CALCULUS I

Practice Problems
Applications of Integrals

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Preface

Here are a set of practice problems for my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

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2. Go to the download page for the site http://tutorial.math.lamar.edu/download.aspx and select the section you'd like solutions for and a link will be provided there.

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Note that some sections will have more problems than others and some will have more or less of a variety of problems. Most sections should have a range of difficulty levels in the problems although this will vary from section to section.

Applications of Integrals

Introduction

Here are a set of practice problems for the Integrals chapter of my Calculus I notes. If you are viewing the pdf version of this document (as opposed to viewing it on the web) this document contains only the problems themselves and no solutions are included in this document. Solutions can be found in a number of places on the site.

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Here is a list of topics in this chapter that have practice problems written for them.

Average Function Value

For problems 1 & 2 determine f_{avg} for the function on the given interval.

1. $f(x) = 8x - 3 + 5\mathbf{e}^{2-x}$ on $[0,2]$

2. $f(x) = \cos(2x) - \sin\left(\frac{x}{2}\right)$ on $\left[-\frac{\pi}{2}, \pi\right]$

3. Find f_{avg} for $f(x) = 4x^2 - x + 5$ on $[-2,3]$ and determine the value(s) of c in $[-2,3]$ for which $f(c) = f_{\text{avg}}$.

Area Between Curves

1. Determine the area below $f(x) = 3 + 2x - x^2$ and above the x-axis.

2. Determine the area to the left of $g(y) = 3 - y^2$ and to the right of $x = -1$.

For problems 3 – 11 determine the area of the region bounded by the given set of curves.

3. $y = x^2 + 2$, $y = \sin(x)$, $x = -1$ and $x = 2$

4. $y = \dfrac{8}{x}$, $y = 2x$ and $x = 4$

5. $x = 3 + y^2$, $x = 2 - y^2$, $y = 1$ and $y = -2$

6. $x = y^2 - y - 6$ and $x = 2y + 4$

7. $y = x\sqrt{x^2 + 1}$, $y = \mathbf{e}^{-\frac{1}{2}x}$, $x = -3$ and the y-axis

8. $y = 4x + 3$, $y = 6 - x - 2x^2$, $x = -4$ and $x = 2$

9. $y = \dfrac{1}{x+2}$, $y = (x+2)^2$, $x = -\dfrac{3}{2}$, $x = 1$

10. $x = y^2 + 1$, $x = 5$, $y = -3$ and $y = 3$

11. $x = \mathbf{e}^{1+2y}$, $x = \mathbf{e}^{1-y}$, $y = -2$ and $y = 1$

Volumes of Solids of Revolution / Method of Rings

For problems 1 – 8 use the method of disks/rings to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $y = \sqrt{x}$, $y = 3$ and the y-axis about the y-axis.

2. Rotate the region bounded by $y = 7 - x^2$, $x = -2$, $x = 2$ and the x-axis about the x-axis.

3. Rotate the region bounded by $x = y^2 - 6y + 10$ and $x = 5$ about the y-axis.

4. Rotate the region bounded by $y = 2x^2$ and $y = x^3$ about the x-axis.

5. Rotate the region bounded by $y = 6\mathbf{e}^{-2x}$ and $y = 6 + 4x - 2x^2$ between $x = 0$ and $x = 1$ about the line $y = -2$.

6. Rotate the region bounded by $y = 10 - 6x + x^2$, $y = -10 + 6x - x^2$, $x = 1$ and $x = 5$ about the line $y = 8$.

7. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $x = 24$.

8. Rotate the region bounded by $y = 2x + 1$, $x = 4$ and $y = 3$ about the line $x = -4$.

Volumes of Solids of Revolution / Method of Cylinders

For problems 1 – 8 use the method of cylinders to determine the volume of the solid obtained by rotating the region bounded by the given curves about the given axis.

1. Rotate the region bounded by $x = \left(y - 2\right)^2$, the x-axis and the y-axis about the x-axis.

2. Rotate the region bounded by $y = \dfrac{1}{x}$, $x = \dfrac{1}{2}$, $x = 4$ and the x-axis about the y-axis.

3. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the y-axis. For this problem assume that $x \ge 0$.

4. Rotate the region bounded by $y = 4x$ and $y = x^3$ about the x-axis. For this problem assume that $x \ge 0$.

5. Rotate the region bounded by $y = 2x + 1$, $y = 3$ and $x = 4$ about the line $y = 10$.

6. Rotate the region bounded by $x = y^2 - 4$ and $x = 6 - 3y$ about the line $y = -8$.

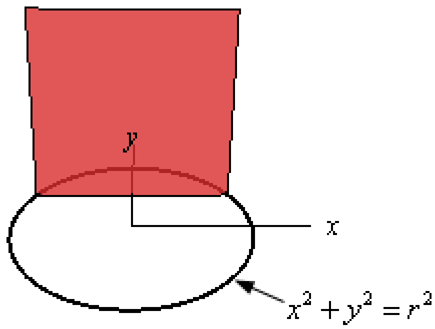
7. Rotate the region bounded by $y = x^2 - 6x + 9$ and $y = -x^2 + 6x - 1$ about the line $x = 8$.

8. Rotate the region bounded by $y = \dfrac{\mathbf{e}^{\frac{1}{2}x}}{x+2}$, $y = 5 - \frac{1}{4}x$, $x = -1$ and $x = 6$ about the line $x = -2$.

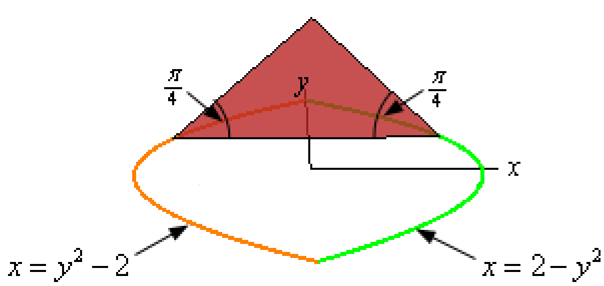
More Volume Problems

1. Find the volume of a pyramid of height h whose base is an equilateral triangle of length L.

2. Find the volume of the solid whose base is a disk of radius r and whose cross-sections are squares. See figure below to see a sketch of the cross-sections.

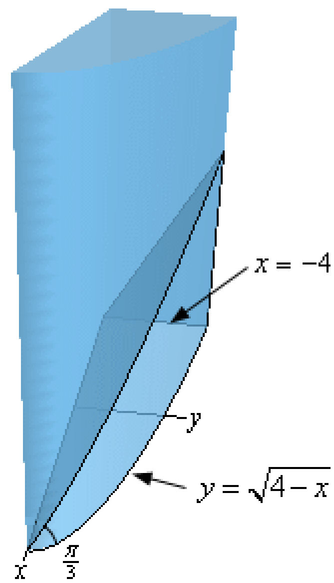


3. Find the volume of the solid whose base is the region bounded by $x = 2 - y^2$ and $x = y^2 - 2$ and whose cross-sections are isosceles triangles with the base perpendicular to the y-axis and the angle between the base and the two sides of equal length is $\frac{\pi}{4}$. See figure below to see a sketch of the cross-sections.

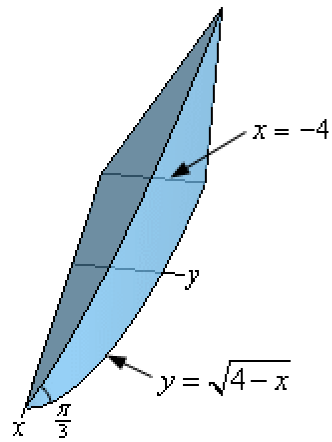


4. Find the volume of a wedge cut out of a "cylinder" whose base is the region bounded by $y = \sqrt{4-x}$, $x = -4$ and the x-axis. The angle between the top and bottom of the wedge is $\frac{\pi}{3}$.

See the figure below for a sketch of the "cylinder" and the wedge (the positive *x*-axis and positive *y*-axis are shown in the sketch – they are just in a different orientation).



"Cylinder"



Wedge

Work

1. A force of $F(x) = x^2 - \cos(3x) + 2$, x is in meters, acts on an object. What is the work required to move the object from $x = 3$ to $x = 7$?

2. A spring has a natural length of 18 inches and a force of 20 lbs is required to stretch and hold the spring to a length of 24 inches. What is the work required to stretch the spring from a length of 21 inches to a length of 26 inches?

3. A cable that weighs ½ kg/meter is lifting a load of 150 kg that is initially at the bottom of a 50 meter shaft. How much work is required to lift the load ¼ of the way up the shaft?

4. A tank of water is 15 feet long and has a cross section in the shape of an equilateral triangle with sides 2 feet long (point of the triangle points directly down). The tank is filled with water to a depth of 9 inches. Determine the amount of work needed to pump all of the water to the top of the tank. Assume that the density of water is 62 lb/ft^3.