3)e^{y^4 - 7y^2} dy$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an “obvious” inside function then there is at least a chance that the “inside” function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = y^4 - 7y^2$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $y$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dy$  as well as the remaining  $y$ 's in the integrand.

$$du = (4y^3 - 14y)dy$$

To help with the substitution let's do a little rewriting of this to get,

$$du = (4y^3 - 14y)dy = -2(7y - 2y^3)dy \quad \Rightarrow \quad (7y - 2y^3)dy = -\frac{1}{2}du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int (7y - 2y^3)e^{y^4 - 7y^2} dy = -\frac{1}{2} \int e^u du = -\frac{1}{2} e^u + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int (7y - 2y^3)e^{y^4 - 7y^2} dy = \boxed{-\frac{1}{2}e^{y^4 - 7y^2} + c}$$

9. Evaluate  $\int \frac{4w + 3}{4w^2 + 6w - 1} dw$ .

Hint : What is the derivative of the denominator?

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 4w^2 + 6w - 1$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $w$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dw$  as well as the remaining  $w$ 's in the integrand.

$$du = (8w + 6)dw$$

To help with the substitution let's do a little rewriting of this to get,

$$du = 2(4w + 3)dw \quad \Rightarrow \quad (4w + 3)dw = \frac{1}{2}du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int \frac{4w + 3}{4w^2 + 6w - 1} dw = \frac{1}{2} \int \frac{1}{u} du = \frac{1}{2} \ln|u| + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int \frac{4w + 3}{4w^2 + 6w - 1} dw = \boxed{\frac{1}{2} \ln|4w^2 + 6w - 1| + c}$$

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10. Evaluate  $\int (\cos(3t) - t^2)(\sin(3t) - t^3)^5 dt$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = \sin(3t) - t^3$$



Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $t$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dt$  as well as the remaining  $t$ 's in the integrand.

$$du = (3 \cos(3t) - 3t^2) dt$$

To help with the substitution let's do a little rewriting of this to get,

$$du = 3(\cos(3t) - t^2) dt \quad \Rightarrow \quad (\cos(3t) - t^2) dt = \frac{1}{3} du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int (\cos(3t) - t^2)(\sin(3t) - t^3)^5 dt = \frac{1}{3} \int u^5 du = \frac{1}{18} u^6 + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int (\cos(3t) - t^2)(\sin(3t) - t^3)^5 dt = \boxed{\frac{1}{18} (\sin(3t) - t^3)^6 + c}$$

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11. Evaluate  $\int 4 \left( \frac{1}{z} - e^{-z} \right) \cos(e^{-z} + \ln z) dz$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = e^{-z} + \ln z$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $z$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dz$  as well as the remaining  $z$ 's in the integrand.

$$du = \left(-e^{-z} + \frac{1}{z}\right) dz$$

Recall that, in most cases, we will also need to do a little manipulation of the differential prior to doing the substitution. In this case we don't need to do that.

Step 3

Doing the substitution and evaluating the integral gives,

$$\int 4 \left( \frac{1}{z} - e^{-z} \right) \cos(e^{-z} + \ln z) dz = \int 4 \cos(u) du = 4 \sin(u) + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int 4 \left( \frac{1}{z} - e^{-z} \right) \cos(e^{-z} + \ln z) dz = \boxed{4 \sin(e^{-z} + \ln z) + c}$$

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12. Evaluate  $\int \sec^2(v) e^{1+\tan(v)} dv$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 1 + \tan(v)$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $v$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dv$  as well as the remaining  $v$ 's in the integrand.

$$du = \sec^2(v) dv$$

Recall that, in most cases, we will also need to do a little manipulation of the differential prior to doing the substitution. In this case we don't need to do that.

Step 3

Doing the substitution and evaluating the integral gives,

$$\int \sec^2(v) e^{1+\tan(v)} dv = \int e^u du = e^u + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int \sec^2(v) e^{1+\tan(v)} dv = \boxed{e^{1+\tan(v)} + c}$$

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13. Evaluate  $\int 10 \sin(2x) \cos(2x) \sqrt{\cos^2(2x) - 5} dx$ .

Hint : Recall that if there is a term in the integrand (or a portion of a term) with an "obvious" inside function then there is at least a chance that the "inside" function is the substitution that we need.

Step 1

In this case it looks like we should use the following as our substitution.

$$u = \cos^2(2x) - 5$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $x$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dx$  as well as the remaining  $x$ 's in the integrand.

$$du = -4 \cos(2x) \sin(2x) dx$$

To help with the substitution let's do a little rewriting of this to get,

$$\begin{aligned} du = -4 \cos(2x) \sin(2x) dx &= -2(2)\left(\frac{5}{5}\right) \cos(2x) \sin(2x) dx \\ &\Rightarrow 10 \cos(2x) \sin(2x) dx = -\frac{5}{2} du \end{aligned}$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int 10 \sin(2x) \cos(2x) \sqrt{\cos^2(2x) - 5} dx = -\frac{5}{2} \int u^{\frac{1}{2}} du = -\frac{5}{3} u^{\frac{3}{2}} + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int 10 \sin(2x) \cos(2x) \sqrt{\cos^2(2x) - 5} dx = \boxed{-\frac{5}{3}(\cos^2(2x) - 5)^{\frac{3}{2}} + c}$$

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14. Evaluate  $\int \frac{\csc(x) \cot(x)}{2 - \csc(x)} dx$ .

Hint : What is the derivative of the denominator?

Step 1

In this case it looks like we should use the following as our substitution.

$$u = 2 - \csc(x)$$

Hint : Recall that after the substitution all the original variables in the integral should be replaced with  $u$ 's.

Step 2

Because we need to make sure that all the  $x$ 's are replaced with  $u$ 's we need to compute the differential so we can eliminate the  $dx$  as well as the remaining  $x$ 's in the integrand.

$$du = \csc(x) \cot(x) dx$$

Recall that, in most cases, we will also need to do a little manipulation of the differential prior to doing the substitution. In this case we don't need to do that.

Step 3

Doing the substitution and evaluating the integral gives,

$$\int \frac{\csc(x) \cot(x)}{2 - \csc(x)} dx = \int \frac{1}{u} du = \ln|u| + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int \frac{\csc(x)\cot(x)}{2-\csc(x)} dx = \boxed{\ln|2-\csc(x)|+c}$$

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15. Evaluate  $\int \frac{6}{7+y^2} dy$ .

Hint : Be careful with this substitution. The integrand should look somewhat familiar, so maybe we should try to put it into a more familiar form.

Step 1

The integrand looks an awful lot like the derivative of the inverse tangent.

$$\frac{d}{du}(\tan^{-1}(u)) = \frac{1}{1+u^2}$$

So, let's do a little rewrite to make the integrand look more like this.

$$\int \frac{6}{7+y^2} dy = \int \frac{6}{7(1+\frac{1}{7}y^2)} dy = \frac{6}{7} \int \frac{1}{1+\frac{1}{7}y^2} dy$$

Hint : One more little rewrite of the integrand should make this look almost exactly like the derivative the inverse tangent and the substitution should then be fairly obvious.

Step 2

Let's do one more rewrite of the integrand.

$$\int \frac{6}{7+y^2} dy = \frac{6}{7} \int \frac{1}{1+(\frac{y}{\sqrt{7}})^2} dy$$

At this point we can see that the following substitution will work for us.

$$u = \frac{y}{\sqrt{7}} \quad \rightarrow \quad du = \frac{1}{\sqrt{7}} dy \quad \rightarrow \quad dy = \sqrt{7} du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int \frac{6}{7+y^2} dy = \frac{6}{7}(\sqrt{7}) \int \frac{1}{1+u^2} du = \frac{6}{\sqrt{7}} \tan^{-1}(u) + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int \frac{6}{7+y^2} dy = \frac{6}{7}(\sqrt{7}) \int \frac{1}{1+u^2} du = \boxed{\frac{6}{\sqrt{7}} \tan^{-1}\left(\frac{y}{\sqrt{7}}\right) + c}$$

Substitutions for inverse trig functions can be a little tricky to spot when you are first start doing them. Once you do enough of them however they start to become a little easier to spot.

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16. Evaluate  $\int \frac{1}{\sqrt{4-9w^2}} dw$ .

Hint : Be careful with this substitution. The integrand should look somewhat familiar, so maybe we should try to put it into a more familiar form.

Step 1

The integrand looks an awful lot like the derivative of the inverse sine.

$$\frac{d}{du}(\sin^{-1}(u)) = \frac{1}{\sqrt{1-u^2}}$$

So, let's do a little rewrite to make the integrand look more like this.

$$\int \frac{1}{\sqrt{4-9w^2}} dw = \int \frac{1}{\sqrt{4(1-\frac{9}{4}w^2)}} dw = \frac{1}{2} \int \frac{1}{\sqrt{1-\frac{9}{4}w^2}} dw$$

Hint : One more little rewrite of the integrand should make this look almost exactly like the derivative the inverse sine and the substitution should then be fairly obvious.

Step 2

Let's do one more rewrite of the integrand.

$$\int \frac{1}{\sqrt{4-9w^2}} dw = \frac{1}{2} \int \frac{1}{\sqrt{1-(\frac{3w}{2})^2}} dw$$

At this point we can see that the following substitution will work for us.

$$u = \frac{3w}{2} \quad \rightarrow \quad du = \frac{3}{2} dw \quad \rightarrow \quad dw = \frac{2}{3} du$$

Step 3

Doing the substitution and evaluating the integral gives,

$$\int \frac{1}{\sqrt{4-9w^2}} dw = \frac{1}{2} \left( \frac{2}{3} \right) \int \frac{1}{\sqrt{1-u^2}} du = \frac{1}{3} \sin^{-1}(u) + c$$

Hint : Don't forget that the original variable in the integrand was not  $u$ !

Step 4

Finally, don't forget to go back to the original variable!

$$\int \frac{1}{\sqrt{4-9w^2}} dw = \boxed{\frac{1}{3} \sin^{-1}\left(\frac{3w}{2}\right) + c}$$

Substitutions for inverse trig functions can be a little tricky to spot when you are first start doing them. Once you do enough of them however they start to become a little easier to spot.

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17. Evaluate each of the following integrals.

(a)  $\int \frac{3x}{1+9x^2} dx$

(b)  $\int \frac{3x}{(1+9x^2)^4} dx$

(c)  $\int \frac{3}{1+9x^2} dx$

Hint : Make sure you pay attention to each of these and note the differences between each integrand and how that will affect the substitution and/or answer.

(a)  $\int \frac{3x}{1+9x^2} dx$

Solution

In this case it looks like the substitution should be

$$u = 1 + 9x^2$$

Here is the differential for this substitution.

$$du = 18x dx \quad \Rightarrow \quad 3x dx = \frac{1}{6} du$$

The integral is then,

$$\int \frac{3x}{1+9x^2} dx = \frac{1}{6} \int \frac{1}{u} du = \frac{1}{6} \ln|u| + c = \boxed{\frac{1}{6} \ln|1+9x^2| + c}$$

(b)  $\int \frac{3x}{(1+9x^2)^4} dx$

**Solution**

The substitution and differential work for this part are identical to the previous part.

$$u = 1+9x^2 \quad du = 18x dx \quad \Rightarrow \quad 3x dx = \frac{1}{6} du$$

Here is the integral for this part,

$$\int \frac{3x}{(1+9x^2)^4} dx = \frac{1}{6} \int \frac{1}{u^4} du = \frac{1}{6} \int u^{-4} du = -\frac{1}{18} u^{-3} + c = \boxed{-\frac{1}{18} \frac{1}{(1+9x^2)^3} + c}$$

Be careful to not just turn every integral of functions of the form of 1/(something) into logarithms! This is one of the more common mistakes that students often make.

(c)  $\int \frac{3}{1+9x^2} dx$

**Solution**

Because we no longer have an x in the numerator this integral is very different from the previous two. Let's do a quick rewrite of the integrand to make the substitution clearer.

$$\int \frac{3}{1+9x^2} dx = \int \frac{3}{1+(3x)^2} dx$$

So, this looks like an inverse tangent problem that will need the substitution.

$$u = 3x \quad \rightarrow \quad du = 3dx$$

The integral is then,

$$\int \frac{3}{1+9x^2} dx = \int \frac{1}{1+u^2} du = \tan^{-1}(u) + c = \boxed{\tan^{-1}(3x) + c}$$


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