Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Derivatives

Product and Quotient Rule

1. Use the Product Rule to find the derivative of $f\left(t\right)=\left(4t^2-t\right)\left(t^3-8t^2+12\right)$.

Solution

There isn't much to do here other than take the derivative using the product rule.

$$f'\left(t\right)=\left(8t-1\right)\left(t^3-8t^2+12\right)+\left(4t^2-t\right)\left(3t^2-16t\right)=20t^4-132t^3+24t^2+96t-12$$

Note that we multiplied everything out to get a "simpler" answer.

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2. Use the Product Rule to find the derivative of $y = \left(1 + \sqrt{x^3}\right)\left(x^{-3} - 2\sqrt[3]{x}\right)$.

Solution

There isn't much to do here other than take the derivative using the product rule. We'll also need to convert the roots to fractional exponents.

$$y = \left(1 + x^{\frac{3}{2}}\right)\left(x^{-3} - 2x^{\frac{1}{3}}\right)$$

The derivative is then,

$$\frac{dy}{dx} = \left(\frac{3}{2}x^{\frac{1}{2}}\right)\left(x^{-3} - 2x^{\frac{1}{3}}\right) + \left(1 + x^{\frac{3}{2}}\right)\left(-3x^{-4} - \frac{2}{3}x^{-\frac{2}{3}}\right) = -3x^{-4} - \frac{3}{2}x^{-\frac{5}{2}} - \frac{2}{3}x^{-\frac{2}{3}} - \frac{11}{3}x^{\frac{5}{6}}$$

Note that we multiplied everything out to get a "simpler" answer.

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3. Use the Product Rule to find the derivative of $h(z) = \left(1 + 2z + 3z^2\right)\left(5z + 8z^2 - z^3\right)$.

Solution

There isn't much to do here other than take the derivative using the product rule.

$$\begin{aligned} h'(z) &= (2 + 6z)\left(5z + 8z^2 - z^3\right) + \left(1 + 2z + 3z^2\right)\left(5 + 16z - 3z^2\right) \\ &= 5 + 36z + 90z^2 + 88z^3 - 15z^4 \end{aligned}$$

Note that we multiplied everything out to get a "simpler" answer.

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4. Use the Quotient Rule to find the derivative of $g(x) = \dfrac{6x^2}{2 - x}$.

Solution

There isn't much to do here other than take the derivative using the quotient rule.

$$g'(x) = \frac{12x(2 - x) - 6x^2(-1)}{(2 - x)^2} = \boxed{\frac{24x - 6x^2}{(2 - x)^2}}$$

5. Use the Quotient Rule to find the derivative of $R(w) = \dfrac{3w + w^4}{2w^2 + 1}$.

Solution

There isn't much to do here other than take the derivative using the quotient rule.

$$R'(w) = \frac{(3 + 4w^3)(2w^2 + 1) - (3w + w^4)(4w)}{(2w^2 + 1)^2} = \boxed{\frac{4w^5 + 4w^3 - 6w^2 + 3}{(2w^2 + 1)^2}}$$

6. Use the Quotient Rule to find the derivative of $f(x) = \dfrac{\sqrt{x} + 2x}{7x - 4x^2}$.

Solution

There isn't much to do here other than take the derivative using the quotient rule.

$$f'(x) = \frac{\left(\frac{1}{2}x^{-\frac{1}{2}} + 2\right)(7x - 4x^2) - \left(x^{\frac{1}{2}} + 2x\right)(7 - 8x)}{(7x - 4x^2)^2}$$

7. If $f(2) = -8$, $f'(2) = 3$, $g(2) = 17$ and $g'(2) = -4$ determine the value of $(f\,g)'(2)$.

Solution

We know that the product rule is,

$$(f\,g)'(x) = f'(x)\,g(x) + f(x)\,g'(x)$$

Now, we want to know the value of this at $x = 2$ and so all we need to do is plug this into the derivative. Doing this gives,

$$(f\,g)'(2) = f'(2)\,g(2) + f(2)\,g'(2)$$

Now, we were given values for all these quantities and so all we need to do is plug these into our "formula" above.

$$(f\,g)'(2) = (3)(17) + (-8)(-4) = \boxed{83}$$

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8. If $f(x) = x^3 g(x)$, $g(-7) = 2$, $g'(-7) = -9$ determine the value of $f'(-7)$.

Hint : Even though we don't know what $g(x)$ is we can still use the product rule to take the derivative and then we can use the given information to get the value of $f'(-7)$.

Solution

Even though we don't know what $g(x)$ is we do have a product of two functions here and so we can use the product rule to determine the derivative of $f(x)$.

$$f'(x) = 3x^2 g(x) + x^3 g'(x)$$

Now all we need to do is plug $x = -7$ into this and use the given information to determine the value of $f'(-7)$.

$$f'(-7) = 3(-7)^2 g(-7) + (-7)^3 g'(-7) = 3(49)(2) + (-343)(-9) = \boxed{3381}$$

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9. Find the equation of the tangent line to $f(x) = \left(1 + 12\sqrt{x}\right)\left(4 - x^2\right)$ at $x = 9$.

Solution

Step 1
We know that the derivative of the function will give us the slope of the tangent line so we'll need the derivative of the function. We'll use the product rule to get the derivative.

$$f'(x) = \left(6x^{-\frac{1}{2}}\right)\left(4 - x^2\right) + \left(1 + 12\sqrt{x}\right)(-2x) = \left(\frac{6}{\sqrt{x}}\right)\left(4 - x^2\right) - 2x\left(1 + 12\sqrt{x}\right)$$

Step 2
Note that we didn't bother to "simplify" the derivative (other than converting the fractional exponent back to a root) because all we really need this for is a quick evaluation.

Speaking of which here are the evaluations that we'll need for this problem.

$$f(9) = (37)(-77) = -2849 \qquad\qquad f'(9) = (2)(-77) - 18(37) = -820$$

Step 3

Now all that we need to do is write down the equation of the tangent line.

$$y = f(9) + f'(9)(x-9) = -2849 - 820(x-9) \qquad \rightarrow \qquad \boxed{y = -820x + 4531}$$

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10. Determine where $f(x) = \dfrac{x - x^2}{1 + 8x^2}$ is increasing and decreasing.

Solution

Step 1

We'll first need the derivative, which will require the quotient rule, because we know that the derivative will give us the rate of change of the function. Here is the derivative.

$$\boxed{f'(x) = \frac{(1-2x)(1+8x^2) - (x - x^2)(16x)}{(1+8x^2)^2} = \frac{1 - 2x - 8x^2}{(1+8x^2)^2}}$$

Step 2

Next, we need to know where the function is not changing and so all we need to do is set the derivative equal to zero and solve. In this case it is clear that the denominator will never be zero for any real number and so the derivative will only be zero where the numerator is zero. Therefore, setting the numerator equal to zero and solving gives,

$$1 - 2x - 8x^2 = -(8x^2 + 2x - 1) = -(4x-1)(2x+1) = 0$$

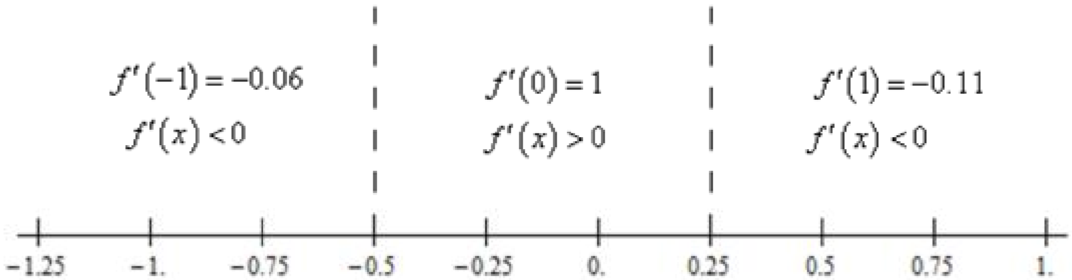
From this it is pretty easy to see that the derivative will be zero, and hence the function will not be changing, at,

$$\boxed{x = -\frac{1}{2} \qquad\qquad x = \frac{1}{4}}$$

Step 3

To get the answer to this problem all we need to know is where the derivative is positive (and hence the function is increasing) or negative (and hence the function is decreasing). Because the derivative is continuous we know that the only place it can change sign is where the derivative is zero. So, as we did in this section a quick number line will give us the sign of the derivative for the various intervals.

Here is the number line for this problem.



From this we get the following increasing/decreasing information.

$$\boxed{\begin{array}{l} \text{Increasing : } -\frac{1}{2} < x < \frac{1}{4} \\ \text{Decreasing : } -\infty < x < -\frac{1}{2}, \;\; \frac{1}{4} < x < \infty \end{array}}$$

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11. Determine where $V(t) = (4 - t^2)(1 + 5t^2)$ is increasing and decreasing.

Solution

Step 1
We'll first need the derivative, for which we will use the product rule, because we know that the derivative will give us the rate of change of the function. Here is the derivative.

$$\boxed{V'(t) = (-2t)(1 + 5t^2) + (4 - t^2)(10t) = 38t - 20t^3 = 2t(19 - 10t^2)}$$

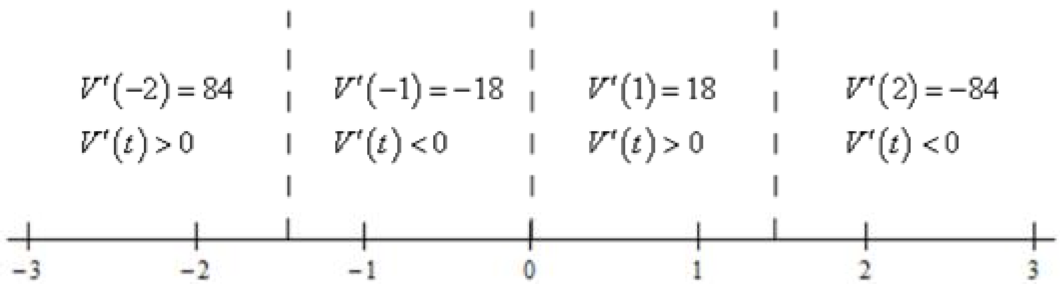
Step 2
Next, we need to know where the function is not changing and so all we need to do is set the derivative equal to zero and solve. From the factored form of the derivative it is easy to see that the derivative will be zero at,

$$\boxed{t = 0 \qquad\qquad t = \pm\sqrt{\frac{19}{10}} = \pm 1.3784}$$

Step 3

To get the answer to this problem all we need to know is where the derivative is positive (and hence the function is increasing) or negative (and hence the function is decreasing). Because the derivative is continuous we know that the only place it can change sign is where the derivative is zero. So, as we did in this section a quick number line will give us the sign of the derivative for the various intervals.

Here is the number line for this problem.



From this we get the following increasing/decreasing information.

$$\boxed{\begin{array}{l}\text{Increasing : } -\infty < t < -\sqrt{\dfrac{19}{10}}, \quad 0 < t < \sqrt{\dfrac{19}{10}} \\ \text{Decreasing : } -\sqrt{\dfrac{19}{10}} < t < 0, \quad \sqrt{\dfrac{19}{10}} < x < \infty\end{array}}$$