Preface

Here are the solutions to the practice problems for my Calculus I notes. Some solutions will have more or less detail than other solutions. The level of detail in each solution will depend up on several issues. If the section is a review section, this mostly applies to problems in the first chapter, there will probably not be as much detail to the solutions given that the problems really should be review. As the difficulty level of the problems increases less detail will go into the basics of the solution under the assumption that if you've reached the level of working the harder problems then you will probably already understand the basics fairly well and won't need all the explanation.

This document was written with presentation on the web in mind. On the web most solutions are broken down into steps and many of the steps have hints. Each hint on the web is given as a popup however in this document they are listed prior to each step. Also, on the web each step can be viewed individually by clicking on links while in this document they are all showing. Also, there are liable to be some formatting parts in this document intended for help in generating the web pages that haven't been removed here. These issues may make the solutions a little difficult to follow at times, but they should still be readable.

Limits At Infinity, Part I

1. For $f\left(x\right)=4x^7-18x^3+9$ evaluate each of the following limits.

(a) $\lim\limits_{x\to-\infty} f\left(x\right)$ **(b)** $\lim\limits_{x\to\infty} f\left(x\right)$

(a) $\lim\limits_{x\to-\infty} f\left(x\right)$

To do this all we need to do is factor out the largest power of x from the whole polynomial and then use basic limit properties along with Fact 1 from this section to evaluate the limit.

$$\begin{aligned}\lim_{x\to-\infty}\left(4x^7-18x^3+9\right) &= \lim_{x\to-\infty}\left[x^7\left(4-\frac{18}{x^4}+\frac{9}{x^7}\right)\right]\\ &= \left(\lim_{x\to-\infty}x^7\right)\left[\lim_{x\to-\infty}\left(4-\frac{18}{x^4}+\frac{9}{x^7}\right)\right]=\left(-\infty\right)\left(4\right)=\boxed{-\infty}\end{aligned}$$

(b) $\lim\limits_{x\to\infty} f\left(x\right)$

For this part all of the mathematical manipulations we did in the first part did not depend upon the limit itself and so don't need to be redone here. We can pick up the problem right before we actually took the limits and then proceed.

$$\lim_{x\to\infty}\left(4x^7-18x^3+9\right)=\left(\lim_{x\to\infty}x^7\right)\left[\lim_{x\to\infty}\left(4-\frac{18}{x^4}+\frac{9}{x^7}\right)\right]=(\infty)(4)=\boxed{\infty}$$

---

2. For $h(t)=\sqrt[3]{t}+12t-2t^2$ evaluate each of the following limits.

(a) $\lim\limits_{t\to-\infty}h(t)$ **(b)** $\lim\limits_{t\to\infty}h(t)$

(a) $\lim\limits_{t\to-\infty}h(t)$

To do this all we need to do is factor out the largest power of x from the whole polynomial and then use basic limit properties along with Fact 1 from this section to evaluate the limit.

Note as well that we'll convert the root over to a fractional exponent in order to allow it to be easier to deal with. Also note that this limit is a perfectly acceptable limit because the root is a cube root and we *can* take cube roots of negative numbers! We would only have run into problems had the index on the root been an even number.

$$\lim_{t\to-\infty}\left(t^{\frac{1}{3}}+12t-2t^2\right)=\lim_{t\to-\infty}\left[t^2\left(\frac{1}{t^{\frac{5}{3}}}+\frac{12}{t}-2\right)\right]$$

$$=\left(\lim_{t\to-\infty}t^2\right)\left[\lim_{t\to-\infty}\left(\frac{1}{t^{\frac{5}{3}}}+\frac{12}{t}-2\right)\right]=(\infty)(-2)=\boxed{-\infty}$$

(b) $\lim\limits_{t\to\infty}h(t)$

For this part all of the mathematical manipulations we did in the first part did not depend upon the limit itself and so don't need to be redone here. We can pick up the problem right before we actually took the limits and then proceed.

$$\lim_{t\to\infty}\left(t^{\frac{1}{3}}+12t-2t^2\right)=\left(\lim_{t\to\infty}t^2\right)\left[\lim_{t\to\infty}\left(\frac{1}{t^{\frac{5}{3}}}+\frac{12}{t}-2\right)\right]=(\infty)(-2)=\boxed{-\infty}$$

---

3. For $f\left(x\right)=\dfrac{8-4x^2}{9x^2+5x}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x\to-\infty} f\left(x\right)$.

(b) Evaluate $\lim\limits_{x\to\infty} f\left(x\right)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x\to-\infty} f\left(x\right)$.

To do this all we need to do is factor out the largest power of x that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

$$\lim_{x\to-\infty}\frac{8-4x^2}{9x^2+5x}=\lim_{x\to-\infty}\frac{x^2\left(\frac{8}{x^2}-4\right)}{x^2\left(9+\frac{5}{x}\right)}=\lim_{x\to-\infty}\frac{\frac{8}{x^2}-4}{9+\frac{5}{x}}=\boxed{\frac{-4}{9}}$$

(b) Evaluate $\lim\limits_{x\to\infty} f\left(x\right)$.

For this part all of the mathematical manipulations we did in the first part did not depend upon the limit itself and so don't really need to be redone here. However, it is easy enough to add them in so we'll go ahead and include them.

$$\lim_{x\to\infty}\frac{8-4x^2}{9x^2+5x}=\lim_{x\to\infty}\frac{x^2\left(\frac{8}{x^2}-4\right)}{x^2\left(9+\frac{5}{x}\right)}=\lim_{x\to\infty}\frac{\frac{8}{x^2}-4}{9+\frac{5}{x}}=\boxed{\frac{-4}{9}}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

We know that there will be a horizontal asymptote for $x\to-\infty$ if $\lim\limits_{x\to-\infty} f\left(x\right)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x\to\infty$ if $\lim\limits_{x\to\infty} f\left(x\right)$ exists and is a finite number.

Therefore, from the first two parts, we can see that we will get the horizontal asymptote,

$$y=-\frac{4}{9}$$

for both $x\to-\infty$ and $x\to\infty$.

4. For $f(x) = \dfrac{3x^7 - 4x^2 + 1}{5 - 10x^2}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x \to -\infty} f(x)$.

(b) Evaluate $\lim\limits_{x \to \infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x \to -\infty} f(x)$.

To do this all we need to do is factor out the largest power of x that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

$$\lim_{x \to -\infty} \frac{3x^7 - 4x^2 + 1}{5 - 10x^2} = \lim_{x \to -\infty} \frac{x^2\left(3x^5 - 4 + \frac{1}{x^2}\right)}{x^2\left(\frac{5}{x^2} - 10\right)} = \lim_{x \to -\infty} \frac{3x^5 - 4 + \frac{1}{x^2}}{\frac{5}{x^2} - 10} = \frac{-\infty}{-10} = \boxed{\infty}$$

(b) Evaluate $\lim\limits_{x \to \infty} f(x)$.

For this part all of the mathematical manipulations we did in the first part did not depend upon the limit itself and so don't really need to be redone here. However, it is easy enough to add them in so we'll go ahead and include them.

$$\lim_{x \to \infty} \frac{3x^7 - 4x^2 + 1}{5 - 10x^2} = \lim_{x \to \infty} \frac{x^2\left(3x^5 - 4 + \frac{1}{x^2}\right)}{x^2\left(\frac{5}{x^2} - 10\right)} = \lim_{x \to \infty} \frac{3x^5 - 4 + \frac{1}{x^2}}{\frac{5}{x^2} - 10} = \frac{\infty}{-10} = \boxed{-\infty}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

We know that there will be a horizontal asymptote for $x \to -\infty$ if $\lim\limits_{x \to -\infty} f(x)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x \to \infty$ if $\lim\limits_{x \to \infty} f(x)$ exists and is a finite number.

Therefore, from the first two parts, we can see that this function will have **no horizontal asymptotes** since neither of the two limits are finite.

---

5. For $f(x) = \dfrac{20x^4 - 7x^3}{2x + 9x^2 + 5x^4}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x\to-\infty} f\left(x\right)$.

(b) Evaluate $\lim\limits_{x\to\infty} f\left(x\right)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x\to-\infty} f\left(x\right)$.

To do this all we need to do is factor out the largest power of x that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

$$\lim_{x\to-\infty}\frac{20x^4-7x^3}{2x+9x^2+5x^4}=\lim_{x\to-\infty}\frac{x^4\left(20-\frac{7}{x}\right)}{x^4\left(\frac{2}{x^3}+\frac{9}{x^2}+5\right)}=\lim_{x\to-\infty}\frac{20-\frac{7}{x}}{\frac{2}{x^3}+\frac{9}{x^2}+5}=\frac{20}{5}=\boxed{4}$$

(b) Evaluate $\lim\limits_{x\to\infty} f\left(x\right)$.

For this part all of the mathematical manipulations we did in the first part did not depend upon the limit itself and so don't really need to be redone here. However, it is easy enough to add them in so we'll go ahead and include them.

$$\lim_{x\to\infty}\frac{20x^4-7x^3}{2x+9x^2+5x^4}=\lim_{x\to\infty}\frac{x^4\left(20-\frac{7}{x}\right)}{x^4\left(\frac{2}{x^3}+\frac{9}{x^2}+5\right)}=\lim_{x\to\infty}\frac{20-\frac{7}{x}}{\frac{2}{x^3}+\frac{9}{x^2}+5}=\frac{20}{5}=\boxed{4}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

We know that there will be a horizontal asymptote for $x\to-\infty$ if $\lim\limits_{x\to-\infty} f\left(x\right)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x\to\infty$ if $\lim\limits_{x\to\infty} f\left(x\right)$ exists and is a finite number.

Therefore, from the first two parts, we can see that we will get the horizontal asymptote,

$$y=4$$

for both $x\to-\infty$ and $x\to\infty$.

---

6. For $f\left(x\right)=\dfrac{x^3-2x+11}{3-6x^5}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x\to-\infty} f\left(x\right)$.

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

To do this all we need to do is factor out the largest power of *x* that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

$$\lim_{x\to-\infty}\frac{x^3-2x+11}{3-6x^5}=\lim_{x\to-\infty}\frac{x^5\left(\frac{1}{x^2}-\frac{2}{x^4}+\frac{11}{x^5}\right)}{x^5\left(\frac{3}{x^5}-6\right)}=\lim_{x\to-\infty}\frac{\frac{1}{x^2}-\frac{2}{x^4}+\frac{11}{x^5}}{\frac{3}{x^5}-6}=\frac{0}{-6}=\boxed{0}$$

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

For this part all of the mathematical manipulations we did in the first part did not depend upon the limit itself and so don't really need to be redone here. However, it is easy enough to add them in so we'll go ahead and include them.

$$\lim_{x\to\infty}\frac{x^3-2x+11}{3-6x^5}=\lim_{x\to\infty}\frac{x^5\left(\frac{1}{x^2}-\frac{2}{x^4}+\frac{11}{x^5}\right)}{x^5\left(\frac{3}{x^5}-6\right)}=\lim_{x\to\infty}\frac{\frac{1}{x^2}-\frac{2}{x^4}+\frac{11}{x^5}}{\frac{3}{x^5}-6}=\frac{0}{-6}=\boxed{0}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

We know that there will be a horizontal asymptote for $x\to-\infty$ if $\lim\limits_{x\to-\infty} f(x)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x\to\infty$ if $\lim\limits_{x\to\infty} f(x)$ exists and is a finite number.

Therefore, from the first two parts, we can see that we will get the horizontal asymptote,

$$y=0$$

for both $x\to-\infty$ and $x\to\infty$.

---

7. For $f(x)=\dfrac{x^6-x^4+x^2-1}{7x^6+4x^3+10}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

To do this all we need to do is factor out the largest power of x that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

$$\lim_{x\to-\infty}\frac{x^6-x^4+x^2-1}{7x^6+4x^3+10}=\lim_{x\to-\infty}\frac{x^6\left(1-\frac{1}{x^2}+\frac{1}{x^4}-\frac{1}{x^6}\right)}{x^6\left(7+\frac{4}{x^3}+\frac{10}{x^6}\right)}=\lim_{x\to-\infty}\frac{1-\frac{1}{x^2}+\frac{1}{x^4}-\frac{1}{x^6}}{7+\frac{4}{x^3}+\frac{10}{x^6}}=\boxed{\frac{1}{7}}$$

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

For this part all of the mathematical manipulations we did in the first part did not depend upon the limit itself and so don't really need to be redone here. However, it is easy enough to add them in so we'll go ahead and include them.

$$\lim_{x\to\infty}\frac{x^6-x^4+x^2-1}{7x^6+4x^3+10}=\lim_{x\to\infty}\frac{x^6\left(1-\frac{1}{x^2}+\frac{1}{x^4}-\frac{1}{x^6}\right)}{x^6\left(7+\frac{4}{x^3}+\frac{10}{x^6}\right)}=\lim_{x\to\infty}\frac{1-\frac{1}{x^2}+\frac{1}{x^4}-\frac{1}{x^6}}{7+\frac{4}{x^3}+\frac{10}{x^6}}=\boxed{\frac{1}{7}}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

We know that there will be a horizontal asymptote for $x\to-\infty$ if $\lim\limits_{x\to-\infty} f(x)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x\to\infty$ if $\lim\limits_{x\to\infty} f(x)$ exists and is a finite number.

Therefore, from the first two parts, we can see that we will get the horizontal asymptote,

$$y=\frac{1}{7}$$

for both $x\to-\infty$ and $x\to\infty$.

---

8. For $f(x)=\dfrac{\sqrt{7+9x^2}}{1-2x}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x\to-\infty} f\left(x\right)$.

To do this all we need to do is factor out the largest power of x that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

In this case the largest power of x in the denominator is just x and so we will need to factor an x out of both the denominator and the numerator. Recall as well that this means we'll need to factor an x^2 out of the root in the numerator so that we'll have an x in the numerator when we are done.

So, let's do the first couple of steps in this process to get us started.

$$\lim_{x\to-\infty}\frac{\sqrt{7+9x^2}}{1-2x}=\lim_{x\to-\infty}\frac{\sqrt{x^2\left(\frac{7}{x^2}+9\right)}}{x\left(\frac{1}{x}-2\right)}=\lim_{x\to-\infty}\frac{\sqrt{x^2}\sqrt{\frac{7}{x^2}+9}}{x\left(\frac{1}{x}-2\right)}=\lim_{x\to-\infty}\frac{\left|x\right|\sqrt{\frac{7}{x^2}+9}}{x\left(\frac{1}{x}-2\right)}$$

Recall from the discussion in this section that,

$$\sqrt{x^2}=\left|x\right|$$

and we *do* need to be careful with that.

Now, because we are looking at the limit $x\to-\infty$ it is safe to assume that $x<0$. Therefore, from the definition of the absolute value we get,

$$\left|x\right|=-x$$

and the limit is then,

$$\lim_{x\to-\infty}\frac{\sqrt{7+9x^2}}{1-2x}=\lim_{x\to-\infty}\frac{-x\sqrt{\frac{7}{x^2}+9}}{x\left(\frac{1}{x}-2\right)}=\lim_{x\to-\infty}\frac{-\sqrt{\frac{7}{x^2}+9}}{\frac{1}{x}-2}=\frac{-\sqrt{9}}{-2}=\boxed{\frac{3}{2}}$$

(b) Evaluate $\lim\limits_{x\to\infty} f\left(x\right)$.

For this part all of the mathematical manipulations we did in the first part up to dealing with the absolute value did not depend upon the limit itself and so don't really need to be redone here. So, up to that part we have,

$$\lim_{x\to\infty}\frac{\sqrt{7+9x^2}}{1-2x}=\lim_{x\to\infty}\frac{\left|x\right|\sqrt{\frac{7}{x^2}+9}}{x\left(\frac{1}{x}-2\right)}$$

In this part we are looking at the limit $x \to \infty$ and so it will be safe to assume in this part that $x > 0$. Therefore, from the definition of the absolute value we get,

$$|x| = x$$

and the limit is then,

$$\lim_{x\to\infty} \frac{\sqrt{7+9x^2}}{1-2x} = \lim_{x\to\infty} \frac{x\sqrt{\frac{7}{x^2}+9}}{x\left(\frac{1}{x}-2\right)} = \lim_{x\to\infty} \frac{\sqrt{\frac{7}{x^2}+9}}{\frac{1}{x}-2} = \frac{\sqrt{9}}{-2} = \boxed{-\frac{3}{2}}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

We know that there will be a horizontal asymptote for $x \to -\infty$ if $\lim\limits_{x\to-\infty} f(x)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x \to \infty$ if $\lim\limits_{x\to\infty} f(x)$ exists and is a finite number.

Therefore, from the first two parts, we can see that we will get the horizontal asymptote,

$$y = \frac{3}{2}$$

for $x \to -\infty$ and we have the horizontal asymptote,

$$y = -\frac{3}{2}$$

for $x \to \infty$.

---

9. For $f(x) = \dfrac{x+8}{\sqrt{2x^2+3}}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

To do this all we need to do is factor out the largest power of x that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

For the denominator we need to be a little careful. The power of x in the denominator needs to be outside of the root so it can cancel against the x's in the numerator. The largest power of x outside of the root that we can get (and leave something we can deal with in the root) will be just x. We get this by factoring an x^2 out of the root.

So, let's do the first couple of steps in this process to get us started.

$$\lim_{x\to-\infty}\frac{x+8}{\sqrt{2x^2+3}}=\lim_{x\to-\infty}\frac{x\left(1+\frac{8}{x}\right)}{\sqrt{x^2\left(2+\frac{3}{x^2}\right)}}=\lim_{x\to-\infty}\frac{x\left(1+\frac{8}{x}\right)}{\sqrt{x^2}\sqrt{2+\frac{3}{x^2}}}=\lim_{x\to-\infty}\frac{x\left(1+\frac{8}{x}\right)}{|x|\sqrt{2+\frac{3}{x^2}}}$$

Recall from the discussion in this section that,

$$\sqrt{x^2}=|x|$$

and we *do* need to be careful with that.

Now, because we are looking at the limit $x\to-\infty$ it is safe to assume that $x<0$. Therefore, from the definition of the absolute value we get,

$$|x|=-x$$

and the limit is then,

$$\lim_{x\to-\infty}\frac{x+8}{\sqrt{2x^2+3}}=\lim_{x\to-\infty}\frac{x\left(1+\frac{8}{x}\right)}{-x\sqrt{2+\frac{3}{x^2}}}=\lim_{x\to-\infty}\frac{1+\frac{8}{x}}{-\sqrt{2+\frac{3}{x^2}}}=\boxed{\frac{1}{-\sqrt{2}}}$$

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

For this part all of the mathematical manipulations we did in the first part up to dealing with the absolute value did not depend upon the limit itself and so don't really need to be redone here. So, up to that part we have,

$$\lim_{x\to\infty}\frac{x+8}{\sqrt{2x^2+3}}=\lim_{x\to\infty}\frac{x\left(1+\frac{8}{x}\right)}{|x|\sqrt{2+\frac{3}{x^2}}}$$

In this part we are looking at the limit $x\to\infty$ and so it will be safe to assume in this part that $x>0$. Therefore, from the definition of the absolute value we get,

$$|x|=x$$

and the limit is then,

$$\lim_{x\to\infty}\frac{x+8}{\sqrt{2x^2+3}}=\lim_{x\to\infty}\frac{x\left(1+\frac{8}{x}\right)}{x\sqrt{2+\frac{3}{x^2}}}=\lim_{x\to\infty}\frac{1+\frac{8}{x}}{\sqrt{2+\frac{3}{x^2}}}=\boxed{\frac{1}{\sqrt{2}}}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.
We know that there will be a horizontal asymptote for $x\to-\infty$ if $\lim\limits_{x\to-\infty} f(x)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x\to\infty$ if $\lim\limits_{x\to\infty} f(x)$ exists and is a finite number.

Therefore, from the first two parts, we can see that we will get the horizontal asymptote,

$$y=-\frac{1}{\sqrt{2}}$$

for $x\to-\infty$ and we have the horizontal asymptote,

$$y=\frac{1}{\sqrt{2}}$$

for $x\to\infty$.

---

10. For $f(x)=\dfrac{8+x-4x^2}{\sqrt{6+x^2+7x^4}}$ answer each of the following questions.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

(b) Evaluate $\lim\limits_{x\to\infty} f(x)$.

(c) Write down the equation(s) of any horizontal asymptotes for the function.

(a) Evaluate $\lim\limits_{x\to-\infty} f(x)$.

To do this all we need to do is factor out the largest power of x that is in the denominator from both the denominator *and* the numerator. Then all we need to do is use basic limit properties along with Fact 1 from this section to evaluate the limit.

For the denominator we need to be a little careful. The power of x in the denominator needs to be outside of the root so it can cancel against the x's in the numerator. The largest power of x outside of the root that we can get (and leave something we can deal with in the root) will be just x^2. We get this by factoring an x^4 out of the root.

So, let's do the first couple of steps in this process to get us started.

$$\lim_{x\to-\infty}\frac{8+x-4x^2}{\sqrt{6+x^2+7x^4}}=\lim_{x\to-\infty}\frac{x^2\left(\frac{8}{x^2}+\frac{1}{x}-4\right)}{\sqrt{x^4\left(\frac{6}{x^4}+\frac{1}{x^2}+7\right)}}$$

$$=\lim_{x\to-\infty}\frac{x^2\left(\frac{8}{x^2}+\frac{1}{x}-4\right)}{\sqrt{x^4}\sqrt{\frac{6}{x^4}+\frac{1}{x^2}+7}}=\lim_{x\to-\infty}\frac{x^2\left(\frac{8}{x^2}+\frac{1}{x}-4\right)}{\left|x^2\right|\sqrt{\frac{6}{x^4}+\frac{1}{x^2}+7}}$$

Recall from the discussion in this section that,

$$\sqrt{x^2}=|x|$$

So in this case we'll have,

$$\sqrt{x^4}=\left|x^2\right|=x^2$$

and note that we can get rid of the absolute value bars because we know that $x^2\geq 0$. So, let's finish the limit up.

$$\lim_{x\to-\infty}\frac{8+x-4x^2}{\sqrt{6+x^2+7x^4}}=\lim_{x\to-\infty}\frac{x^2\left(\frac{8}{x^2}+\frac{1}{x}-4\right)}{x^2\sqrt{\frac{6}{x^4}+\frac{1}{x^2}+7}}=\lim_{x\to-\infty}\frac{\frac{8}{x^2}+\frac{1}{x}-4}{\sqrt{\frac{6}{x^4}+\frac{1}{x^2}+7}}=\boxed{\frac{-4}{\sqrt{7}}}$$

(b) Evaluate $\lim\limits_{x\to\infty}f(x)$.

Unlike the previous two problems with roots in them all of the mathematical manipulations in this case did not depend upon the actual limit because we were factoring an x^2 out which will always be positive and so there will be no reason to redo all of that work.

Here is this limit (with most of the work excluded),

For this part all of the mathematical manipulations we did in the first part up to dealing with the absolute value did not depend upon the limit itself and so don't really need to be redone here. So, up to that part we have,

$$\lim_{x\to\infty}\frac{8+x-4x^2}{\sqrt{6+x^2+7x^4}}=\lim_{x\to\infty}\frac{x^2\left(\frac{8}{x^2}+\frac{1}{x}-4\right)}{x^2\sqrt{\frac{6}{x^4}+\frac{1}{x^2}+7}}=\lim_{x\to\infty}\frac{\frac{8}{x^2}+\frac{1}{x}-4}{\sqrt{\frac{6}{x^4}+\frac{1}{x^2}+7}}=\boxed{\frac{-4}{\sqrt{7}}}$$

(c) Write down the equation(s) of any horizontal asymptotes for the function.

We know that there will be a horizontal asymptote for $x \to -\infty$ if $\lim_{x \to -\infty} f(x)$ exists and is a finite number. Likewise we'll have a horizontal asymptote for $x \to \infty$ if $\lim_{x \to \infty} f(x)$ exists and is a finite number.

Therefore, from the first two parts, we can see that we will get the horizontal asymptote,

$$y = -\frac{4}{\sqrt{7}}$$

For both $x \to -\infty$ and $x \to \infty$.

